Investigations in Number, Data, and Space®

Standards for Mathematical Practice in *Investigations in Number, Data, and Space*

Investigations in Number, Data and Space is a coherent and focused K-5 mathematics curriculum that can be used to implement the philosophy and content described by the Common Core State Standards for Mathematics[1] (CCSSM).

The CCSS include Standards for Mathematical Practice that "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years." (CCSSM, p. 8.) These Standards for Mathematical Practice are closely aligned with the six major goals that guided the development of the *Investigations* curriculum:

- "Support students to make sense of mathematics and learn that they can be mathematical thinkers.
- Focus on computational fluency with whole numbers as a major goal of the elementary grades.
- Provide substantive work in important areas of mathematics - rational numbers, geometry, measurement, data, and early algebra- and connections among them.
- Emphasize reasoning about mathematical ideas.
- Communicate mathematics content and pedagogy to teachers.
- Engage the range of learners in understanding mathematics." (Page 6 of any unit.)

The CCSS Mathematical Practices are deeply embedded in the fabric of the *Investigations* curriculum and facilitate the teaching and learning of mathematics. Virtually every lesson in *Investigations* includes one or more of these math practices.

Mathematical Practice 1: Make sense of problems and persevere in solving them.
Mathematical Practice 2: Reason abstractly and quantitatively.

The first two Practices are the foundation of the *Investigations* curriculum. They support the expectation that *all* students can, and do, make sense of and reason about mathematics.

The first principle that guided the development of *Investigations* states: "Students have mathematical ideas. Students come to school with ideas about numbers, shapes, measurements, patterns, and data. If given the opportunity to learn in an environment that stresses making sense of mathematics, students build on the ideas they already have and learn about new mathematics they have never encountered. They learn mathematical content and develop fluency and skill that is well grounded in meaning. Students learn that they are capable of having mathematical ideas, applying what they know to new situations, and thinking and reasoning about unfamiliar problems." (Page 6 of any unit.)

[1] http://www.corestandards.org/the-standards/mathematics

These two practices are embedded in most lessons in *Investigations*. Students are expected to make sense of, and solve, problems, and to reason mathematically about the problems posed, as illustrated by the following examples from the curriculum.

Students solve story problems throughout the curriculum. In K-2, a predictable routine helps focus students on making sense of problems, and on using what they know to reason about and solve them. First the teacher tells a story, and asks students to visualize the action. Several students retell the story and the class thinks about whether there will be more or less at the end. Students then solve the problem, which involves using materials, drawings, and/or numbers and notation to model the problem. Finally, strategies and notation are shared, recorded, and compared. Such discussions highlight reasoning strategies (e.g. "To do 10-4, I did 10-2 and then I minused 2 more.") Follow-up problems are often related, providing another opportunity to explore important relationships (e.g.10-4 followed by 4+6).

In Grade 3, Unit 6, *Stories, Tables, and Graphs*, students create a 12-cube train composed of a repeating pattern of red, blue, and green cubes. Numbers are associated with the elements of the pattern, and students are asked to determine the element of the pattern associated with a particular counting number. For example, students are asked the color of the 13th cube, the 20th cube, and so on.

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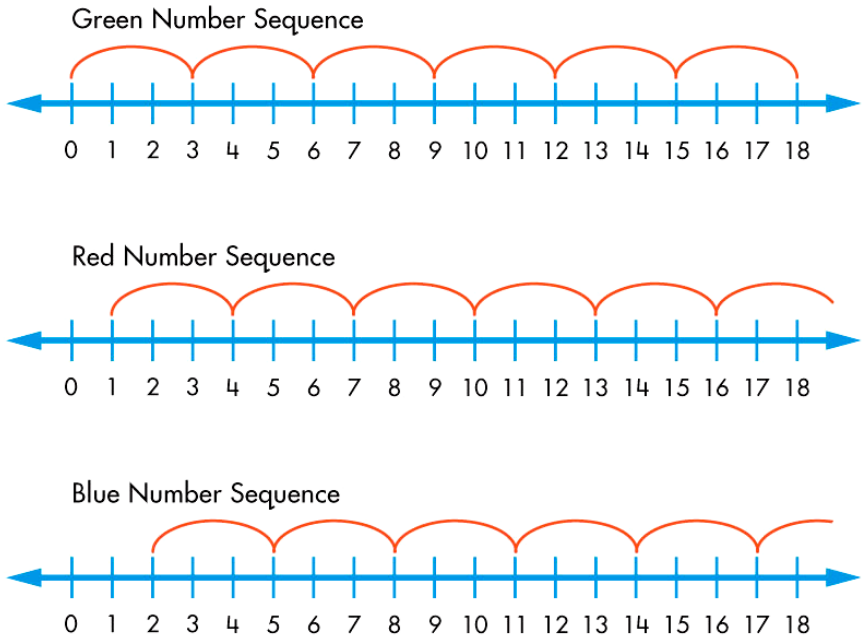
Through discussion, students determine that all green cubes in the pattern are multiples of 3.



Grade 3, Unit 6, page 71

Students are asked to discuss how they found the color of the 53rd cube, without counting by 1s. Strategies include:

> "I think it's blue because 54 is green—it is a counting by 3 number—and you minus one, so it's blue."
> "I did 12 and 12 and 12 and 12, that got me to 48. Then I counted up 5 and got blue." (3U6, p. 71)

Mathematical Practice 4: Model with mathematics.
Mathematical Practice 5: Use appropriate tools strategically.
Mathematical Practice 6: Attend to precision.

Choosing appropriate tools and representations, and the ability to accurately solve problems and communicate those solutions to others, are critical components of the mathematical work in *Investigations*. Whole group discussions often focus students' attention on number patterns and encourage argument based on mathematical reasoning.

"Tools and materials are used throughout the curriculum. Students of all ages benefit from being able to use materials to model problems and explain their thinking. It is important to encourage all students to use tools and materials. If materials are used only when someone is having difficulty, students may get the mistaken idea that using materials is a less sophisticated and less valued way of solving the problem. Encourage students to talk about how they use certain materials. They should see how different people, including the teacher, use a variety of materials in solving the same problem." (*Implementing* Investigations *at Grade X*, p. 10.)

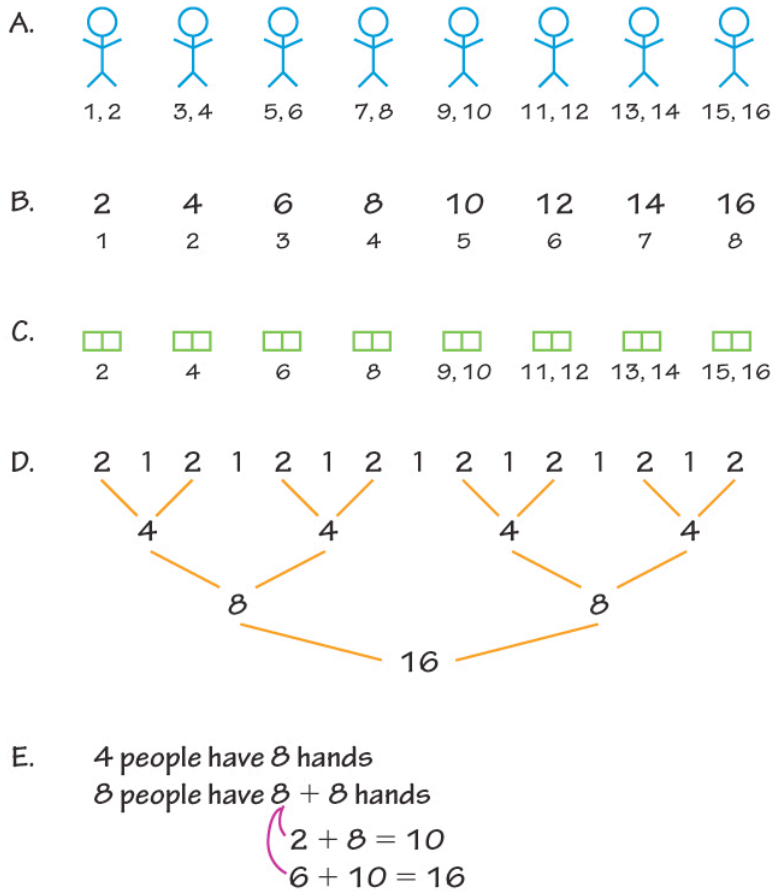
"Throughout the *Investigations* curriculum, students use representations and contexts to help them visualize, describe, and analyze mathematical relationships. Thinking with representations and contexts allows students to express and further their ideas and enables students to engage with each other's ideas. Whether solving a multiplication problem, finding the area of a rectangle, describing the relationship between two variables, or ordering fractions, students use representations and contexts to investigate and explain." (*Implementing* Investigations *at Grade 4*, p. 47)

In *Investigations*, students are expected to accurately solve problems and communicate their thinking to others. Consider the following examples from Grades 1 and 5.

In Grade 1, Unit 8, *Twos, Fives, and Tens*, students figure out how many hands there are in a group of 8 people. To solve this problem, students use drawings, cubes, and numerical reasoning. In a whole group discussion, the teacher asks students to explain their solutions:

> "I drew 8 kids and then counted their hands."
> "I counted by 2's 8 times."
> "I used cubes. I made 8 groups of 2 and then counted them by 2's."
> "I added eight 2's. I combined 2's to get 4's, then 4's to get 8's… and then 2 8's is 16."
> "I know 4 people have 8 hands, So 8 people have 8+8 hands. To solve 8+8 I broke one of the 8s into a 2 and a 6. Then I did 8 + 2, which is 10, and 6 more." (1U8, p. 57.)

As students explain their solutions, the teacher records the information on a chart, demonstrating ways to record their work.



Grade 1, Unit 8, page 57

In Grade 5, Unit 4, *What's That Portion?*, students compare fractions such as 5/6 and 7/8. They use fraction and percent equivalents, the relationship of a fraction to ½ or to 1, and representations such as rectangles or other drawings, to determine which fraction is greater. Student solutions include:

> "I drew two pizzas and then divided one into sixths and one into eighths. I knew right away the 7/8 was bigger, because the one piece of pizza missing from 7/8 is a smaller piece than the piece missing from 5/6."

> "7/8 is equal to ¾ plus 1/8, so I knew that was 75% + 12 ½%, and that's 87 ½%. Then I figured 5/6 is equal to 3/6 + 2/6, so that's 50% + 33 1/3%, and that's 83 1/3%, so 7/8 is more." (5U4, p. 64.)

Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.
Mathematical Practice 7: Look for and make use of structure.
Mathematical Practice 8: Look for and express regularity in repeated reasoning.

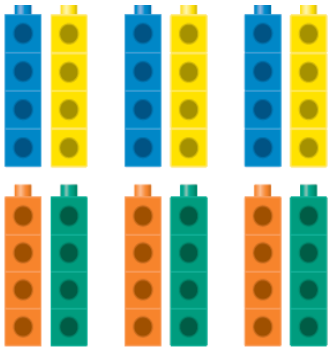
In *Investigations* classrooms, students are expected to make sense of the mathematics, and reason about what they notice. "As students develop strategies for solving problems, they are involved in three major tasks: understanding the structure of the problem, developing strategies for solving the problem, and communicating their solutions and strategies orally and in writing. Elementary students typically progress through a series of strategies for solving problems.
Learning how to communicate their thinking, both orally and in writing, is another important piece of students work in the elementary grades." (2U1, p. 197.)

"As students engage in activities of the curriculum, they frequently find numerical relationships as they work. Part of the teacher's work is helping students notice those relationships, verbalize them, and consider such questions as, 'Does this hold for all numbers? How can we know?'" (2U8, p. 153.) *Investigations* students are constantly asked what they notice about the way numbers and/or operations "behave" and to articulate, represent, and justify generalizations about numbers and operations. Consider the following examples from Kindergarten and Grade 4.

In Kindergarten, Unit 2, *Counting and Comparing*, the teacher poses a question about order during a discussion about counting a set of red and blue tiles. "I noticed that [Abby] counted the reds first and then the blues. I'm wondering if we'll get the same total if someone else counts the blues first, then the reds. What do you think will happen when we switch the order?" (KU2, p. 53.) Activities and discussions across K-2 build on this foundation, as students explore whether order matters when adding two numbers, three numbers, larger numbers, and when subtracting.

In Grade 4, Unit 1, *Factors, Multiples, and Arrays*, students are asked to reason about numbers and their factors. They find factors of multiples of 100 (100, 200, 300 . . .) and list ideas about factors of multiples of 100. For example, students quickly notice that factors of 100 are all factors of multiples of 100. Students use what they notice to make representations that show that a factor of a number is also a factor of its multiples. Students use these representations to communicate with, and convince, classmates. Student solutions for "proving" that all the factors of 16 (e.g. 4, 8) are also factors of 48 include:

> "See—there are 4s in every 16. (and 3 16's in 48) If you can split one of the 16s into 4s, then you can split all of the 16s into 4s. It doesn't matter how many 16s you have, you can just keep splitting them into 4s." (4U1, p. 109.)



Grade 4, Unit 1, page 109.

Conclusion: While the CCSS Content Standards describe what mathematics students should be able to understand and do, the mathematical practices describe how students should engage with these mathematical concepts and skills. The *Investigations* curriculum is intentionally designed to promote a deep understanding of mathematics and develop mathematically proficient students who can think, reason, model and solve problems.