Language and Representation

Count and Compare: A Visual Representation for Multiplication

In this case, Katrina Sajak reflects on an extension she made to the game Count and Compare *to maximize the use of arrays as a visual representation for multiplication and to address the diverse needs of the students in her classroom.*

Recently, a third-grade colleague and I were discussing some concerns about the array game *Count and Compare.* When her students played the game at home, some parents had reported that their children were "bored" because they already "knew their multiplication facts." However, this game helps students develop and understand a visual representation for multiplication and is about much more than merely practicing multiplication facts. I wondered if we were taking full advantage of the opportunities for the learning this game offers.

I shared a modification I made to *Count and Compare* to add an element of challenge for the range of learners in my classroom. Students who were using the game to help them learn and practice the factor pairs were supported by the spatial comparisons offered by the basic game. However, I wanted to ensure that all students were using the game to deepen their understanding of multiplication. I decided to ask each pair of students to come to the discussion with an "interesting comparison" they had worked with. I particularly pushed those students who I knew were already fluent with the factor pairs represented by the arrays by asking them to describe the relationship between the arrays they were comparing. (For this work, I paired together students who had a similar level of familiarity with factor pairs.)

As students played, I observed and took notes on what they said. The result was an amazing explosion of ideas that I wanted to be sure we considered as a group. Therefore, as students shared their "interesting comparisons" in the discussion at the end of the session, we grouped them into categories on a piece of chart paper. The categories were as follows:

Arrays we found with equivalent area

Example: a 3×8 equals a 2×12

Arrays for which the area of one is half or double the area of the other

Example: two 3×6s make a 6×6

Arrays for which the area of one is one-fourth or four times the area of the other

Example: four 3×3s make a 6×6 or a 3×3 is one fourth of a 6×6

Arrays we found that were "close calls"

Example: a 7×7 and a 12×4 have almost the same area

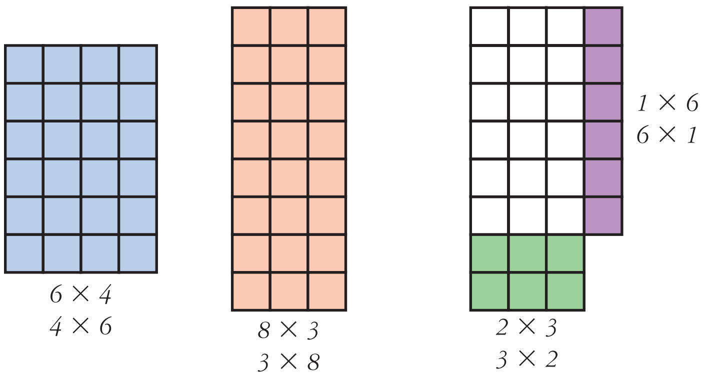
Arrays for which one big array is equal to two or more smaller arrays

Example: a 10×5 equals a 3×5 plus a 3×5 plus a 4×5

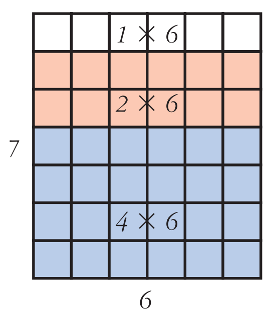
These comparisons provided ways to clarify some things about the meaning and characteristics of multiplication and how it differs from other operations. For example, when comparing a 6×6 array with a 5×7 array, some students initially thought they must have equal area because one dimension "went up by one and the other went down by one." As they talked about this relationship that had worked for addition ($6 + 6 = 5 + 7$), they realized that the same didn't hold true for multiplication ($6 \times 6 \neq 5 \times 7$).

We were out of time for the day, but I wanted the ideas on the "interesting comparisons" chart to be accessible to as many students as possible. I asked a few students to be prepared at the start of the next math class to use array cards to represent some of the relationships they had found as they played *Count and Compare.* This was the result.

Tom and Lisa had contributed 6 × 4 = 3 × 8 to our list of equivalent arrays. They placed the 6 × 4 array on top of the 3 × 8 array and observed that the “sticking out” part of the 6 × 4 was an extra 6 × 1, and the “sticking out” piece of the 3 × 8 was an extra 2 × 3. They could see the equivalence because 2 × 3 = 1 × 6, since both equal six squares.



Rashawn and Cynthia observed that 7 × 6 = (4 × 6) + (2 × 6) + (1 × 6). They represented this relationship for the class by building the (4 × 6) + (2 × 6) + (1 × 6) with three array cards right on top of the 7 × 6 array card. This pair tried the same approach with another pair of arrays and came up with this equation: (7 × 3) + (7 × 4) = 7 × 7.



Jill and Nora observed that “four 2 × 2s make a 4 × 4.” They demonstrated to the class how four 2 × 2 arrays would be required to cover a 4 × 4 array. They also reported that “a 2 × 2 is a third of a 2 × 6” and demonstrated how three 2 × 2 arrays would cover a 2 × 6 array.

My students also began to think of *generalizations about comparing arrays* that you can make by comparing their dimensions. They put it this way:

- “If both dimensions of one array are bigger than both dimensions of the other array, then the array with the bigger dimensions is larger.” This was demonstrated by placing the array with the bigger dimensions over the smaller one and showing that “no parts were sticking out.”
- “If one dimension of an array is the same as one dimension of another array, then whichever array has the largest other dimension is the bigger one.” For example, when comparing a 3 × 8 and a 4 × 8, students reasoned: “8 = 8. Since 4 > 3, then 4 × 8 > 3 × 8.”
- “When you are comparing two arrays that are the same size (area), the one that has the biggest dimension will also have the smallest dimension. The other array will be closer to a square.”

As I looked back, I was satisfied that we had taken full advantage of the mathematical opportunities inspired by *Count and Compare.* Students who needed practice with multiplication facts were provided with a new way of visualizing the factor pairs and their products. In addition, they and all students were able to use this activity to explore the relationships between different factor pairs as well as the properties of the operation of multiplication.

This case demonstrates Ms. Sajak’s awareness of two things: the differing needs of the students in her classroom and the richness of the mathematical ideas that can be explored through the game Count and Compare. *By asking students to pay greater attention to the comparisons, to be explicit about what they were noticing, and to represent the relationships they found, she was able to address those diverse needs while maximizing the opportunity for all students to explore the mathematical ideas. The challenge remains, for her and her colleague, to effectively communicate this use of the game to the parents of their students.*

Questions for Discussion

1. Ms. Sajak asked her students to find an interesting comparison as they played the game *Count and Compare.* What important mathematical ideas emerged as the students shared, discussed, and represented the comparisons they found? Which ideas might be particularly fruitful to pursue further?

2. How could Ms. Sajak and her colleague highlight the important ideas in *Count and Compare* for parents? How can you communicate to parents the opportunities for learning provided by a game or activity and support such use at home?