Language and Representation

How Many Tens and Ones? Finding Out What a Student Knows

Ian is a second grader who is described by his classroom teacher as having trouble distinguishing between tens and ones. In the process of solving a subtraction problem, for example, he frequently transposes the tens and ones, without recognizing that he has dramatically altered the original problem. In an effort to clarify what he knows and where his understanding breaks down, math resource teacher Stephanie Clements decides to work with Ian individually.

I placed two handfuls of cubes on the table and asked Ian to count them. After he counted all the cubes, he declared accurately that there were 36 cubes. I asked him to write 36 on a piece of paper. Then I circled the 3 in 36 and asked him to use the cubes to show me what the 3 represents—to arrange the cubes so that it would be easy to see what the 3 means in 36. Ian made three towers of 10 and one stick of 6 cubes.

Ian: [pointing to the three towers of 10] This is 3 tens.

Teacher: How many cubes are there in 3 tens?

Ian: [counting] 10, 20, 30. Thirty cubes.

Teacher: Show me what the 6 represents in the 36.

Ian: [pointing to the stick of 6 cubes] This is 6 ones.

Teacher: So how many tens and ones are there in 36?

Ian: There are 3 tens and 6 ones.

Teacher: And how many more cubes do you need to get to 40? [Ian counted on his fingers and then said, "Four cubes." I wrote 40 on a piece of paper and asked what the 4 in 40 represents. He added 4 more cubes on to the stick of 6 cubes, to make a 10.]

Ian: Now there are 4 tens, and it is 40.

I wrote the number 59 on a piece of paper and asked him what he knows about that number. He took some connecting cubes and made 5 tens and 9 ones.

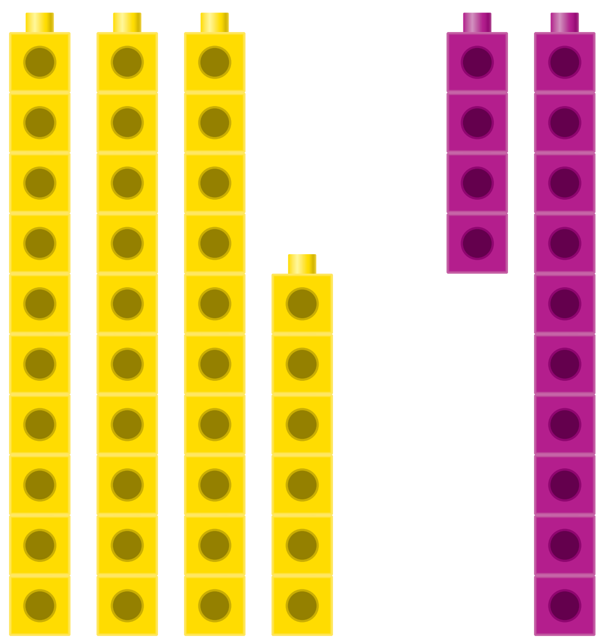
Ian: [counting the cubes quietly] 10, 20, 30, 40, 50, and 9, so it's 59.

Teacher: Is there anything else you know about 59?

Ian: One more and it's 60.

I wrote a few more numbers (47, 64, 82, 55) on a piece of paper and for each number asked him how many more he needed to get to the next multiple of 10. He answered correctly each time. I also asked him to tell me what the digits in each position meant, and he was able to tell me without using the cubes to demonstrate.

Next, I asked him to make an array using 50 cubes. He made 5 rows of 10. Then I split the array into two groups: 3 tens and 6 ones (36) and 1 ten and 4 ones (14).



Teacher: If I have 36 cubes, how many more do I need to get to 50?

Ian: 14!

Teacher: How did you get 14?

Ian: I counted 10, 20, 30; and 6 plus 4 gets to 40, and 10 more is 50. So 10 + 4 equals 14 more to get to 50.

Teacher: How about 18? How many cubes do you need to get to 50?

Ian: 10 and 8 plus 2 equals 20; 30, 40, 50.
So 30 + 2 = 32.

At this point, Ian is making good use of his understanding of multiples of 10 to calculate the difference between a number and 50: round the number up to the next multiple of 10, count by tens to 50, and then add on the amount it took to get to the first multiple of 10.

I then gave him a 100 chart and told him that our next step was to solve problems that get to 100. I wanted to see if he could transfer his new strategy and if he could use the 100 chart, in place of cubes, to solve the problem. I put a marker on 74 and asked him how far it was from 74 to 100.

Ian: 10, 20, 30, 40, 50, 60, 70; 4 plus 6 is 80, 90, 100. So 20 + 6 = 26.

I found it interesting that Ian had correctly determined the difference between 74 and 100 but had found it necessary to count the tens and ones in 74 first. I decided to give him another problem, so I put a marker on 43 and asked him how far that number was from 100. Once again, Ian successfully determined the difference but started by counting the tens and ones in 43.

Ian: 10, 20, 30, 40; 3 plus 7 is 50; 60, 70, 80, 90, 100. So 50 + 7 = 57.

I gave him more problems by putting markers on different numbers on the 100 chart and asking him how far each number was from 100. Each time Ian was successful, though each time he counted the tens and ones in the number first.

Ian seemed confident about implementing his strategy using the 100 chart, so I asked him if he thought he was ready to solve a problem without the chart. He agreed to try it. I said to Ian, "If I have 58, how far am I from 100?"

Ian: 58 plus 2 is 60.

Ian then raised his hand and counted his fingers as he said, "70, 80, 90, 100," then, smiling, he raised two fingers on his other hand, and exclaimed, "plus 2 equals 42!"

Ian had successfully solved the problem without cubes or a 100 chart. He had a big smile on his face. I gave him a few more problems, and he solved them successfully as well. And each time, he added on from the number I had given him to 100 without going back and counting all of the tens and ones in that starting number.

In this case, Ms. Clements works with Ian, in an effort to both assess his understanding of place value and to apply that understanding to finding the difference between two numbers. She deliberately sets up and guides him through a progression of tasks, drawing out what he already knows and using it as a basis for their work together. Ms. Clements's strategy illuminates the way a mathematical idea can be represented in multiple ways.

Working in this incremental and explicit way, Ms. Clements is able to make a precise assessment of what Ian understands while also helping him organize and develop his ideas.

Questions for Discussion

1. What questions did Ms. Clements pose and what experiences did she offer Ian to both assess what he knows and then build on his understanding of place value? How might the use of multiple representations have helped to move his understanding forward?

2. What understandings did Ian demonstrate in his interaction with Ms. Clements? What questions are you left with about what Ian does and does not understand?

3. Have you had a student like Ian in your classroom? What might be your next steps with Ian or a student like him? For example, is there a particular strategy or representation that you would like to see the student use?