The Revision of *Investigations in Number, Data and Space*

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Introduction

The 1st edition of the K-5 mathematics curriculum *Investigations in Number, Data, and Space* was developed at TERC from 1990-1998, through a grant from the National Science Foundation (NSF). A major revision, supported by NSF, TERC, and Pearson Scott Foresman, began in 2001, and the 2nd edition was completed for use in school year 2007-2008. While fundamental principles did not change from one edition to the next, the embodiment and implications of those principles have been revisited, refined, and improved.

Goals and Guiding Principles

Since beginning work on this curriculum 18 years ago, six major goals have guided our work:

- to support students to make sense of mathematics and learn that they can be mathematical thinkers
- to focus on computational fluency with whole numbers as a major goal of the elementary grades
- to provide substantive work in important areas of mathematics—rational numbers, geometry, measurement, data, and early algebra—and connections among them
- to emphasize reasoning about mathematical ideas
- to communicate mathematics content and pedagogy to teachers
- to engage the range of learners in understanding mathematics.

Underlying these goals are three guiding principles that are our touchstones as we approach both students and teachers as agents of their own learning:

1. *Students have mathematical ideas*. The curriculum must support all students in developing and expanding those ideas.

2. *Teachers are engaged in ongoing learning* about mathematics content and about how students learn mathematics. Even the best preservice preparation cannot provide enough background in mathematics and student learning. The curriculum must be a professional development tool for teachers.

3. *Teachers collaborate with students and the curriculum materials* to create the curriculum as enacted in the classroom. The only way for a good curriculum to be used well is for teachers to be active participants in continually modifying it based on their ongoing observations of and conversations with students.

In the next sections, selected aspects of the 2nd edition are described.

Aspects of the Revision

1. Computational Fluency

Developing computational fluency with whole numbers must be a lynchpin of the elementary curriculum. This development includes the building blocks of computation: understanding the base ten number system and its place value notation, understanding the meaning of the operations and their relationships, knowing the basic addition and multiplication number combinations (the "facts") and their counterparts for subtraction and division, estimating reasonable results, interpreting problems embedded in contexts, and practicing and consolidating accurate and efficient strategies for computing. It also includes developing curiosity about the characteristics of numbers and operations and learning to articulate, represent, and justify generalizations.

One principle of the *Investigations* curriculum is that time and focus on the building blocks of computational fluency precedes practice and consolidation. Extended time across several grades is spent on each operation. Let's take subtraction as an example of this process. In Kindergarten and grade 1, students solve subtraction problems by modeling the action of subtraction. By grade 2, students use the inverse relationship between addition and subtraction to add up to solve problems. During grades 2 and 3, they become fluent with the subtraction "facts," and model and solve a variety of types of subtraction problems, including comparison and missing part problems. In grade 3, they use their understanding of place value to solve problems with larger numbers. In grades 3 and 4, they articulate, represent, and justify important generalizations about subtraction, for example: if you add the same amount to each number in a subtraction expression, the difference does not change, as in the equation 483 - 197 = 486 - 200. They analyze and compare strategies for solving subtraction problems. They expand their command of computation procedures with multidigit numbers. At this point, they are also in a position to appreciate the short-cut notation of the U.S. "traditional" or "borrowing" algorithm for subtraction, analyze how it works, and compare it to other algorithms.

The account above gives only a glimpse of the work that helps students develop an understanding of subtraction. Each operation has similar complexity. When we think of entering first graders who are coordinating written and spoken numbers with their quantitative meaning, second graders who are uncovering the relationship between ten tens and one hundred, and fourth graders who are becoming flexible in their use of a number of algorithms to solve problems, we begin to get some sense of how much work there is to do in these grades.

In the revision of *Investigations*, as we strengthened the coherence and rigor of the number and operations strand, we were determined not to sacrifice the time and depth required for careful development of ideas in this strand. Maintaining a focus on depth and meaning required us to navigate the morass of varying state standards while keeping ourselves grounded in the experience of real students in real classrooms.

In order to give the attention needed to number and operations we had to make hard decisions about how much time can be spent on other important mathematical content: geometry, measurement, data, and patterns and functions. We also considered more carefully how work in these other content areas can connect to and support work in number and operations. For example, a greater emphasis on the foundations of algebra across the grades opens up important opportunities to strengthen work with number and operations. By creating a strong, coherent content strand in patterns and functions across grades K-5, we were able to connect the work primary students do with repeating patterns to the later work on functions. The work on functions provides interesting problem contexts in which students' work on ratio and on constant change connect to and support their work on multiplication. Geometry and measurement provide contexts in which students revisit multiplication and fractions. Within the number and operations units themselves, articulation, representation, and justification of general claims about the operations (an aspect of early algebraic thinking) strengthen students' understanding of the operations.

The choices we made about content in the *Investigations* curriculum are based on knowledge from research and practice, including our own extensive field testing. Choice, balance, sequence, and pace of content are based on:

- the centrality of number and operations for elementary school students
- the importance of exposing all elementary students to a range of mathematics content, including work with geometry, measurement, data, patterns, functions, and the foundations of algebra
- the development of foundational ideas that students need in order to build understanding of mathematics
- what we learned from several years of in depth work with many, diverse students and their teachers.

2. What's the Math?: Explicitness and Coherence

Considering how to make the purpose, flow, and connections in the curriculum more explicit was central to the revision process. As *Investigations* was implemented during its first decade of use, and we supported professional development and implementation in a variety of ways, we were often surprised to find that what was clear to us about the mathematical focus of a unit, a session, or an activity, and about the connection and sequence of the sessions and activities, was not always so clear to teachers. Sometimes this happened because the meanings of words were not shared. For example, a teacher might say, "but there's not enough work on place value," but after a conversation, it might become clear that what this teacher meant by "place value" was "expanded notation," rather than a deeper understanding of the base ten system. At other times, a teacher might say, "I didn't understand the point of this activity, so I skipped it," when we considered the activity crucial. Or a discussion in the curriculum, as implemented, might become a listing of student ideas with little focus or direction. Although "mathematical emphases" were listed at the beginning of every class session in the 1st edition, the words used in these emphases did not always help teachers understand the session's focus.

Design decisions for the 2nd edition resulting from these observations included: 1) more careful choice and wording of the mathematical focus of each Investigation and each session; 2) the development of student benchmarks for each unit; 3) clear statement of the focus for each class discussion; 4) a Teacher Note, with examples of student work, for each assessment, related directly to the benchmarks for the unit, including analysis of work that meets the benchmark(s), partially meets the benchmark(s), or does not meet the benchmark(s); and 5) more careful choices about the activities included in each Investigation within a unit. Explicitness and coherence help teachers to:

- understand the mathematical "story line" of each unit
- make connections between the mathematical focus of the unit and each activity and assessment
- set mathematical goals for each unit and for the school year.

3. Classroom Discussion: Clarity and Focus

In a curriculum that stresses students' development, investigation, and articulation of mathematical ideas, learning to communicate about those ideas is key. Through talk, as well as through writing, students develop their mathematical reasoning and justify their ideas. Class discussion is one site for this work.

We have become more aware of teachers' need for help in focusing class discussion so that this precious time can be productive and useful for students. There are often several possible avenues to pursue in a class discussion. The authors must make clear the focus of the discussion, clarify the reason for that discussion in the sequence of mathematical development, provide ways to frame the discussion, suggest questions to ask that are generative and provoke student thinking, suggest ways that students can be prepared to participate in the discussion, define possible outcomes of the discussion, and indicate what ideas are likely *not* to be resolved at the end of the discussion.

In the development of the 2nd edition, we worked to make the purpose and course of each discussion clearer by providing a clear statement of each discussion's focus, initial and follow-up questions that can support student thinking, and a description of possible courses the discussion might take, including examples of possible student responses. Our revised Dialogue Boxes also have more commentary to help teachers notice particular aspects of the dialogue.

Classroom discussion is a critical component of the curriculum and must be supported in the text so that teachers:

- focus the discussion on key mathematical ideas
- ask questions that engage student thinking
- know how the discussion connects to the work that precedes and follows it.

4. Support for the Range of Learners

One of the goals that guided our curriculum design process is "to engage the range of learners in understanding mathematics." This goal required us to reflect on whether the curriculum engages all students in significant mathematical work and how to support teachers in making modifications to meet their students' needs. The 2nd edition includes two new elements that focus on this goal.

First, throughout the units, many of the activities include a section called "Supporting the Range of Learners." These sections offer suggestions gleaned from classroom experience about how to modify and/or extend activities for the variety of students in the class while maintaining focus on the important mathematical ideas. Second, a set of cases at each grade level, written by teachers and framed with commentary written by staff, provides examples of how teachers think through the issues of diversity in their classrooms. These cases are not meant to be exemplars nor can they provide ready-made solutions for other classrooms; rather, they show the ways in which teachers consider how the needs and strengths of their students vary, how they pose questions about their own teaching, and how they examine the results of different teaching strategies. The cases are organized into the following categories: 1) Setting up the mathematical community; 2) Accommodations for Learning, and 3) Language and representation. These supports in the curriculum share thinking, based on ideas from teachers in diverse classrooms, to help users of the curriculum to:

- facilitate the active participation of all students in mathematics
- acknowledge and build on the strengths students bring to the classroom from their own communities, cultures, and languages
- respect the intellectual work of all students, including those who are struggling
- develop modifications when students are having difficulty or need more challenge.

The *Investigations* curriculum continues to maintain a focus on the development of meaning in mathematics. The curriculum supports students in becoming mathematically literate, proficient, and confident, and in seeing mathematics as accessible, interesting, and useful. The curriculum supports teachings in understanding mathematics content and in attending to and building on their students' ideas.

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