Investigations in Number, Data, and Space®

Math Content by Strand

The Base-Ten Number System: Place Value

Introduction[1]

Learning about whole number computation must be closely connected to learning about the base-ten number system. The base-ten number system is a "place value" system. That is, any numeral, say 2, can represent different values, depending on where it appears in a written number: it can represent 2 ones, 2 tens, 2 hundreds, 2 thousands, as well as 2 tenths, 2 hundredths, and so forth. Understanding our place value system requires coordinating the way we write the numerals that represent a particular number (e.g., 217) and the way we name numbers in words (e.g., two hundred seventeen) with how those symbols represent quantities.

The heart of this work is relating the written numerals to the quantity and to how the quantity is composed. It builds from work on tens and ones in grades 1 and 2, to work on numbers in the 100s and 1,000s in grade 3, to work on numbers in the 1,000s to 10,000 in Grade 4, to a focus on 10,000s to 100,000 and beyond in Grade 5. This is not simply a matter of saying that 217 "has two hundreds, 1 ten, and 7 ones," which we know students can easily learn to do as a pattern without much meaning attached to it. Students must learn to visualize how 217 is built up from hundreds, tens, and ones, in a way that helps them relate its value to other quantities. Understanding the place value of a number such as 217 entails knowing, for example, that 217 is closer to 200 than to 300, that it is 100 more than 117, that it is 17 more than 200, that it is 3 less than 220, and that it is composed of 21 tens and 7 ones.

A thorough understanding of the base-ten number system is one of the critical building blocks for developing computational fluency. The composition of numbers from multiples of 1, 10, 100, 1000, and so forth, is the basis of most of the strategies students will adopt for whole number operations.

First, the understanding of place value is at the heart of estimation. For example, consider adding two different quantities to 32:

32 + 30 = ____ 32 + 3 = ____

---

[1] Introduction excerpted from the Teacher Note, Computational Fluency and Place Value, *Implementing Investigations in Grade (Kindergarten, 1, 2, 3, 4, and 5).*

How much will 32 increase in each case? Students think about how the first sum will now have 6 tens, but the ones will not change, whereas in the second sum, the ones will change, but the tens remain the same. Considering the place value of numbers that are being added, subtracted, multiplied, or divided provides the basis for developing a reasonable estimate of the result.

Students' computational algorithms and procedures depend on knowledge about decomposing numbers and about the effects of operating with multiples of ten. For example, one of the most common algorithms for addition is adding by place. Each number is decomposed into ones, tens, hundreds, and so forth; these parts are then combined, for example:

326 + 493
300 + 400 = 700
20 + 90 = 110
6 + 3 = 9
700 + 110 + 9 = 819

In order to carry out this algorithm fluently, students must know a great deal about place value, not just how to decompose numbers. They must also be able to apply their knowledge of single-digit sums such as 3 + 4 and 2 + 9 to sums such as 300 + 400 and 20 + 90. In other words, they know how to interpret the place value of numbers *as they operate with them*—in this case that just as 2 ones plus 9 ones equals 11 ones, 2 tens plus 9 tens equals 11 tens, or 110.

As with addition, algorithms for multidigit multiplication also depend on knowledge about how the place value of numbers is interpreted as numbers are multiplied. Again, students must understand how they can apply knowledge of single-digit combinations such as 3 x 4 to solve problems such as 36 x 42, for example:

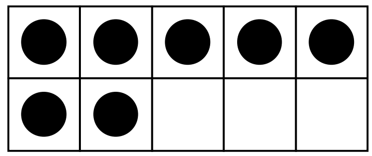
30 x 40 = 1200
30 x 2 = 60
6 x 40 = 240
6 x 2 = 12
1200 + 240 + 60 + 12 = 1512

After students work on activities in which they represent what happens when numbers are multiplied by 10, they apply what they've learned to more difficult problems. For example, they gradually come to understand how knowledge of 3 x 4 helps them solve 30 x 4, 3 x 40, 30 x 40, 3 x 400, and so forth.

Place Value Across the Grades

Kindergarten
A main focus in Kindergarten is counting, which is the basis for understanding the number system and for almost all the number work in the primary grades.

Students hear and use the counting sequence (the number names, in order) in a variety of contexts. Students engage in repeated practice with counting and develop visual images for quantities to 10. For example, work with Ten Frames helps students develop visual images of 10 and how that number is composed and decomposed.



As students are developing accurate counting strategies they are also building an understanding of how the numbers in the counting sequence are related: Each number is one more (or one less) than the number before (or after) it.

Students develop an understanding of the concepts of greater than, less than, and equal to, and develop language for describing quantitative comparisons (e.g. bigger, more, smaller, fewer, less, same, equal) as they count and compare quantities.

Related Emphases

Counting and Quantity

- Developing an understanding of the magnitude and position of numbers
- Developing strategies for accurately counting a set of objects by ones

Related Benchmarks

- Count a set of up to 20 objects
- Figure out what is one more or one fewer than a number

Grade 1

Because counting is the basis for understanding the number system and for almost all of the number work in the primary grades, first grade students work on developing strategies for accurately counting a group of up to 50 objects, both by ones and by numbers other than one (2s, 5s, and 10s). As students are developing accurate counting strategies they continue to build an understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after) it.

As students build this understanding, they compare and order quantities and develop a sense of the relative size of numbers and the quantities they represent.

First grade students also focus on understanding the quantity 10, including how it is composed and its importance in grouping and keeping track of quantities. Students are expected to develop fluency with the combinations of 10 by the end of the school year.

This work with the combinations of 10 also lays the foundation for exploring new ideas such as counting by 10s, the place value of the teen numbers (e.g., the teen numbers are made up of one group of ten and some number of 1s), and generating equivalent expressions for numbers under 20 (e.g., 8 + 5 = 10 + 3).

Related Emphases

Counting and Quantity
- Developing an understanding of the magnitude and position of numbers
- Developing strategies for accurately counting a set of objects by ones and groups

Number Composition
- Composing Numbers up to 20 with two addends

Computational Fluency
- Knowing addition combinations of 10

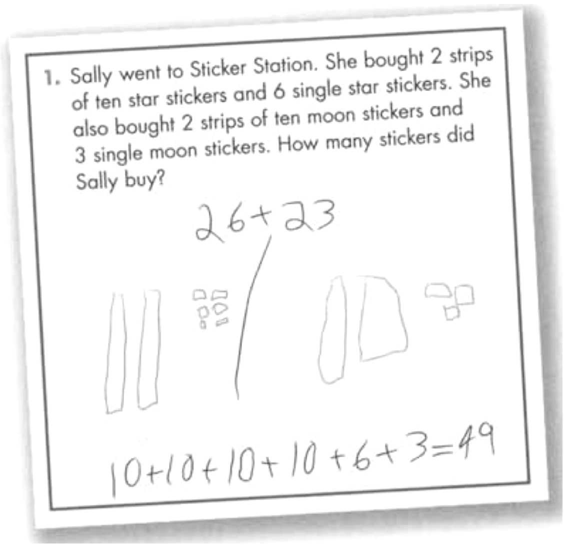
Related Benchmarks

- Interpret and solve problems about the number of tens and ones in a quantity
- Demonstrate fluency with the two-addend combinations of 10
- Identify, read, write, and sequence numbers up to 105

Grade 2

At the beginning of the school year, students have varied opportunities to count sets of objects by ones, write the number sequence, and explore and compare representations of the counting numbers on the number line and the 100 chart. As the school year progresses, most second graders shift from thinking and working primarily with ones to thinking and working with *groups* of ones. To help them make this shift, students have many opportunities to develop strategies for grouping and for counting by groups. The focus is first on contexts that encourage counting by groups of 2, 5, or 10 and then specifically on *groups of 10* and the base ten structure of our number system.

In Grade 2, students work extensively with contexts and models that represent the place value structure of our base-ten number system. They use a "sticker" context where stickers come in singles, strips of 10, and sheets of 100.



Sample Student Work

They also work with money, focusing specifically on pennies, dimes, and dollars. They use these contexts to build and visualize how two-digit numbers are composed. For example, 33 cents can be composed of 3 dimes and 3 pennies or 2 dimes and 13 pennies or 1 dimes and 23 pennies.

Students also work with other models, including the number line, 100 chart, and cubes organized into towers of ten. The purpose of these models is to help students build mental images that they can use in visualizing and solving problems.

In Grade 2, students' work with place value plays a foundational role in their development of strategies for adding and subtracting 2-digit numbers. Students' use and understanding of the two strategies for addition, adding by place and adding one number in parts, and the strategy for subtraction of subtracting one number in parts, depend on an understanding of how to decompose numbers into tens and ones.

Adding tens and ones

14 + 32 = <u>46</u>
10 + 30 = 40
4 + 2 = 6
40 + 6 = 46

Adding on one number in parts

14 + 32 = <u>46</u>
32 + 10 = 42
42 + 4 = 46

Subtracting in parts

46 – 32 = 14
46 – 2 = 44
44 – 10 = 34
34 – 10 = 24
24 – 10 = 14

Related Emphases

The Base Ten Number System

- Understanding the equivalence of one group and the discrete units that comprise it

Counting and Quantity

- Developing an understanding of the magnitude and sequence of numbers up to 100

Related Benchmarks

- Interpret and solve problems about the number of tens and ones in a quantity
- Demonstrate fluency with addition combinations to 10 + 10
- Count by 2s, 5s, and 10s, up to 110

Grade 3

In Grade 3, students build an understanding of the base-ten number system to 1,000 by using 100 grids to construct a class and individual 1,000 charts and by thinking about how 2- and 3-digit numbers relate to 1,000. Students use base-ten contexts (stickers arranged in sheets of 100, strips of 10, and singles and dollars, dimes, and pennies) to represent the place value of two-digit and three-digit numbers. Students identify the hundreds digit as representing how many 100s are in the number, the tens digit as representing how many 10s, and the ones digit as representing how many 1s. They also break numbers into 100s, 10s, and 1s in different ways. For example, the number 137 can be composed from 1 hundred, 3 tens, and 7 ones; 13 tens and 7 ones; 12 tens and 17 ones; 11 tens and 27 ones and so on. Decomposing numbers flexibly supports students' work on developing computational fluency.

The development of accurate and efficient computation strategies involves the ability to add multiples of 10 to and subtract them from any number. In Grade 3, students use the place value context of stickers arranged in sheets of 100, strips of 10, and singles to examine what happens when 10 or a multiple of 10 is added to or subtracted from a 2- or 3-digit number. They look at what happens to the digit in the ones place and explain why the value of this digit doesn't change. They extend this work as they add 100 to numbers; looking at what happens to the tens and ones digits and explaining why these digits remain the same.

To deepen their understanding of place value and develop more efficiency in computation, students then extend this work to adding or subtracting multiples of 10 and 100 (i.e., 20, 50, 70, 200, 500, 700) to or from any number. They continue this work throughout the year in the Ten- Minute Math activity, *Practicing Place Value*.

In Grade 3, students apply their understanding of place value and adding and subtracting 10s as they solve a variety of addition and subtraction problems, using strategies that involve adding by place or adding or subtracting one number in parts. Carrying out these strategies depends on an understanding of how to break numbers into hundreds, tens, and ones.

Adding by Place

349 + 175 =

300 + 100 = 400

40 + 70 = 110

9 + 5 = 14

400 + 110 + 14 = 524

Adding one number on in parts

349 + 175 =

349 + 100 = 449

449 + 50 = 499

499 + 25 = 524

Subtracting in parts

451 – 187 =

451 – 100 = 351

351 – 80 = 271

271 – 7 = 264

In Grade 3, students begin their formal study of multiplication and division. Just as adding and subtracting multiples of 10 is a key part of developing computational fluency in addition and subtraction, learning to multiply and divide by 10 (and later multiples of 10) is a key part of developing computational fluency in multiplication and division. Part of the work of third grade involves developing an understanding of the relationship between skip counting and multiplication. Most third graders can easily skip count by 10. Their work is to connect that skip counting sequence to the meaning of 1 x 10, 2 x 10, 3 x 10, etc. as one group of 10 or 10, two groups of 10 or 20, three groups of 10 or 30, and so forth. As students learn the x10 combinations through 100, they can use these combinations to find the products of other less familiar combinations [e.g., if 10 x 4 = 40, then 9 x 4 = (10 x 4) – 4 = 36; if 10 x 4 = 40, then 11 x 4 = (10 x 4) + 4 = 44].

In Grade 3, students determine, describe, and compare sets of multiples. They apply their knowledge of multiples of 10 to solving other multiplication problems. For example, they highlight and look for patterns in the multiples of 5 and 10 on 100 charts and then examine the multiples of these two numbers together. In comparing these sets of multiples, they see that there are two multiples of 5 for each multiple of 10 on the 100 chart; it takes three jumps of 10 to get to 30, but 6 jumps of 5. Such observations lead to an investigation of how knowing the multiples of one number can help determine the multiples of a number that is double or half that number: how can a student use the known product of 10 x 8 to help solve 5 x 8?

Related Emphases

The Base Ten Number System

- Understanding the equivalence of one group and the discrete units that comprise it Extending knowledge of the number system to 1,000

Computational Fluency

- Adding and subtracting accurately and efficiently
- Learning the multiplication combinations with products to 50 fluently

Whole Number Operations

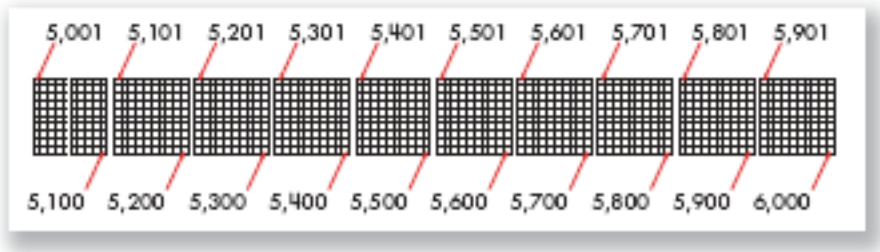
- Developing strategies for division based on understanding the inverse relationship between multiplication and division

Related Benchmarks

- Read, write, and sequence numbers to 1,000
- Identify the value of each digit in a 3-digit number (100s, 10s, and 1s)
- Identify how many groups of 10 are in a 3-digit number (e.g. 153 has 15 groups of 10, plus 3 ones)
- Break up 3-digit numbers less than 200 into 100s, 10s, and 1s in different ways (e.g. 153 equals 1 hundred, 5 tens, and 3 ones; 15 tens and 3 ones; 14 tens and 13 ones, etc.)Add multiples of 10 (up to 100) to and subtract them from 2-digit and small 3-digit numbers
- Add multiples of 10 and 100 (to 1,000) to and subtract them from any 3-digit number
- Solve addition problems with 3-digit numbers (to 400) using strategies that involve breaking numbers apart, either by place value or by adding one number in parts
- Solve multiplication combinations and related division problems by using skip counting or known combinations

Grade 4

In Grade 4, students extend their knowledge of the base-ten number system, working with numbers up to 10,000. Their work focuses on understanding the structure of 10,000 and how numbers are related within that structure, recognizing the place value of digits in large numbers, and using place value to determine the magnitude of numbers.



In Grade 4, students extend and apply their understanding of place value and develop more efficiency in computation as they consider what happens when 100 and 1,000 and multiples of 100 and 1,000 are added to or subtracted from whole numbers. They consider what happens when they add these multiples as well as multiples of one tenth to decimal numbers (e.g., 356.4 + 20, 356.4 – 20, 356.4 + 200, 356.4 – 200, 56.4 + 0.1, 56.4 + 0.3, 56.4 – 0.1). Students compare each sum or difference with the starting number and identify how the value of each place has or has not changed and why. Students practice these skills throughout the year in the Ten-Minute math activity, *Practicing Place Value.*

In another variation of *Practicing Place Value*, students decompose numbers by place as they answer the question, How Many 10s? (100s?, 1,000s?) are in a given number. This goes beyond naming the number in each place; it includes understanding, for example, that while 335 is composed of 3 hundreds, 3 tens, and 5 ones, it can also be decomposed into 2 hundreds, 13 tens, and 5 ones, or 1 hundred, 23 tens, and 5 ones, or 33 tens and 5 ones. Decomposing numbers flexibly supports students' continued work on developing computational fluency.

In Grade 4, learning to multiply by 10 and multiples of 10 is a key component of the work on developing computational fluency in multiplication. Fourth graders consider what happens after a number is multiplied by 10 or 100.

For example:

8 x 1 =	8 ones =	8
8 x 10 =	8 tens =	80
8 x 100 =	8 hundreds =	800
8 x 1,000 =	8 thousands =	8,000

Through work with arrays and other representations, students come to understand that what they may describe as "adding a zero" actually represents the fact that each number is ten times greater than the previous number: or 80 (8 tens) is 10 times greater than 8 (8 ones), 800 (8 hundreds) is ten times greater than 80 (8 tens), etc.

Students then extend this work as they use arrays and other representations to show situations involving multiplying by other multiples of 10, like 40, 60, or 90. This work helps them understand, for example, that the product of 3 x 40 is ten times the product of 3 x 4.

Fourth grade students commonly use strategies to solve multiplication problems that involve breaking numbers apart by place. These strategies are based on an understanding of the effect of multiplying by multiples of 10.

Breaking numbers apart by addition

48 x 42 =

40 x 40 = 1,600

40 x 2 = 80

8 x 40 = 320

8 x 2 = 16

1,600 + 80 + 320 + 16 = 2,016

48 x 42 =

48 x 40 = 1,920

48 x 2 = 96

1,920 + 96 = 2,016

In Grade 4, students work on understanding division as making groups of the divisor. They use the inverse relationship between multiplication and division to solve division problems. As in multiplication, working with multiples of 10 is key. Many students think about division problems in terms of "taking out groups." In the problem 242 ÷ 22, they think, "How many groups of 22 are in 242?" Then they can make decisions about how many groups they can "take out" all at once. In this case, 10 groups of 22 is 220, a number very close to the dividend. Students can then consider what part of the dividend remains: "If I take 10 groups of 22 (220) out of 242, how many more groups do I need to take out in order to finish the problem?"

Related Emphases

The Base Ten Number System

- Extending knowledge of the number system to 10,000

Computational Fluency

- Adding and subtracting accurately and efficiently
- Solving multiplication problems with 2-digit numbers

Whole Number Operations

- Understanding division as making groups of the divisor

Related Benchmarks

- Read, write, and sequence numbers to 10,000
- Add and subtract multiples of 10 (including multiples of 100 and 1,000) fluently
- Read, write, and interpret decimal fractions in tenths and hundredths
- Multiply by 10 and multiples of 10
- Multiply 2-digit numbers by one-digit and small 2-digit numbers (e.g. 12, 15, 20), using strategies that involve breaking the numbers apart
- Solve division problems (2-digit and small 3-digit numbers divided by 1-digit numbers), including some that result in a remainder
- Solve division problems with 1- digit and small 2-digit divisors using at least one strategy efficiently

Grade 5

In Grade 5, students extend their knowledge of the base ten number system, working with numbers in the hundred thousands and beyond. While much of their place value work with whole numbers is similar to the work of fourth grade (adding and subtracting multiples of tens and explaining the results), students work with numbers to the ten thousands. This helps them develop reasonable estimates for sums and differences when solving problems with large numbers.

Fifth graders also consider what happens when they add these multiples and multiples of tenths to decimal numbers (e.g., 356.4 + 20, 356.4 – 20, 356.4 + 200, 356.4 – 200, 356.4 + 0.1, 356.4 + 0.3, 356.4 + 0.03, 356.4 – 0.03). Students compare each sum or difference with the starting number and identify how the value of the places has changed and why. Students practice these skills throughout the year in the Ten-Minute math activity, *Practicing Place Value*.

In another variation of *Practicing Place Value*, students decompose numbers by place as they answer the question, How Many 10s? (100s?, 1,000s?) are in a given number. This goes beyond naming the number in each place; it includes understanding, for example, that while 9,435 is composed of 9 thousands, 4 hundreds, 3 tens, and 5 ones, it can also be decomposed as 8 thousand, 14 hundreds, 3 tens, and 5 ones or as 94 hundreds and 35 ones. Decomposing numbers flexibly supports students' continued work on developing computational fluency.

Students continue to consolidate and refine their multiplications strategies as they solve multiplication problems with 2- and 3-digit numbers. One common strategy involves breaking numbers apart by place. Students, efficient use of this strategy is dependent on their understanding of the effects of multiplying by 10 and multiples of 10.

Breaking numbers apart by addition

148 x 42 =	**148 x 42 =**
40 x 100 = 4,000	100 x 42 = 4,200
40 x 40 = 1,600	48 x 40 = 1,920
40 x 8 = 320	48 x 2 = 96
2 x 100 = 200	4,200 + 1,920 + 96 = 6,216
2 x 40 = 80	
2 x 8 = 16	
4,000 + 1,600 + 320 + 200 + 80 + 16 = 6,216	

In Grade 5, students also refine their strategies for division. They solve division problems by relating them to missing factor problems (e.g., 462 ÷ 21 = ____ and ____ x 21 = 462), by building up groups of the divisor, and by using multiples of 10 to solve problems more efficiently. For example, when solving 1,280 ÷ 32, some students start with 10 x 32 = 320, and keep adding ten groups of 32 until they reach 1,280. Students can learn to be more efficient: for example, once they know that 10 x 32 = 320, they can double that (20 x 32 = 640), and double it again (40 x 32 = 1,280) to find the answer.

Related Emphases

The Base Ten Number System
- Extending knowledge of the number system to 100,000 and beyond

Computational Fluency
- Adding and subtracting accurately and efficiently
- Solving Multiplication problems with 2- and 3-digit numbers
- Solving division problems with 2-digit divisors

Whole Number Operations
- Understanding and using the relationship between multiplication and division to solve division problems

Rational Numbers
- Understanding the meaning of decimal fractions
- Comparing decimal fractions
- Adding decimal fractions

Related Benchmarks

- Read, write, and sequence numbers to 100,000
- Solve multiplication problems efficiently

- Read, write, and interpret decimal fractions to thousandths
- Order decimals to the thousandths
- Add decimal fractions through reasoning about place value, equivalents, and representations
- Solve division problems efficiently