Opportunities to Observe

About Assessment

Assessment plays a critical role in teaching and learning, and it is an integral part of *Investigations in Number, Data, and Space,* the comprehensive, K–5 mathematics curriculum developed at TERC. For a teacher using this curriculum, assessment is an ongoing process. Teachers observe students' discussions and explanations of their strategies on a daily basis and examine their work as it evolves. While students are busy recording and representing their work, working on projects, sharing with partners, and playing mathematical games, teachers have many opportunities to observe students' mathematical thinking. What teachers learn through observation guides their decisions about how to proceed. In any of the units, they repeatedly consider questions like these:

- Do students come up with their own strategies for solving problems, or do they expect others to tell them what to do? What do their strategies reveal about their mathematical understanding?
- Do students understand that there are different strategies for solving problems? Do they articulate their strategies and try to understand other students' strategies?
- How effectively do students use materials as tools to help with their mathematical work?
- Do students have effective ideas for keeping track of and recording their work? Does keeping track of and recording their work seem difficult for them?

Teachers' Tools

Each *Investigations* unit includes questions to guide teachers' thinking while observing students at work. There are also built-in assessment tools: Teacher Checkpoints and embedded Assessment activities.

Teacher Checkpoints offer a time to "check in" with individual students, watch them at work, and ask questions that illuminate how they are thinking. Two important ingredients of this process are asking students open-ended questions about their work and showing genuine interest in how they are approaching the task. When students see that a teacher is interested in their thinking and is expecting them to come up with their own ways of solving problems, they may surprise others with the depth of their understanding. Checkpoints also give teachers a chance to pause in the teaching sequence and reflect on how a class is doing overall.

Assessment activities embedded in each *Investigations* unit help teachers examine specific pieces of student work, figure out what it means, and provide feedback. From the students' point of view, these assessment activities are no different from any others. Each is a learning experience in and of itself, as well as an opportunity for teachers to gather evidence about students' mathematical understanding. The embedded assessment activities sometimes involve writing and reflecting; at other times, a discussion or brief interaction between student and teacher; and in still other instances, the creation and explanation of a product.

Students' Strategies

In the following activity, adapted from the *Investigations* unit "Putting Together and Taking Apart: Addition and Subtraction," teachers observe students solving a problem about calculating the distance between two numbers on the 100 chart. This assessment activity follows up on related student games and activities about using the 100 chart and finding the difference between two numbers.

Students may use a variety of strategies and tools to solve the problem. Some may calculate the distance between the two numbers using jumps of 10's and 1's on the 100 chart. Others may count out the distance by 1's using their fingers or other counters. Some students may count up from the lower number, and some may count down from the higher number.

The assessment activity requires that students write down what they did using both words and numbers, so that someone reading their papers can understand how they solved the problem. Having to explain how they worked through a problem helps students be more focused and clear in their mathematical thinking. It also helps them realize that doing mathematics is a process that may involve tentative starts, revising one's approach, taking different paths, and working through ideas. Observing students' explanations, and not just their answers, helps teachers form an overall picture of each student's mathematical progress, as well as the progress of each class as a whole.

how far?

This activity is adapted from "Putting Together and Taking Apart: Addition and Subtraction," a second-grade unit of *Investigations in Number, Data, and Space* (S.J. Russell, C. Tierney, J. Mokros, & K. Economopoulos), published by Cuisenaire-Dale Seymour Publications. Reprinted with permission.

To order, contact Cuisenaire-Dale Seymour Publications, 10 Bank Street, White Plains, NY 10602-5026, (800) 872-1100, info@cuisenaire.com, www.cuisenaire-dsp.com.

Materials

- Student worksheet (1 per student)
- Buttons, paperclips, interlocking cubes, or other counters
- 100 chart

What Happens

In this activity you, the teacher, observe second-grade students as they solve a problem about calculating the distance between two numbers on the 100 chart. The activity provides an opportunity for you to assess students' problem-solving abilities. Observing students as they work can give you a sense of how they approach the problem. Their written work provides information about the strategies they are using and how clearly they are able to record their thinking.

Explain to students that you have been learning about their thinking by watching them solve problems and play games, and by reading their explanations about strategies they use to solve problems.

> *We have been talking about your strategies for solving addition and subtraction problems. Today you will solve a problem that involves figuring out how far away one number is from another. After you have solved the problem, you will need to explain your thinking. It is important to use words and numbers so that someone reading your paper can understand how you figured out the distance between two numbers.*

Distribute a copy of the student sheet, How Far?, to each student. Briefly go over the problem, explaining to students that their task is to figure out how far it is from 38 to 65. Explain to students that you would like to get an idea about how each of them is thinking about these problems and that for this activity each person should work alone.

Counters and 100 charts should be available. Remind students to include these tools in their explanation if they use them.

Observing Students

As you watch students work, make notes about the following questions:

* How do students approach the problem? Do they seem to understand the problem and readily begin to solve it, or are they more hesitant and unsure in their approach?
* Which students can solve the problem fluently using numerical reasoning, without consulting a 100 chart?
* Which students count by 1's to find the distance between the two points on the 100 chart? Which students are using 10 and multiples of 10 in their solutions?
* Do students use other materials to solve the problem? If so, which materials and how are they used?
* Are students counting up from 38 to 65? If so, how? By 1's? By 1's and 10's? Are they counting back from 65 to 38?
* Are students using multiple strategies to check their work?
* How clearly are students able to record their solutions? Do their explanations convey their thinking?

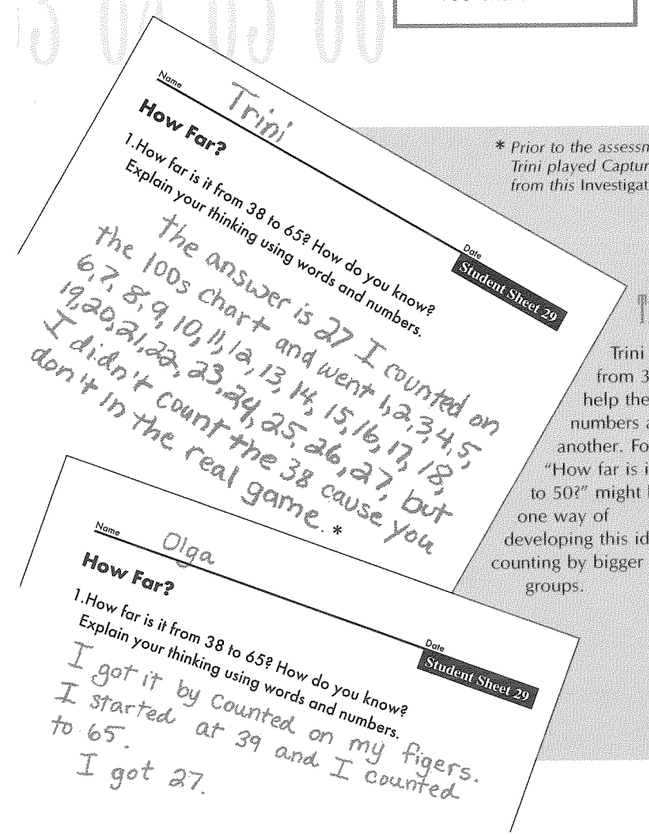
Interpreting Strategies

As students finish working, collect their papers so that you can look at them more closely. Look at how students approach the problems to get a sense of the strategies they are using, particularly which students count by 1's from one number to the other and which students are using groups of 10's and 1's to determine the difference.

What do you notice about the work of the class as a whole? Are most students using similar strategies for solving the problem? For example, do most students count on from 38 by 1's? Is there a group of students who are using 10's and 1's? By sorting the strategies into groups, you can get a sense of how your class as a whole is approaching this type of problem. If you find that students are using predominantly one strategy, say, counting by 1's, you may want to spend time looking at a different strategy as a way of expanding students' experience.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The 100 chart is used extensively throughout the Investigations *curriculum as a concrete model of the number system and how it is organized. Students can see all the numbers between 1 and 100 and their relationship to one another.*


Name Trini
How Far?
1. How far is it from 38 to 65? How do you know? Explain your thinking using words and numbers.
Date Student Sheet 29
the answer is 27 I counted on the 100s chart and went 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27, but I didn't count the 38 cause you don't in the real game. *

* *Prior to the assessment activity Trini played Capture 5, a game from this* Investigations *unit.*

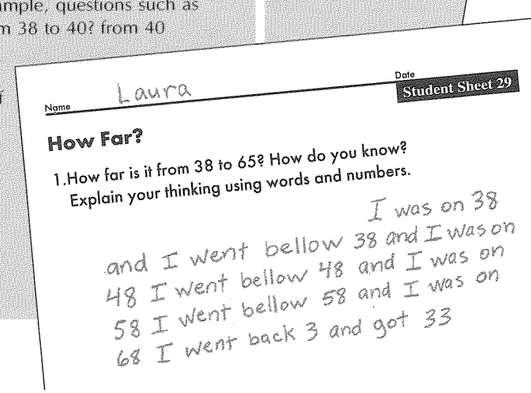
Trini and Olga

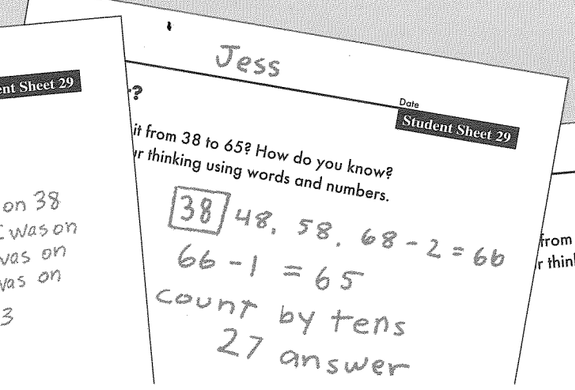
Trini and Olga both counted by 1's from 38 to 65. A next step might be to help them begin to use "chunks" of numbers as they count from one number to another. For example, questions such as "How far is it from 38 to 40? from 40 to 50?" might be one way of developing this idea of counting by bigger groups.

Name Olga
How Far?
1. How far is it from 38 to 65? How do you know? Explain your thinking using words and numbers.
Date Student Sheet 29
I got it by counted on my figers. I started at 39 and I counted to 65. I got 27.

Laura and Jess

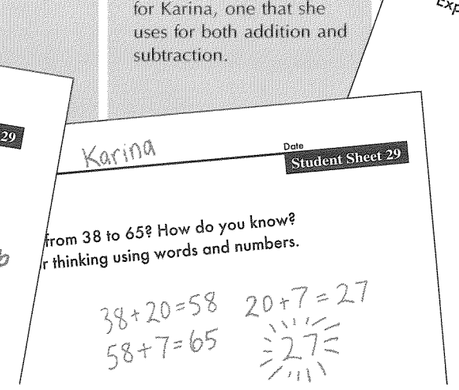
Laura and Jess both counted up from 38 using groups of 10. Both students added on three 10's and then subtracted 3 or "went back," as Laura explained, to get to the target number of 65. However, Laura seems confused by what "going back" means, adding on 3 to the 30 (10 + 10 + 10) to get 33 instead of subtracting to get the answer of 27. This is a common confusion, especially when students are moving around on the 100 chart. Issues of directionality are still slippery ideas for young students. They benefit from talking through their strategies and demonstrating their ideas with each other and in the whole group in order to solidify their thinking.


Name Laura
How Far?
1. How far is it from 38 to 65? How do you know? Explain your thinking using words and numbers.
Date Student Sheet 29
I was on 38 and I went bellow 38 and I was on 48 I went bellow 48 and I was on 58 I went bellow 58 and I was on 68 I went back 3 and got 33


Jess
?
it from 38 to 65? How do you know?
r thinking using words and numbers.
Date Student Sheet 29
38 48, 58, 68 - 2 = 66
66 - 1 = 65
count by tens
27 answer

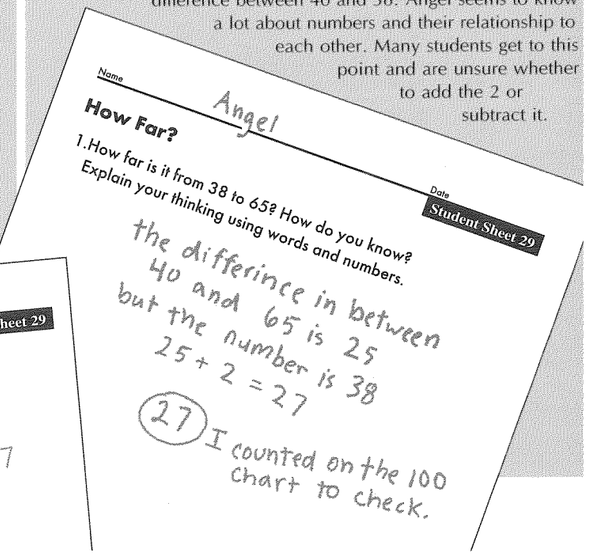
Karina

Karina also counted up from 38, but she did so in two steps using a group of 20 and then a group of 7. This is a consistent strategy for Karina, one that she uses for both addition and subtraction.


Karina
Date Student Sheet 29
from 38 to 65? How do you know?
r thinking using words and numbers.
38+20=58 20+7=27
58+7=65 27

Angel

Angel adjusted the problem a bit in order to work with a multiple of 10. She mentally knew that the difference between 40 and 65 is 25. She then added 2 more to compensate for the difference between 40 and 38. Angel seems to know a lot about numbers and their relationship to each other. Many students get to this point and are unsure whether to add the 2 or subtract it.


Name Angel
How Far?
1. How far is it from 38 to 65? How do you know? Explain your thinking using words and numbers.
Date Student Sheet 29
the differince in between 40 and 65 is 25 but the number is 38 25 + 2 = 27
27 I counted on the 100 chart to check.