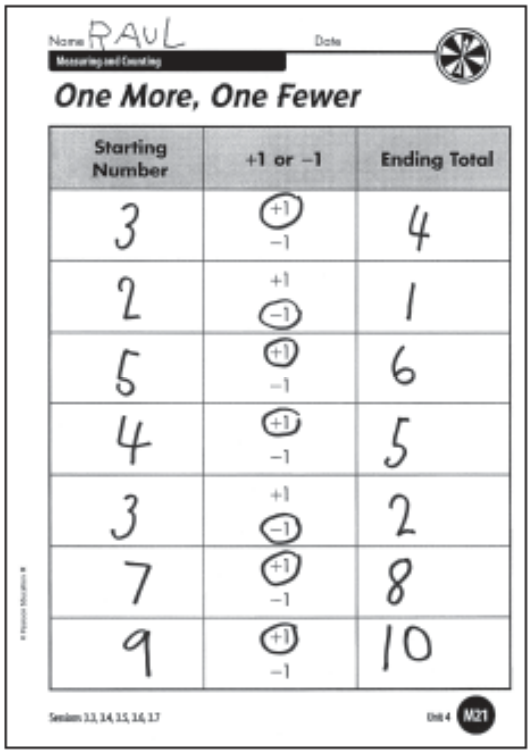
Investigations in Number, Data, and Space®

Math Content by Strand

Number and Operations: Whole Numbers
Addition and Subtraction and the Number System

Kindergarten

Young students develop their understanding of the operations of addition and subtraction by having many opportunities to count, visualize, model, solve and discuss different types of problems. Many of the counting activities in Kindergarten build a bridge to the operations of addition and subtraction, as students add a small amount to a set or remove a small amount from a set and figure out, "How many now?" One of the ways students are introduced to addition and subtraction is via story problems about combining and separating. They retell the stories, act them out, and solve them, by modeling the action involved and using counting strategies. Students also play a variety of games that model the operations of addition and subtraction. They have repeated experiences joining two or more amounts, and removing an amount from a whole.



Name RAUL Date

Measuring and Counting

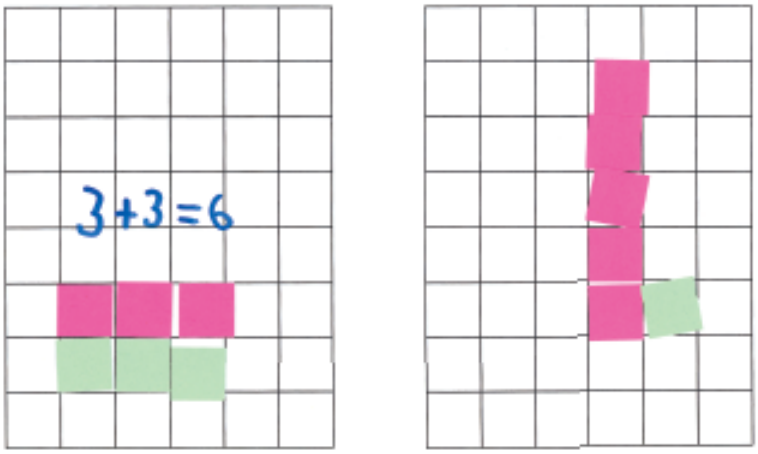
One More, One Fewer

Starting Number	+1 or −1	Ending Total
3	(+1) −1	4
2	+1 (−1)	1
5	(+1) −1	6
4	(+1) −1	5
3	+1 (−1)	2
7	(+1) −1	8
9	(+1) −1	10

Sessions 3.3, 3.4, 3.5, 3.6, 3.7 Unit 4 M21

Later in the year students work with combinations of quantities that they can count fluently. As they find ways to arrange and describe sets of 5-10 square tiles or record combinations of two-color counters, they begin to see that numbers can be composed in different ways. They work on activities that involve

seeing and describing a given quantity (e.g. 6 tiles) as made up of groups (e.g. a group of 4 and a group of 2). They are also asked to decompose quantities (e.g. 6 can be split into 4 and 2) and to find one or more combinations of a quantity (e.g. 6 can be decomposed as 6 and 0, 3 and 3, or 5 and 1.) This work lays the foundation for making meaningful sense of 4 + 2 = 6 and 6 – 4 = 2 in subsequent years.



Students show their arrangements of six tiles and indicate how they know there are six tiles in all.

Example: Record the total number of chips. Toss the chips. Record the number that are red and the number that are yellow.

Total Number: 6

Red	Yellow
2	4
5	1
3	3
3	3
4	2

Students use mathematical tools and representations to model and solve problems to clarify and communicate their thinking. Kindergartners are just beginning to learn how to represent their mathematical work on paper and are encouraged to do so in ways that make sense to them. Many use combinations of pictures, words, and numbers.

The Algebra Connections pages of the units that focus on counting, addition, and subtraction show how students develop ideas about how numbers describe the size of a set–that the number of objects in a set is fixed no matter how it is arranged and counted, and different sets may have the same number of objects. Students' observations about the constancy of the total, no matter what the order of counting a set of objects, lays the foundation for what they will later call the commutative property of addition.

These pages also show how students work on ideas of combining and decomposing quantities and on understanding how addition and subtraction operate. Thus, the generalization Kindergarten students are approaching might be stated as: When adding (with the numbers they know), the resulting amount is greater than you started with. When subtracting (with the numbers they know), the resulting amount is less than you started with.

Emphases

Whole Number Operations

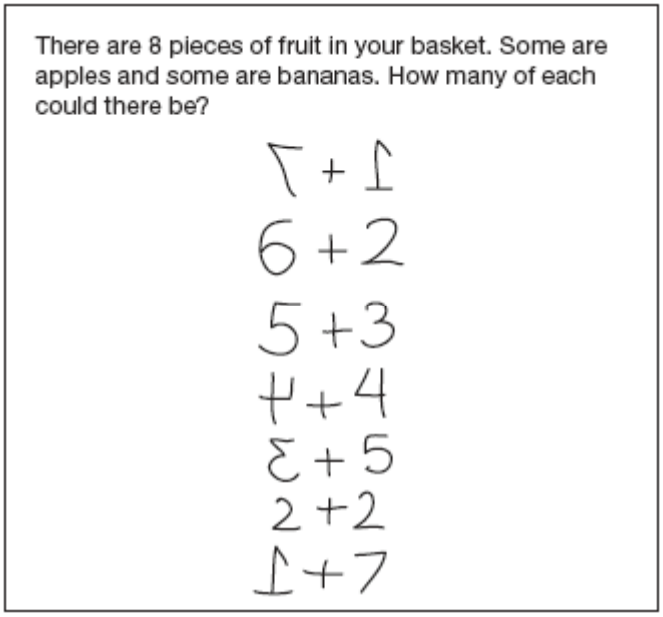
- Making sense of and developing strategies to solve addition and subtraction problems with small numbers
- Using manipulatives, drawings, tools and notation to show strategies and solutions

Benchmarks

- Combine two small quantities
- Figure out what is one more or one fewer than a number

Grade 1

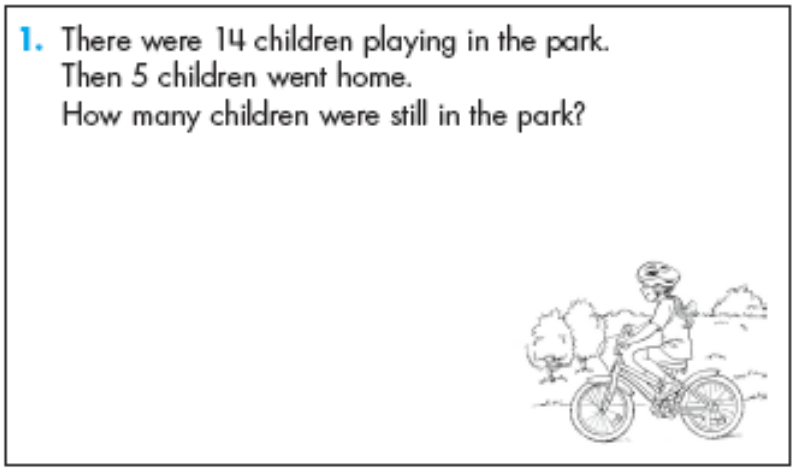
In first grade, students work with the important idea that quantities can be composed and decomposed in different ways, while the quantity remains the same. Students have repeated experiences breaking one number (a whole) into two parts, or combining two parts to form a whole. They consider the relationship between the parts, noticing, for example, that when the whole remains the same, as one part increases the other part decreases. Students work with composing and decomposing numbers to 20, and focus on the addition combinations of 10. Students are expected to develop fluency with the combinations of 10 by the end of the school year.



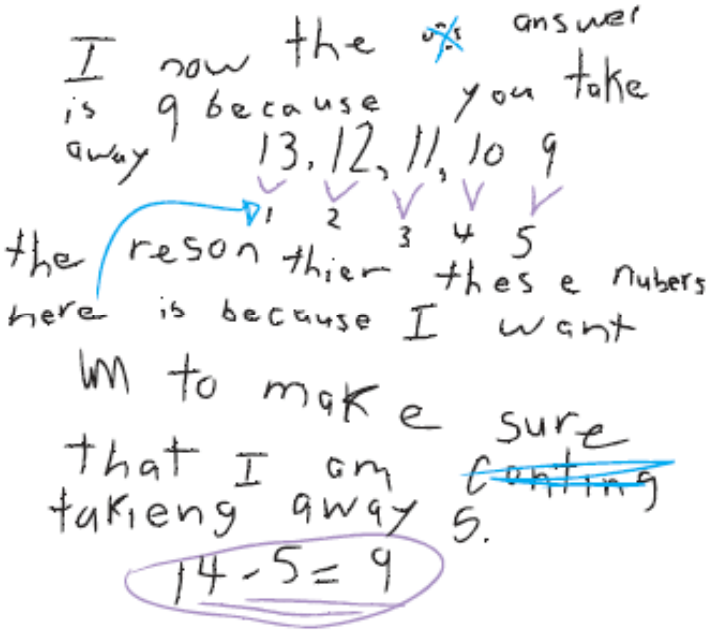
A student uses an ordered list to organize his responses to a How Many of Each? *Problem.*

The addition and subtraction work of first grade focuses on making sense of these operations, practicing adding and subtracting single-digit numbers, and solving addition and subtraction story problems. Many of the games and activities involve students in comparing and combining two amounts or removing one amount away from the other, which offers practice with single-digit addition and subtraction. The goal of the work with story problems is for students to learn to visualize the action of story problems and to solve the problems in ways that make sense to them.

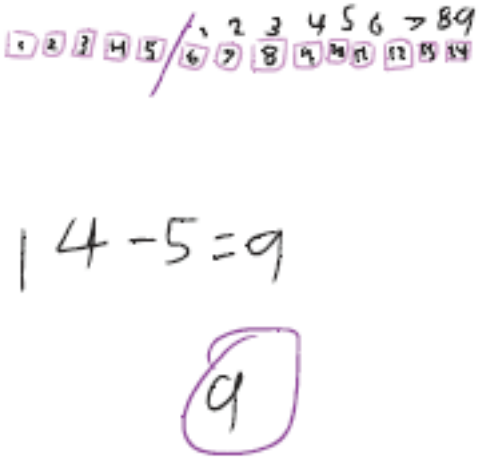
By the end of the year, it is expected that first graders will *count on* to combine two small quantities and that some students will *use a combinations they know* to solve related problems (e.g., 6 + 4 = 10 so 6 + 5 = 11). For subtraction, many students will still *show all, remove some, and count those that remain*. Others will *count back*, *count up*, or *use relationships they know* (e.g., 14 – 5 = 14 – 4 – 1).



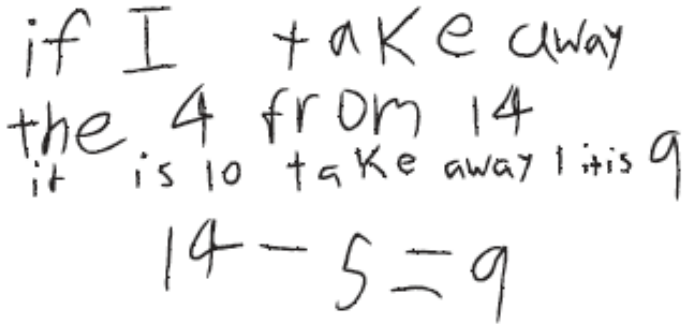
A sample subtraction problem from the Student Activity Book



Count Back



Show all, remove or cross out some, and then count how many are left



Use a combination you know

Students use mathematical tools, such as cubes and counters, and representations, such as the number line and 100 chart, to model and solve addition and subtraction problems and to clarify and communicate their thinking. They are encouraged to represent their work on paper in ways that make sense to them. Many use a combination of pictures, words, numbers and mathematical symbols.

The Algebra Connections pages of each of the four curriculum units that focus on addition and subtraction show how students are applying the commutative property of addition as they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 6 + 4 = 5 + 5 and 8 + 5 = 10 + 3) or when they use addition combinations they know to solve more difficult problems (e.g., since 5 + 5 = 10, 5 + 6 must equal 10 + 1, or 11).

Emphases

Number Composition

- Representing numbers using equivalent expressions
- Composing numbers up to 20 with two addends

Whole Number Operations

- Making sense of and developing strategies to solve addition and subtraction problems with small numbers
- Using manipulatives, drawings, tools and notation to show strategies and solutions

Computational Fluency

- Knowing addition combinations of 10
- Combine two small quantities

Benchmarks

- Find more than one combination of two addends for a number up to 10 (e.g., 7 is 4 and 3 and it's also 5 and 2)
- Find at least 5 two-addend combinations of 10
- Interpret (retell the action and sequence) and solve addition and subtraction story problems
- Find at least five combinations of two addends for a number up to 15
- Subtract one small quantity from another
- Represent numbers using equivalent expressions
- Combine two small quantities by at least counting on
- Demonstrate fluency with the two-addend combinations of 10

Grade 2

Throughout second grade, students work on making sense of the action of different types of addition and subtraction problems and on developing efficient strategies for solving them and for recording their work. They solve addition and subtraction problems in ways that make sense to them and practice using particular strategies.

Students study two particular strategies for addition –adding tens and ones and adding one number in parts. By the end of the school year, students are expected to have at least one strategy that they can use to accurately and efficiently solve an addition problem.

In Grade 2, students' work with place value becomes the basis for the development of strategies for adding and subtracting 2-digit numbers. The two strategies for addition, adding by place and adding one number in parts, and the strategy for subtraction of subtracting one number in parts, depend on an understanding of how to break numbers into tens and ones.

Adding tens and ones

14 + 32 = 46
10 + 30 = 40
4 + 2 = 6
40 + 6 = 46

Adding on one number in parts

14 + 32 = 46
32 + 10 = 42
42 + 4 = 46

Students consider and practice two strategies for subtraction– subtracting in parts and adding up. By the end of the year they are expected to have one strategy that they can use to accurately solve a subtraction problem.

Subtracting in parts

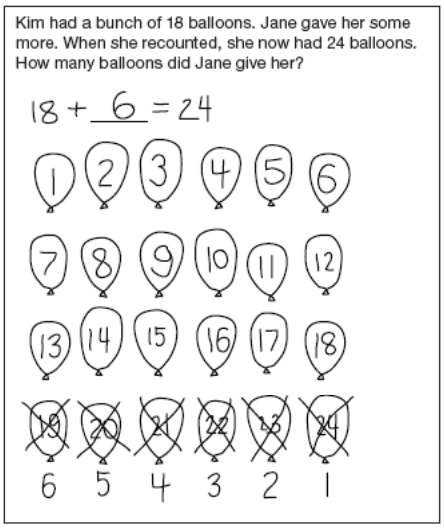
46 – 32 = 14
46 – 2 = 44
44 – 10 = 34
34 – 10 = 24
24 – 10 = 14

Adding up

46 – 32 = 14
32 + 10 = 42
42 + 4 = 46
10 + 4 = 14

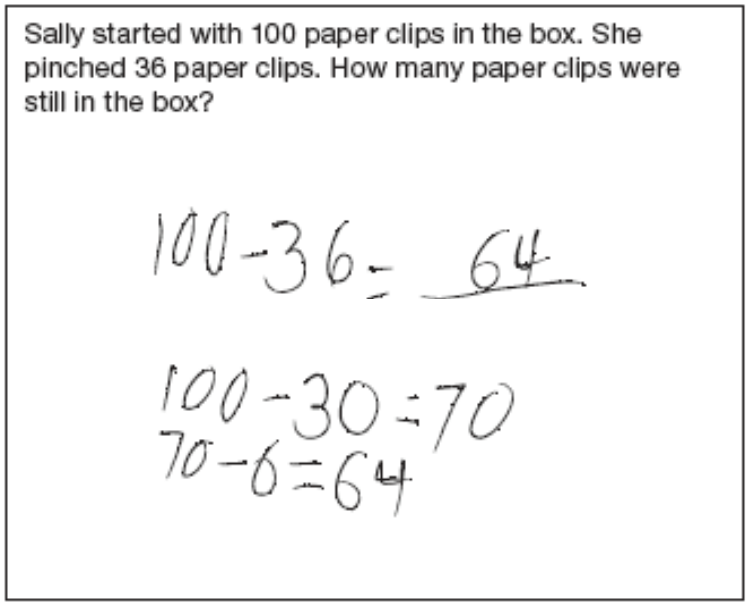
Knowing the single-digit addition combinations helps students as they work to develop efficient strategies for adding and subtracting. Students are expected to be fluent with addition combinations up to 10 + 10 by the end of the year. Students use the relationship between addition and subtraction to solve subtraction problems and to develop fluency with the subtraction expressions related to the addition combinations to 10 + 10.

Students use mathematical tools and representations to model and solve problems to clarify and communicate their thinking. They are encouraged to show their mathematics work on paper in ways that make sense to them; many use some combination of pictures, words, numbers and mathematical symbols and notation.



A student uses pictures, numbers, and notation to show his solution.

Students are expected to use standard notation to write equations to represent addition or subtraction problems. They are also expected to have methods for clearly showing their work, including: sticker notation, numbers, equations, the number line and 100 Chart, or some combinations of these.



A student writes an equation and uses a number line to show her work.

The Algebra Connections pages of each of the four curriculum units that focus on addition and subtraction show how students are applying the commutative and associative properties of addition as they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 5+ 9 = 4 + 10 or 5 + 9 = 5 + 10 - 1). In addition, these pages highlight the work that students do in proving generalizations about adding odd and even numbers.

Emphases

Whole Number Operations

- Using manipulatives, drawings, tools, and notation to show strategies and solutions
- Making sense of and developing strategies to solve addition and subtraction problems with totals to 100
- Understanding the properties of addition and subtraction
- Adding even and odd numbers

Computational Fluency

- Knowing addition combinations to 10+10

Benchmarks (compiled from Units 1, 3, and 8)

- Determine the difference between two numbers (up to 45)
- Interpret addition and subtraction story problems (read a story problem and determine what needs to be figured out)
- Have at least one strategy for solving addition and subtraction (as removal) story problems
- Demonstrate fluency with addition combinations to 10 + 10
- Understand what it means to double a quantity
- Use known combinations to add several numbers in any order
- Interpret and solve subtraction (removal) and unknown change story problems with totals up to 45
- Count on or break numbers apart to add two or more numbers up to a total of 45
- Write an equation that represents an addition or subtraction situation
- Determine the difference between a number and any multiple of 10 up to 100
- Add multiples of 5, up to 100
- Subtract two-digit numbers
- Reason about partners, teams, and leftovers to make and justify generalizations about what happens when even and odd numbers are added
- Add two 2-digit numbers accurately and efficiently

Grade 3

In Grade 3, students build an understanding of the base-ten number system to 1,000 by studying the structure of 1,000 and using a base-ten context to represent the place value of two-digit and three-digit numbers. Students identify the hundreds digit as representing how many 100s are in the number, the tens digit as representing how many 10s, and the ones digit as representing how many 1s. They also break numbers into 100s, 10s, and 1s in different ways:

137 = 1 hundred, 3 tens, and 7 ones

137 = 1 hundred, 2 tens, and 17 ones

137 = 13 tens and 7 ones

137 = 12 tens and 17 ones

137 = 11 tens and 27 ones

. . .

In their work with number and operations in Grade 3, students focus particularly on addition and subtraction. Students solve addition and subtraction problems with two-digit and three-digit numbers, developing computation strategies that are built on adding and subtracting multiples of 10 and finding combinations that add to 100. Addition strategies include breaking the numbers apart and then either adding by place or adding on one number in parts. They also examine problems that lend themselves to changing the numbers in order to make them easier to add. Subtraction strategies include subtracting a number in parts, adding up, and subtracting back.

Addition Strategies

Adding by Place

349 + 175 =

300 + 100 = 400

40 + 70 = 110

9 + 5 = 14

400 + 110 + 14 = 524

Adding on number on in parts

349 + 175 =

349 + 100 = 449

449 + 50 = 499

499 + 25 = 524

Changing the numbers

349 + 175 =

350 + 175 = 525

525 – 1 = 524

Subtraction Strategies

Subtracting in parts

451 – 187 =

451 – 100 = 351

351 – 80 = 271

271 – 7 = 264

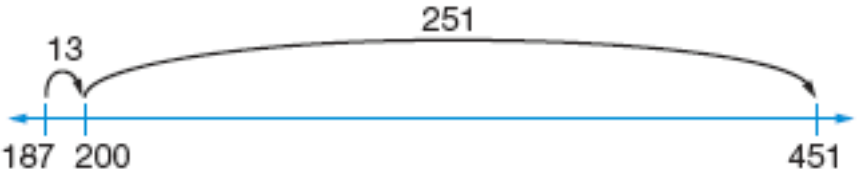
Adding up

451 – 187 =

187 + 13 = 200

200 + 251 = 451

13 + 251 = 264



Subtracting back

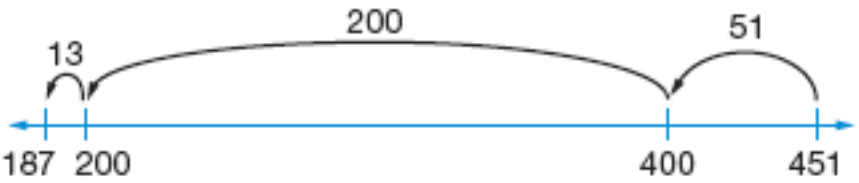
451 – 187 =

451 – 51 = 400

400 – 200 = 200

200 – 13 = 187

51 + 200 + 13 = 264



Students expand their understanding of subtraction to include other problem situations besides removal (or take away)—the type they are probably most familiar with from their work in Grade K-2. These include comparison problems and finding the missing part of a whole:

> If I am collecting 1,000 baseball cards, and I have 250 so far, how many more do I need?

The ability to visualize what is happening in a subtraction situation is an important foundation for understanding the operation and how it works, and for making good decisions about strategies for computation. Students use visual representations (cubes, number lines, 300 charts, and 1,000 Charts) and story contexts. They work on developing fluency with the subtraction problems related to the addition combinations to 10 + 10 (the subtraction facts) so they are able to use these easily when solving subtraction problems with two-digit and three-digit numbers.

The Algebra Connections pages of each of the three curriculum units that focus on addition and subtraction show how students are applying the commutative and associative properties of addition as

they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 48 + 72 = 50 + 70).

Emphases

The Base Ten Number System

- Understanding the equivalence of one group and the discrete units that comprise it
- Extending knowledge of the number system to 1,000

Whole Number Operations

- Understanding different types of subtraction problems
- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers

Computational Fluency

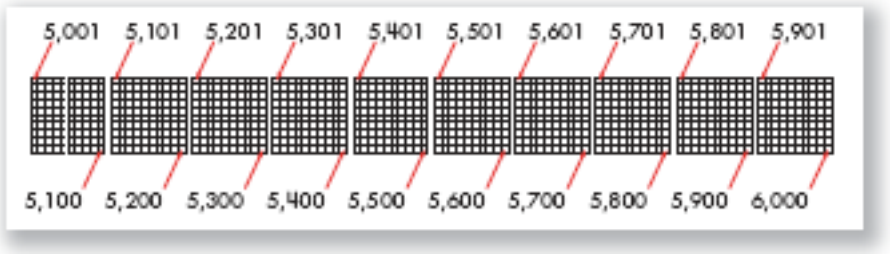
- Adding and subtracting accurately and efficiently

Benchmarks (compiled from Units 1, 3, and 8)

- Demonstrate fluency with the addition combinations up to 10 + 10
- Add multiples of 10 (up to 100) to and subtract them from 2-digit and small 3-digit numbers
- Solve addition problems with 2-digit numbers using strategies involving breaking numbers apart by place or adding one number in parts
- Break up 3-digit numbers less than 200 into 100s, 10s, and 1s in different ways (e.g. 153 equals 1 hundred, 5 tens, and 3 ones; 15 tens and 3 ones; 14 tens and 13 ones, etc.)
- Find combinations of 2-digit numbers that add to 100 or $1.00
- Read, write, and sequence numbers to 1,000
- Identify the value of each digit in a 3-digit number (100s, 10s, and 1s)
- Identify how many groups of 10 are in a 3-digit number (e.g. 153 has 15 groups of 10, plus 3 ones)
- Solve addition problems with 3-digit numbers (to 400) using strategies that involve breaking numbers apart, either by place value or by adding one number in parts
- Solve subtraction story problems in contexts that include removing a part from a whole, comparing two quantities, or finding a missing part
- Solve subtraction problems with 2-digit and 3-digit numbers (to 300) using strategies that involve either subtracting a number in parts, adding up, or subtracting back
- Add multiples of 10 and 100 (to 1,000) to and subtract them from any 3-digit number
- Solve 3-digit addition problems using at least one strategy efficiently
- Demonstrate fluency with problems related to the addition combinations to 10 + 10 (the subtraction facts)
- Solve subtraction problems with 3-digit numbers using strategies that involve either subtracting a number in parts, adding up, or subtracting back

Grade 4

In Grade 4, students extend their knowledge of the base-ten number system, working with numbers up to 10,000. Their work focuses on understanding the structure of 10,000 and how numbers are related within that structure, recognizing the place value of digits in large numbers, and using place value to determine the magnitude of numbers.

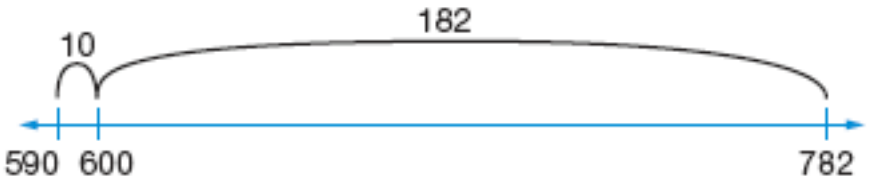


By discussing, refining and comparing their strategies for adding and subtracting 3- and 4-digit numbers, including studying the U.S. algorithm for addition, students continue expanding their understanding of addition and subtraction. Their strategies should involve good mental arithmetic, estimation, clear and concise notation, and a sound understanding of number relationships. By identifying and naming addition and subtraction strategies that they are using, students are adding to the repertoire of strategies they can use for flexible and fluent computation. Further, they consider how and why certain methods work. For example, some students change one or both numbers in an addition or subtraction expression to create an easier problem, then compensate as needed for that change. In this unit, students study *why* certain addition expressions are equivalent (e.g., 457 + 198 = 455 + 200) and *how* certain expressions in subtraction are related (e.g., 782 – 590 and 782 – 600).

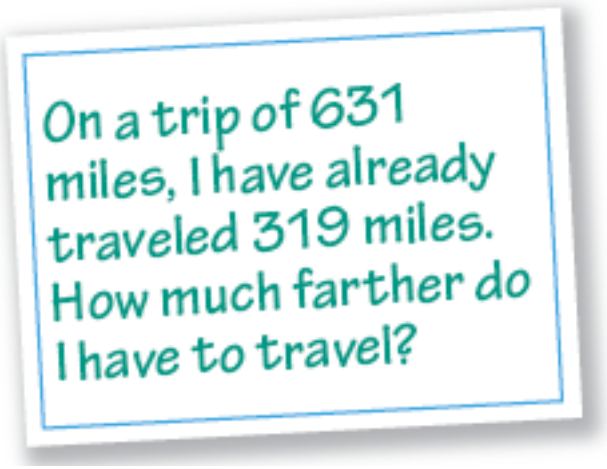
782 – 590 = _____

782 – 600 = 182

182 + 10 = 192



To help them make good decisions about strategies for subtraction and continue to develop their understanding of how subtraction operates, students use visual representations, such as number lines and 100 Charts, and story contexts that include several types of subtraction situations—removal (or take away), comparison, and missing parts. Students focus particularly on missing part problems in the context of distance:



Some students visualize a problem like this one as adding up from the distance traveled to the total distance, while others visualize subtracting the distance traveled from the total distance. This provides another opportunity for students to consider the relationship between addition and subtraction.

319 + ____ = 631

631 – 319 = ____

The Algebra Connections page in the curriculum unit that focuses on addition and subtraction shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic ideas that underlie the generalizations students investigate and articulate when they create equivalent expressions in order to solve a problem (e.g., 124 – 89 = 125 – 90).

Emphases

The Base Ten Number System

- Extending knowledge of the base-ten number system to 10,000

Computational Fluency

- Adding and subtracting accurately and efficiently

Whole Number Operations

- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers
- Understanding different types of subtraction problems

Benchmarks

- Read, write, and sequence numbers to 10,000
- Add and subtract multiples of 10 (including multiples of 100 and 1,000) fluently
- Solve addition problems efficiently, choosing from a variety of strategies
- Solve subtraction problems with 3-digit numbers by using at least one strategy efficiently

Grade 5

In Grade 5, students extend their knowledge of the base ten number system, working with numbers in the hundred thousands and beyond. In their place value work, students focus on adding and subtracting multiples of 100 and 1,000 to multi-digit numbers and explaining the results. This work helps them develop reasonable estimates for sums and differences when solving problems with large numbers. Students apply their understanding of addition to multi-step problems with large numbers. They develop increased fluency as they study a range of strategies and generalize the strategies they understand to solve problems with very large numbers.

90, 945 – 1,000 =

90,945 – 1,200 =

90,945 – 1,210 =

90,945 – 1,310 =

Students practice and refine their strategies for solving subtraction problems. They also classify and analyze the logic of different strategies; they learn more about the operation of subtraction by thinking about how these strategies work. Students consider which subtraction problems can be solved easily by changing one of the numbers and then adjusting the difference. As they discuss and analyze this approach, they visualize important properties of subtraction. By revisiting the steps and notation of the U.S. algorithm for subtraction and comparing it to other algorithms, students think through how regrouping enables subtracting by place, with results that are all in positive numbers.

Examples of Subtraction Strategies

Subtracting in parts

3,451 – 1,287 =

3,451 – 1,200 = 3,251

2,251 – 80 = 2,171

2,171 – 7 = 2,164

Adding up

3,451 – 1,287 =

1,287 + 13 = 1300

1,300 + 2,100 = 3,400

3,400 + 51 = 3,451

13 + 2,100 + 51 = 2,164

Subtracting back

3,451 – 1,287 =

3,451 – 51 = 3,400

3,400 – 2,100 = 1,300

1,300 – 13 = 1,287

51 + 2,100 + 13 = 2,164

Changing the numbers

3,451 – 1,287 =

3,451 – 1,300 = 2,151

2,151 + 13 = 2,164

3,451 – 1,287 =

(add 13 to both number to create an equivalent problem)

3,451 – 1,287 = 3,464 – 1300

= 2,164

The Algebra Connections page in the curriculum unit that focuses on addition and subtraction shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic ideas that underlie the generalizations students investigate and articulate when they create equivalent expressions in order to solve a problem (e.g., 892 – 567 = 895 – 570).

Emphases

The Base Ten Number System

- Extending knowledge of the base-ten number system to 100,000 and beyond

Computational Fluency

- Adding and subtracting accurately and efficiently

Whole Number Operations

- Examining and using strategies for subtracting whole numbers

Benchmarks

- Read, write, and sequence numbers to 100,000
- Solve subtraction problems accurately and efficiently, choosing from a variety of strategies