



Investigations

in Number, Data, and Space®

Unit Guide for Kindergarten, Unit 1:
Who Is In School Today?
Classroom Routines and Materials



Unit Guide for Kindergarten, Unit 1

Who Is In School Today?

Classroom Routines and Materials

Unit Summary:

The processes, structures, and materials that are important features of the Kindergarten math curriculum are introduced in this unit. It also introduces routines common to many Kindergarten classrooms that students will encounter regularly throughout the year. These routines include taking attendance, using the calendar to count and to keep track of time and events, counting sets of objects, and collecting and discussing data about the class. These Classroom Routines offer reinforcement of number concepts that are central to the Kindergarten curriculum. Full write-ups of all Classroom Routines can be found in *Implementing Investigations in Kindergarten* pp. 22-31.

Materials:

Who Is In School Today? (1 copy per person)
Implementing Investigations in Kindergarten (optional)
Geoblocks (one bin per 4 people)
Pattern blocks (one bin per 4 people)
Cubes (one bin per 4 people)
Attribute Blocks (one set per 4 people)
Counting Jar (1 per 4 people; see Materials to Prepare, p. 55)
Scrap paper (1 per person)
Buttons (1 handful per pair)

Do the following activities from *Who is In School Today?*:

1. Identify the mathematics in the unit

To get an overview of the mathematics students will be doing in this unit, refer to these sections in the unit front matter. As you look at these sections, begin thinking about the main mathematical ideas students work on in this unit.

- Turn to pp. 8-9, *Overview of This Unit*. Look at the title of each Investigation and read the summary for each Investigation.
- Review the *Mathematics in This Unit* essay, pp. 10-13. Look at the Mathematical Emphases and Math Focus Points. (The emphases are numbered, and can be found above bulleted lists of Math Focus Points.)
- Read *Assessing the Benchmarks*, p. 15.

Discuss

- What mathematical ideas and skills are students working on in this unit?
- What mathematics are students expected to know at the beginning of the unit? At the end?

2. Exploring Materials (Session 1.1)

Throughout this unit students explore the math materials they will be using throughout the year.

- Read the Activity, *Introducing Math Workshop and Materials* pp. 26-27. Choose one material to explore: Geoblocks, pattern blocks, cubes or attribute blocks. Attribute blocks can be divided into 2 sets, thick and thin, 2 people per set. Consider the following questions as you explore the material you chose and then discuss these questions after everyone has time to explore one or two materials.

Discuss

- What attributes of the material do you notice?
- What relationships do you notice among the objects?
- What do you think students might do with this material?
- What might they notice or learn from exploring this material?

- Look at the Teaching Notes p. 27, “Why Free Exploration?” and “Managing Math Workshop”. Read the Teacher Note, *Supporting Students’ Free Play*, pp. 132-133. (For more ideas about setting up a mathematical community and classroom see the *Setting Up the Investigations Classroom* essay, in *Implementing Investigations in Kindergarten*, pp. 9-14.)

Discuss

- How can you best support student learning as they explore materials?
- How can students’ exploration of these materials help them in their math work later in the year?
- What can you do to help students become familiar with expectations for how they should work during math time?

3. Attendance (Sessions 1.1, 1.2, 1.6, 2.2, and 2.5)

Three Classroom Routines are introduced in this unit, however, only one of them, *Attendance*, is explored in this unit guide. In the *Attendance* routine students establish the number of students present each day by counting the students and using information about who is not present that day. Different parts of the *Attendance* routine are introduced in different sessions in this unit. As for all Classroom Routines, this activity is then continued throughout the year.

- Read the Activity, *Introducing Attendance*, pp. 25-26 in Session 1.1. Take attendance with your group as described in this activity. Read the Activity, *Counting Around the Circle*, p. 33 in Session 1.2 and count around the circle as described in the activity. Read the Activity, *Introducing the Attendance Stick*, pp. 48-49 of Session 1.6 and make an attendance stick for your group as described in this activity.

- Read the Activities, *Using the Attendance Stick*, pp. 65-66 in Session 2.2 and *Attendance: Introducing a Labeled Attendance Stick*, pp. 83-86 in Session 2.5.

Discuss

- How do all the components of this routine work together?
- How will you make this part of your daily routine in your classroom?

- Read the Teaching Notes, p. 25 “The Attendance Routine”, “How Many Are We?” and the Math Note, p. 25 “Learning to Count”; the Teaching Note, “Counting Off”, p. 33, and the Teaching Note, “Attendance Over Time”, p. 85.

Discuss

- What aspects of counting are students working on in this routine?
- What other math ideas are they working on?

4. Counting Jar (Sessions 2.1 and 3.2)

The Counting Jar is an activity that is introduced in this unit and continued throughout the year. The amount of objects, type of objects and composition of the set of objects placed in the jar is varied throughout the year.

- Read the Activity, *Introducing the Counting Jar*, pp. 59-60 in Session 2.1 and the Activity, *Counting Jar: Introducing Recording*, pp. 101-102 in Session 3.2. Do all the components of the Counting Jar activity in pairs or individually (depending on how many Counting Jars you have). (You might also combine this activity with the activity that follows, *Button Match-Up*, so that logistically it is easier for everyone to have a chance to do the Counting Jar. Alternatively, you can have just 3 or 4 people do the Counting Jar as an example for everyone else, then continue with the discussion.)
- Read the Teaching Notes, “Counting Jar”, “The Math of the Counting Jar”, “Handling Mistakes”, p. 59, and “Creating an Equivalent Set”, p. 60.
- Read the Dialogue Box, *You Could Use Dots*, pp. 142-143.

Discuss

- How would you expect students will show on paper how many items are in the Counting Jar?
- What is the teacher trying to highlight about recording the number of balls in the Counting Jar in this Dialogue Box?
- How can each of the components of this activity - counting the objects in the jar, creating an equivalent set of the number of objects in the jar and recording the amount in the jar - help students develop an understanding of counting and help them develop their counting skills?

- Look together at the Resource Master M5, *Assessment Checklist: Counting*, pictured in the sidebar, p. 60 and the example of a filled in *Assessment Checklist: Counting*, p. 61.

Discuss

- What might you observe and record on this checklist the first time students do the Counting Jar?
- What would you expect to observe by the end of this unit? What would you expect to observe later in the year, after students have done the Counting Jar multiple times as well as other counting activities?

5. Button Match-Up (Session 2.3)

In this activity, *Button Match-Up*, along with *Attribute Block Match-Up*, *Sorting People* and *Sorting Attribute Blocks*, students begin some work on sorting objects by their attributes. This work is continued in the pattern and data units.

- Read the Activity, *Introducing Button Match-Up*, pp. 71-72. Play *Button Match-Up* in pairs or fours.
- Read the Teacher Note, *Sorting and Identifying Attributes*, p. 136.

Discuss

- How can work with sorting and identifying attributes be useful to students in their mathematical learning?
- What challenges do you foresee Kindergarten students having in identifying attributes and sorting by attributes?

6. Wrap Up

- Look back at the unit overview, pp. 8-9.

Discuss

- How do the activities done during this unit study fit into the rest of the mathematics of the unit?

Other Key Features of *Who is Here Today?*

- Classroom Routines in This Unit, p. 16
 - Attendance*
 - Calendar*
 - Today's Question*
- Assessment
 - Counting (Introduced in Session 2.1)
 - Resource Master M5, Assessment Checklist
 - Teacher Note, *Observing Kindergarteners as They Count*, p. 135



Investigations

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Unit Guide for Grade 1, Unit 1:
How Many of Each?
Addition, Subtraction, and the Number System 1



Unit Guide for Grade 1, Unit 1

How Many of Each?

Addition, Subtraction, and the Number System 1

Unit Summary:

Students count, order, compare quantities, and work with the operation of addition. Students also work on finding addition combinations up to 10 and learn to make sense of and solve story problems. They are introduced to several of the classroom routines that practice and reinforce work with counting, developing visual images of number, collecting data, and working with concepts of time (calendar/clock).

Materials:

How Many of Each? (1 copy per person)

Student Activity Book p. 14, Ordering Numbers (1 per person)

Primary Number Cards (1 deck per pair, use manufactured set or Resource Masters M13-M16, see Materials to Prepare, p. 51)

Connecting cubes (about 40 per pair)

Student Activity Book p. 21, How Many Pencils? (1 per person)

Student Activity Book p. 33, Seven Peas and Carrots (1 per person)

Cubes or counters in two colors (7 of each color per person)

Paper plates (1 per person)

Crayons or markers in the same colors as cubes or counters (1 of each color per pair)

Do the following activities from *How Many of Each?*:

1. Identify the mathematics in the unit

To get an overview of the mathematics students will be doing in this unit, refer to these sections in the unit front matter. As you look at these sections, begin thinking about the main mathematical ideas students work on in this unit.

- Turn to pp. 8-9, *Overview of This Unit*. Look at the title of each Investigation and read the summary for each Investigation.
- Review the *Mathematics in This Unit* essay, pp. 10-13. Look at the Mathematical Emphases and Math Focus Points. (The emphases are numbered, and can be found above bulleted lists of Math Focus Points.)
- Read the “Benchmarks in This Unit” in the table on p. 15, *Assessing the Benchmarks*.

Discuss

- What mathematical ideas and skills are students working on in this unit?
- What mathematics are students expected to know at the beginning of the unit? At the end?

2. Introducing Start With/Get To (Session 2.1)

Throughout Investigation 1 of this unit, students are introduced to the yearlong *Morning Meeting Classroom Routine*. During this session, students are introduced to another yearlong routine, *Start With/Get To*, which uses the number line as a mathematical tool for counting.

- Read the Activity, *Introducing Start With/Get To*, pp. 55-56.
- Read the Dialogue Box, *Start With/Get To*, p. 225.

Discuss

- As students do the *Start With/Get To* activity, what information can you attain about their understanding of the counting sequence, number names, written numbers, and/or the distance between numbers?
- How might this activity benefit students as they work on other math activities?

3. Ordering Numbers (Session 2.7)

Students order four numbers or quantities and discuss the notation that mathematicians use to compare numbers.

- Read the Activities, *Introducing Ordering Numbers*, pp. 89-90 and *Ordering Numbers*, pp. 90-91. Complete Student Activity Book p. 14, Ordering Numbers.

Discuss

- As you observe students working on this activity, what information about their understanding of numbers and sequencing can you collect?

- Read the Discussion, *Comparing and Ordering Numbers*, pp. 91-93, and the Teacher Note, *Introducing Notation, Part 1* pp. 198-199.

Discuss

- Why is it important to match your recording to what the students say?
- What is the purpose of modeling standard notation in Grade 1?

4. Solving an Addition Story Problem and Sharing Strategies (Session 3.3)

During this Investigation, students have been playing several games about combining two quantities. For this session, students are introduced to, and solve, addition story problems.

- Read the Activity, *Solving an Addition Story Problem*, pp. 113-116 and the Discussion, *Sharing Strategies*, pp. 116-118. As you read the text, discuss the importance of connecting the context of the story to understanding the operation of addition.

Discuss

- Examine the sample student work on pp. 115-116. How are these students making sense of this addition story problem?

- Read the Teacher Note, *Recording Strategies for a Combining Problem*, pp. 204-205.

Discuss

- How does the teacher help each student to clearly describe their strategy and a way for the student to record their work?

5. Seven Peas and Carrots (Session 4.1)

In this session, students are introduced to a new type of problem in which they determine one or more combinations they could have to make 7 in all. This is the first of several *How Many of Each?* problems students will solve over the course of the year.

- Complete Student Activity Book p. 33, Seven Peas and Carrots. Discuss with a partner how you know you have found all the combinations.
- Read the Activities, *Introducing Seven Peas and Carrots*, p. 149, and *Solving Seven Peas and Carrots*, pp. 150-151.
- Read the Teacher Note, *About How Many of Each? Problems*, pp. 211-212.

Discuss

- What math ideas are students working on as they solve *How Many of Each?* problems?
- How do these problems help meet a range of learners within a classroom?

- Read the Teacher Note, *When the Teacher Records Students' Solutions* p. 215.

Discuss

- During class discussions, why is it important to vary the recording method and to record solutions in the order that students give them?

6. Wrap Up

- Look back at the unit overview, pp. 8-9.

Discuss

- How do the activities done during this unit study fit into the mathematics of the rest of the unit?

Other Key Features of *How Many of Each?*

- Algebra Connections in This Unit, pp. 16-19
- Classroom Routines in This Unit, p. 20
 - Morning Meeting*
 - Start With/Get To*
 - Quick Images*
- Assessment
 - Assessment: Counting 20 (Sessions 1.2-1.4)
Resource Master M3, Assessment Checklist
 - Assessment: Counting 20 (Session 2.6)
Teacher Note, pp. 194-197
 - Assessment: Double Compare (Session 3.6)
Teacher Note, pp. 208-210
 - End-of-Unit Assessment (Session 4.7)
Teacher Note, pp. 216-223



Investigations

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Unit Guide for Grade 2, Unit 1:
Counting, Coins, and Combinations
Addition, Subtraction, and the Number System 1



Unit Guide for Grade 2, Unit 1

Counting, Coins, and Combinations

Addition, Subtraction, and the Number System 1

Unit Summary:

Students count and compare quantities, compose and decompose numbers, and deepen understandings about the operations of addition and subtraction. They develop strategies for comparing, combining, and doubling quantities, as well as taking one quantity away. Students achieve fluency with three sets of addition combinations (10s, + 1, + 2). During this first unit of the year, students are introduced to several year-long classroom routines that offer regular practice with composing and decomposing numbers; developing visual images of quantities; counting, collecting, and analyzing data; and telling time.

Materials:

Counting, Coins, and Combinations (1 copy per person)

Class number line and 2 clothespins or clips (See Materials to Prepare, p. 25)

Do the following activities from *Counting, Coins, and Combinations*:

1. Identify the mathematics in the unit

To get an overview of the mathematics students will be doing in this unit, refer to these sections in the unit front matter. As you look at these sections, begin thinking about the main mathematical ideas students work on in this unit.

- Turn to pp. 8-9, *Overview of This Unit*. Look at the title of each Investigation and read the summary for each Investigation.
- Review the *Mathematics in This Unit* essay, pp. 10-13. Look at the Mathematical Emphases and Math Focus Points. (The emphases are numbered, and can be found above bulleted lists of Math Focus Points.)
- Read the “Benchmarks in This Unit” in the table on p. 15, *Assessing the Benchmarks*.

Discuss

- What mathematical ideas and skills are students working on in this unit?
- What mathematics are students expected to know at the beginning of the unit? At the end?

2. Guess My Number on the Number Line (Session 1.3)

In Investigation 1, students are introduced to math tools they will be using, and two of the Classroom Routines they will be revisiting, throughout the year. This session focuses on the number line, which students use to play *Guess My Number*.

- Read the Activity, *Guess My Number on the Number Line*, pp. 42-44, including the Teaching Notes, “Using the Number Line” and “Mathematical Symbols” p. 42, and “Guessing the Number,” p. 43.
- Play several rounds of *Guess My Number* with a partner/small group.

Discuss

- What strategies and questions did you use as you played?
- What strategies and questions would you expect of your students?
- How do you anticipate student strategies will change as they become more familiar with the game? How might you adapt this game as the year progresses?

- Read the Teacher Note, *The Number Line and the 100 Chart: Two Models of Our Number System*, pp. 184-185.

Discuss

- How is a number line similar to, and different from, a 100 chart?
- How comfortable are your students with the number line and 100 chart? Do they prefer one to the other? Choose the tool based on the situation or problem? What issues typically arise with each?

3. Enough for the Class? (Session 2.1)

In addition to introducing another Classroom Routine, Investigation 2 focuses on counting and coins. In this session, students solve and discuss the first of several *Enough for the Class?* problems, which involve finding the difference between two quantities.

- Read the Activity, *Enough for the Class*, pp. 65-66. (See Materials to Prepare, p. 59 for a description of Counting Bag A.)
- Read the Dialogue Box, *Are There Enough for the Class?*, pp. 210-211.

Discuss

- Discuss the range of strategies discussed in the Dialogue Box. What tools or representations did students use to solve the problem?
- How did the teacher in the Dialogue Box structure the discussion? What mathematical ideas did (s) he choose to highlight? What tools and representations/?

4. Addition Combinations (Session 3.5)

In Investigation 3, the final *Classroom Routine* is introduced, and students focus in particular on combinations that make 10. In this session, students are introduced to how they will work on the addition combinations this year.

- Read the Discussion, *Addition Combinations*, p. 130; and the activities *Introducing the Addition Cards*, p. 131 and *Which Combinations Do I Know?*, pp. 131-132. Read the Teaching Note “Addition Combinations” p. 132, and the Algebra Note “Remembering the Combinations”, p. 132.

Discuss

- How does this process for learning and practicing “the facts” compare to your experience when you were a student?

- Read the Teacher Note, *Strategies for Learning the Addition Combinations*, pp. 191-192.

Discuss

- What is the benefit to learning the addition combinations in groups?
- As you work with families, what key ideas about the addition combinations would you hope to convey?

5. An Addition Story Problem (Session 4.1)

Investigation 4 focuses on solving addition and subtraction story problems and doubling. In this session, students solve a problem about combining two groups, record their work, and share their strategies.

- Read the first part of the Activity, *An Addition Story Problem*, p. 139 (up to the red line).

Discuss

- How does asking students to imagine what is happening in the problem, retell the story the story in their own words, and think about whether the answer will be more or less than the starting amount, support students in solving story problems? In understanding addition and subtraction?
- How would you expect your students to solve this problem? To show their work?

- Read the rest of the Activity, *Ongoing Assessment: Observing Students at Work and Differentiation: Supporting the Range of Learners*, pp. 139-140.

- Read the Dialogue Box, *An Addition Story Problem*, pp. 218-219.

Discuss

- What strategies are students using to solve the story problem? To show their work?
- How does the teacher in the Dialogue Box help students differentiate between the tools (e.g., the number line, cubes, the 100 chart) and the strategy (e.g., counted all, counting on) used to solve a problem? Why is this differentiation important?
- How does (s) he encourage students to make complete explanations?

6. Wrap Up

- Look back at the unit overview, pp. 8-9.

Discuss

- How do the activities done during this unit study fit into the mathematics of the rest of the unit?

Other Key Features of *Counting, Coins, and Combinations*

- Algebra Connections in This Unit, pp. 16-19
- Classroom Routines in This Unit, p. 20
 - What Time Is It?*
 - Today's Number*
 - Quick Images*
- Assessment
 - Counting Pennies (Introduced in Session 2.2)
 - Resource Master M12, Assessment Checklist
 - Enough for the Class? (Session 2.8)
 - Teacher Note, pp. 193-194
 - How Many Cans? (Session 4.8)
 - Teacher Note, pp. 199-201
 - End-of-Unit Assessment (Session 4.9)
 - Teacher Note, pp. 202-208



Investigations

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Unit Guide for Grade 3, Unit 1:
Trading Stickers, Combining Coins
Addition, Subtraction, and the Number System 1



Unit Guide for Grade 3, Unit 1

Trading Stickers, Combining Coins

Addition, Subtraction, and the Number System 1

Unit Summary:

Students solve addition problems with two and small three-digit numbers, solve subtraction problems involving two-digit numbers, find combinations of numbers that add to 100, and work with coins and coin values. Their understanding of place value develops as they add and subtract 10s to and from three-digit numbers, break three-digit numbers into hundreds, tens, and ones in different ways, and solve story problems involving hundreds, tens, and ones.

Materials:

Trading Stickers, Combining Coins (1 copy per person)

Resource Master M4, 100 Chart (1 per person)

Student Activity Book pp. 15-16, How Many More Stickers to Get 100? (1 per person)

Resource Master M12, 100 Grids (1 per person)

Resource Master M13, Capture 5 (1 per pair)

Resource Masters M14-M15, Change Cards (1 deck per pair, see Materials to Prepare, p. 25)

Resource Master M16, Capture 5 Recording Sheet

Resource Master M30, *Close to 100* (1 per pair)

Student Activity Book p. 39, Close to 100 Score Sheet (1 per pair)

Digit Cards (1 deck per pair) Use manufactured decks or Resource Masters M33-M35, see Materials to Prepare, p. 97

Student Activity Book p. 49, Story Problems 4 (1 per person)

Do the following activities from *Trading Stickers, Combining Coins*:

1. Identify the mathematics in the unit

To get an overview of the mathematics students will be doing in this unit, refer to these sections in the unit front matter. As you look at these sections, begin thinking about the main mathematical ideas students work on in this unit.

- Turn to pp. 8-9, *Overview of This Unit*. Look at the title of each Investigation and read the summary for each Investigation.
- Review the *Mathematics in This Unit* essay, pp. 10-13. Look at the Mathematical Emphases and Math Focus Points. (The emphases are numbered, and can be found above bulleted lists of Math Focus Points.)

- Read the “Benchmarks in This Unit” in the table on p. 15, *Assessing the Benchmarks*.

Discuss

- What mathematical ideas and skills are students working on in this unit?
- What mathematics are students expected to know at the beginning of the unit? At the end?

2. Introducing How Many More Stickers to Get 100? (Session 1.4)
How Many More Stickers to Get 100? (Session 1.4)

(Note: In Grade 2, a context used for representing the place-value of 2- and 3-digit numbers is stickers sold at a “Sticker Station” in sheets of 100, strips of 10s and singles. This context is also used in Grade 3.)

In this session, students consider a context in which stickers are stored in books with 100 to a page. Students solve missing addend problems in which they determine how many more stickers are needed to make 100. They use 100 grids and number lines as tools for solving the problems and representing their strategies.

- Read the Activity, *Introducing How Many More Stickers to Get 100?*, pp. 53-55. Solve the problems on Student Activity Book pp. 15-16, *How Many More Stickers to Get 100?*

Discuss

- How does the context of Sticker Station help students understand the structure of our base-ten number system, particularly 100?
- What strategies do you expect to see your students using to solve these problems?

- Read the Discussion, *Representing Strategies on the Number Line*, pp. 56-59, including the Algebra Note, *The Relationship Between Addition and Subtraction*, p. 57, and the Math Note, *Representing Subtraction on the Number Line*, p. 58.

- Use an unmarked number line to represent the strategies you used to solve the first two problems on Student Activity Book p. 15.

Discuss

- Compare how people in your group used the number line to represent their solution. What was the same and different?
- How does using the unmarked number line in these problems highlight the relationship between addition and subtraction?
- How can the use of the number line as a representation support students’ understanding of subtraction as the difference between two numbers?

3. Capture 5 (Session 1.5)

Capture 5 is a game that involves students in adding and subtracting multiples of 10 and 1 to and from numbers up to 100.

- Play the game *Capture 5*, following the directions on Resource Master M13, including recording your equations on Resource Master, M16, Capture 5 Recording Sheet.

Discuss

- What strategies did you use as you played *Capture 5*?
- How can you support students in developing more efficient strategies as they play this game, for example moving from counting by 1s to making moves in larger chunks? (Refer to the *Ongoing Assessment: Observing Students at Work* and *Differentiation: Supporting the Range of Learners*, pp. 63-64.)

- Read the Discussion, *Strategies for Capture 5*, on pp. 72-74 in Session 1.7, and the Dialogue Box, *Strategies for Capture 5*, p. 171.

Discuss

- How does *Capture 5* highlight the relationship between addition and subtraction?

4. Close to 100 (Session 2.2)

Close to 100 is a game that involves using knowledge of place value and known combinations with sums of 100 ($20 + 80$, $25 + 75$, $50 + 50$, etc.) to find pairs of 2-digit numbers that add to 100 or close to 100.

- Play the game *Close to 100*, following the directions on Resource Master M30. Keep score on Student Activity Book p. 39, Close to 100 Recording Sheet.

Discuss

- What strategies did you use as you played *Close to 100*? For example, did you use familiar combinations (e.g., $50 + 50$ or $75 + 25$) to help you? If so, explain how.
- How did your understanding of place value help you choose two numbers with a sum of 100 or close to 100?

- Read the *Differentiation: Supporting the Range of Learners*, p. 108, and the Discussion, *Strategies for Close to 100*, pp. 113-115, in Session 2.3.

Discuss

- How can you support students who are making combinations randomly?
- How can you scaffold the activity for them to help them consider combinations they know that make 100 or to encourage them to consider the sum of the tens digits when picking two numbers?

5. Story Problems (Session 2.5)
Story Problem Strategies (Session 2.6)

Students revisit and extend the work they have done in this unit solving story problems. In this session, they solve problems that involve sums over 100.

- Solve Problems 1 and 3 on Student Activity Book p. 49, Story Problems 4. Read the Activity *Story Problems*, pp. 123-124.

Discuss

- What strategies did you use to solve these two problems?
- What will you look for as your students do this work?
- What tools or representations might you encourage students to use to help them make sense of these problems and/or become more efficient in solving them?

- Read the Discussion, *Story Problem Strategies*, pp. 126-130, in Session 2.6, and the Teacher Note, *Addition Strategies*, pp. 149-151.

Discuss

- As the students in this discussion share their strategies, the teacher refers to the problem context as she questions them. How does this help the students keep track of the parts of the problem that have been solved and the parts that remain to be solved?
- How does the use of cubes help the students visualize the strategy of adding by place?
- How does the unmarked number line help them visualize and compare Gil and Kim's strategies of adding one number in parts?

6. Wrap Up

- Look back at the unit overview, pp. 8-9.

Discuss

- How do the activities done during this unit study fit into the overall mathematical storyline of the unit?

Other Key Features of *Trading Stickers, Combining Coins*

- Algebra Connections in this Unit, pp. 16-17
- Classroom Routine and Ten-Minute Math in This Unit, p. 18
 - What's the Temperature?*
 - Practicing Place Value*
 - More or Less?*
- Assessment
 - Adding and Subtracting 10s (Sessions 1.6-1.7)
 - Resource Master, M3 Assessment Checklist
 - Hundreds, Tens, and Ones (Session 1.9)
 - Teacher Note, pp. 153-158
 - Addition Combinations (Session 2.5)
 - Teacher Note, pp. 161-163
 - End-of-Unit Assessment (Session 2.8)
 - Teacher Note, pp. 164-169



Investigations

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Unit Guide for Grade 4, Unit 1:
Factors, Multiples, and Arrays
Multiplication and Division 1



Unit Guide for Grade 4 Unit 1

Factors, Multiples, and Arrays

Multiplication and Division 1

Unit Summary:

Students deepen their understanding of the operation of multiplication. Students use rectangular arrays to represent the relationship between factors and multiples, use what they know to solve problems that increase in size, and focus on solving problems efficiently. They continue to develop fluency with multiplication combinations (facts up to 12×12).

Materials:

Factors, Multiples and Arrays (1 per person)

Resource Master M2, One Centimeter Grid Paper (2-3 per person)

12 x 18 construction paper (1 per person)

Student Activity Book p. 19, Multiple Turn Over Recording Sheet (1 per person)

Resource Master M45, Multiple Turn Over (1 per pair)

Multiple Cards (1 set per pair, use manufactured decks or Resource Masters M46-M49, see materials to Prepare, p. 55)

Calculators (optional)

Student Activity Book pp. 39-40, Factors of 16 and 48 (1 per pair)

Do the following activities from *Factors, Multiples, and Arrays*:

1. Identify the mathematics in the unit

To get an overview of the mathematics students will be doing in this unit, refer to these sections in the unit front matter. As you look at these sections, begin thinking about the main mathematical ideas students work on in this unit.

- Turn to pp. 8-9, *Overview of This Unit*, pp. 8-9. Look at the title of each Investigation and read the summary for each Investigation.
- Review the *Mathematics in This Unit* essay, pp. 10-13. Look at the Mathematical Emphases and Math Focus Points. (The emphases are numbered, and can be found above bulleted lists of Math Focus Points.)
- Read the “Benchmarks in This Unit” in the table on p. 15, *Assessing the Benchmarks*.

Discuss

- What mathematical ideas and skills are students working on in this unit?
- What mathematics are students expected to know at the beginning of the unit? At the end?

2. Making Arrays (Session 1.2)

In this Investigation, students use arrays to represent multiplication situations. In this session, they use what they know about multiplication to find all the arrays for given numbers.

- Read the Activity, *Introducing Making Arrays*, pp. 33-34, to understand how students are introduced to arrays in Grade 4. Read the Activity, *Making Arrays*, pp. 34-37, and choose one set of numbers to work on. (Make sure that each person in your group chooses a different pair of numbers from the list on p. 34.)
- Read the Teaching Note, “Factors of a Number and Its Multiples,” p. 34.

Discuss

- What are the smallest and largest factors you found for your number? Could there be anything smaller or larger?
- Were there numbers that you knew wouldn’t work? Why?
- How do you know you have all the arrays for a number?
- How does the array model support students in learning multiplication combinations?

3. Playing Multiple Turn Over (Session 2.3)

Students have been working on their multiplication facts and identifying two factors that multiply together to make a multiple (for example, $8 \times 6 = 48$; 48 is a multiple of both 6 and 8). In this session, students are introduced to the game *Multiple Turn Over* to work on the relationship between factors and multiples and to continue to practice multiplication facts up to 12×12 .

- Read the Activities, *Introducing Multiple Turn Over*, pp. 72-73, *Playing Multiple Turn Over*, pp. 73-75. Play 3 or 4 rounds of the game with a partner.

Discuss

- What knowledge of multiples are you using? For example, do you know that even numbers are multiples of 2, or that multiples of 5 end in 0 or 5?
- How did you determine the factors of each multiple? Did you “just know” them, and/or use reasoning?
- What strategies might students use in choosing a multiple on the board to cover?

- Read the Dialogue Box, *Identifying Factors and Multiples in Multiple Turn Over*, p. 133.

Discuss

- What knowledge of factors and multiples are these students using as they explain their strategies?

4. Factors of 16 and 48 (Session 3.3)

In this Investigation students use what they know about factors of one number to find factors of other numbers. In this session, they explore the concept that the factors of a number are also factors of a multiple of that number.

- Read the Activity, *Factors of 16 and 48*, pp. 106-108. Work with a partner to complete Student Activity Book pp. 39-40, Factors of 16 and 48, using any representations that might be useful.

Discuss

- How did you explain why the factors of 16 are also the factors of 48?
- How are the factors of 16 and 48 related? That is, how many 3s fit in to 16? How many 3s fit in to 48? Why is that?
- How might students use arrays to demonstrate the relationship between the factors of 16 and 48?

- Read the section *Finding Factors*, p. 17, *Algebra Connections in This Unit*.
- Read the Discussion: *Are the Factors of 16 also Factors of 48?*, pp. 108-110, to see examples of student's explanations.

Discuss

- How would you describe the relationship between the factors of 16 and 48?
- Is this relationship true with any other factors? How do you know?
- How does this activity support students in extending their knowledge of factors?

5. Wrap Up

- Look back at the unit overview, pp. 8-9.

Discuss

- How do the activities done during this unit study fit into the overall mathematical storyline of the unit?

Other Key Features of *Factors, Multiples and Arrays*

- Algebra Connections in This Unit, pp. 16-17
- Ten-Minute Math in This Unit, p. 18
 - Today's Number*
 - Quick Images*
 - Counting Around the Class*
- Assessment
 - Representing 8×6 (Session 1.5)
 - Resource Master M32, Assessment Checklist
 - Multiplication Combinations (Session 2.5)
 - Teacher Note, pp. 120-122
 - End-of-Unit Assessment (Session 3.4)
 - Teacher Note, pp. 123-127



Investigations

in Number, Data, and Space®

Unit Guide for Grade 5, Unit 1:
Number Puzzles and Multiple Towers
Multiplication and Division 1



Unit Guide for Grade 5, Unit 1

Number Puzzles and Multiple Towers

Multiplication and Division 1

Unit Summary:

Students refine and gain fluency in solving two-digit by two-digit multiplication problems, develop strategies for division problems with one- and two-digit divisors, and increase their knowledge of multiplication relationships by learning about prime factorization (e.g., $36 = 4 \times 9 = (2 \times 2) \times 9 = 2 \times 2 \times 3 \times 3$).

Materials:

Number Puzzles and Multiples Towers (1 copy per person)
Resource Master M15, Centimeter Grid Paper (2-3 per person)
Student Activity Book pp. 1-2, Number Puzzles: 1 Clue. (1 per person)
Color tiles (1 set available)
12" x 18" Construction paper (1 per person)
Student Activity Book p. 35, Multiplication Cluster Problems (1 per person)
Student Activity Book pp. 49-50, Problems about Multiples of 21 (1 per person)

Do the following activities from *Number Puzzles and Multiple Towers*:

1. Identify the mathematics in the unit

To get an overview of the mathematics students will be doing in this unit, refer to these sections in the unit front matter. As you look at these sections, begin thinking about the main mathematical ideas students work on in this unit.

- Turn to pp. 8-9, *Overview of This Unit*. Look at the title of each Investigation and read the summary for each Investigation.
- Review the *Mathematics in This Unit* essay, pp. 10-13. Look at the Mathematical Emphases and Math Focus Points. (The emphases are numbered, and can be found above bulleted lists of Math Focus Points.)
- Read the “Benchmarks in This Unit” in the table on p. 15, *Assessing the Benchmarks*.

Discuss

- What mathematical ideas and skills are students working on in this unit?
- What mathematics are students expected to know at the beginning of the unit? At the end?

2. Building and Using Arrays (Session 1.1)

In this Investigation, students use arrays and number puzzles to learn about properties of numbers. In this session, students build rectangular arrays of 2-digit numbers and identify factors and multiples of these numbers.

- Read and do the Activity, *Building Arrays*, p. 31-33. Members of your group should work on different numbers, choosing from 24, 28, 36 or 42.

Discuss

- How did you find arrays for 24 (28, 36 or 42)?
- What relationships do you notice among the arrays for (24)? Did you use the dimensions of one array to help you find the dimensions of another?
- Do you think you found all the arrays for your number? How do you know?

- Read the Activity, *Number Puzzles: 1 Clue*, pp. 34-35. Complete Student Activity Book pp. 1-2, *Number Puzzles: 1 Clue*. Read the Algebra Note, “Models,” p. 34.

Discuss

- How do arrays show different properties (e.g. odd, even, prime, square, etc.) of numbers?
- How does the work of finding arrays and solving number puzzles help students better understand the operations of multiplication and division?

3. Solving 35×28 (Session 2.1)

In this Investigation, students continue to develop strategies for solving 2-digit by 2-digit multiplication problems. In this session, students solve a two-digit multiplication problem and represent their solution.

- Working by yourself:
 - Create a story problem for 35×28 .
 - Solve the problem, showing your solution clearly.
 - Draw a quick representation for the problem, using cubes, arrays, groups, etc.
- With a partner:
 - Create a poster that shows one (or both) of your strategies, including the representation.
 - Share posters with the group.
- Read the Activity, *Solving 35×28* , pp. 71-74.
- Read the Teaching Notes, “Story Contexts” p. 71, and the Math Note, “Clear and Concise Notation”, p. 73.

Discuss

- How are the strategies and representations your group used the same and different from those shown in the session?
- How do the story context and the representation help students answer the following questions about each strategy: What part of the problem has been solved with each step? What remains to be solved? Where is the 35 in your solution? Where is the 28? Where is the answer?
- How do the questions above help students achieve computational fluency in multiplication?

4. Multiplication Cluster Problems (Session 2.4)

Students are introduced to *Cluster Problems*, which are sets of problems that help students to use what they know to find the solution to harder problems.

- Read the Activity, *Introduction to Cluster Problems*, pp. 92-93. Complete Student Activity Book p. 35, Multiplication Cluster Problems.

Discuss

- What are the relationships among the problems in each set of cluster problems?
- How did you use the problems in the cluster to solve the larger problem?
- How do cluster problems help students efficiently solve multiplication problems? How do they help students understand the operation of multiplication?

- Read the Teacher Note, *Multiplication Strategies*, pp. 161-162.

Discuss

- Look again at the cluster problems on SAB p. 35. Which multiplication strategy is suggested by each set of clusters?

5. Using Multiple Towers (Session 3.2)

In this Investigation, students develop various strategies for solving division problems. In this session, students list a sequence of multiples and use it to solve multiplication and division problems.

- Read the Activities, *Introducing Multiple Towers*, pp. 122-124, and *Using Multiple Towers*, pp. 124-125. Use the multiple tower shown on p. 124 to solve the problems on Student Activity Book pp. 49-50, Problems about Multiples of 21.

Discuss

- What numbers and/or relationships on the Multiple Towers did you use to solve the problems?
- How does the solution to one problem help to solve the others?

- Read the Teacher Note, *The Relationship Between Multiplication and Division*, p. 169.

Discuss

- How is the relationship between multiplication and division highlighted in the Multiple Tower work? (That is, how would you describe the relationship between $126 \div 21 = \underline{\quad}$ and $\underline{\quad} \times 21 = 126$?)
- How does this work help students solve multiplication and division problems?

6. Wrap Up

- Look back at the unit overview, pp. 8-9.

Discuss

- How do the activities done during this unit study fit into the overall mathematical storyline of the unit?

Other Key Features of *Number Puzzles and Multiple Towers*

- Algebra Connections in This Unit, pp. 16-19
- Ten-Minute Math, p. 20
 - Quick Images*
 - Number Puzzles*
- Assessment:
 - Number Puzzles (Session 1.5)
 - Resource Master M34, Assessment Checklist
 - What Is the Answer? (Session 2.7)
 - Teacher Note: Assessment, pp. 166-168
 - End-of-Unit Assessment (Session 3.8)
 - Teacher Note, pp. 174-177