Investigations in Number, Data, and Space®

Grade 2 Math Content [1]

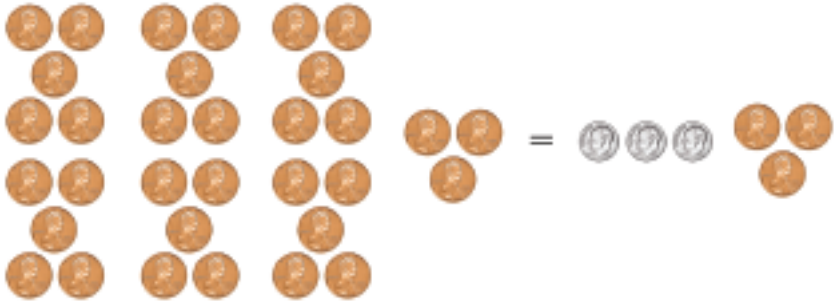
Number and Operations: Whole Numbers

Counting and the Number System

At the beginning of the school year, students have varied opportunities to count sets of objects by ones, write the number sequence, and explore and compare representations of the counting numbers on the number line and the 100 chart. As the school year progresses, most second graders shift from thinking and working primarily with ones to thinking and working with groups of ones. To help them make this shift, students have many opportunities to develop strategies for grouping and for counting by groups. The focus is first on contexts that encourage counting by groups of 2, 5, or 10 and then specifically on groups of 10 and the base ten structure of our number system.

Example: There are 6 people in our classroom. How many legs are there?

Example: If I have 33 pennies and I trade them for as many dimes as I can, how many dimes will I have? How many pennies?



3 groups of 10 pennies and 1 group of 3 pennies equals 3 dimes and 3 pennies.

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

In Grade 2, students work extensively with contexts and models that represent the place value structure of our base-ten number system. They use these contexts to build and visualize how two-digit numbers are composed. For example, 33 cents can be composed of 3 dimes and 3 pennies or 2 dimes and 13 pennies or 1 dimes and 23 pennies.

As an extention of their work with number composition, students investigate even and odd numbers through the context of partners (groups of two) and teams (two equal groups) and then develop definitions of even and odd numbers.

Example: If there are 12 students, can everyone have a partner?

Can there be 2 equal teams?

Is 12 even or odd? Why do you think so?

A student might respond: "Even because everyone would have a partner, and there would be two teams with no leftovers."

Emphases

Counting and Quantity:

- Developing strategies for accurately counting a set of objects by ones and groups
- Developing an understanding of the magnitude and sequence of numbers up to 100
- Counting by equal groups

The Base Ten Number System

- Understanding the equivalence of one group and the discrete units that comprise it

Benchmarks

- Count a set of objects up to 60 in at least one way
- Define even and odd numbers in terms of groups of two or two equal groups
- Recognize and identify coins and their values
- Interpret and solve problems about the number of tens and ones in a quantity
- Know coin equivalencies for nickel, dime, and quarter
- Count by 2s, 5s, and 10s, up to a number

Addition and Subtraction and the Number System

Throughout second grade, students work on making sense of the action of different types of addition and subtraction problems and on developing efficient strategies for solving them and for recording their work. They solve addition and subtraction problems in ways that make sense to them and practice using particular strategies.

Students study two particular strategies for addition –adding tens and ones and adding one number in parts. By the end of the school year, students are expected to have at least one strategy that they can use to accurately and efficiently solve an addition problem.

In Grade 2, students' work with place value becomes the basis for the development of strategies for adding and subtracting 2-digit numbers. The two strategies for addition, adding by place and adding one number in parts, and the strategy for subtraction of subtracting one number in parts, depend on an understanding of how to break numbers into tens and ones.

Adding tens and ones

14 + 32 = 46
10 + 30 = 40
4 + 2 = 6
40 + 6 = 46

Adding on one number in parts

14 + 32 = 46
32 + 10 = 42
42 + 4 = 46

Students consider and practice two strategies for subtraction– subtracting in parts and adding up. By the end of the year they are expected to have one strategy that they can use to accurately solve a subtraction problem.

Subtracting in parts

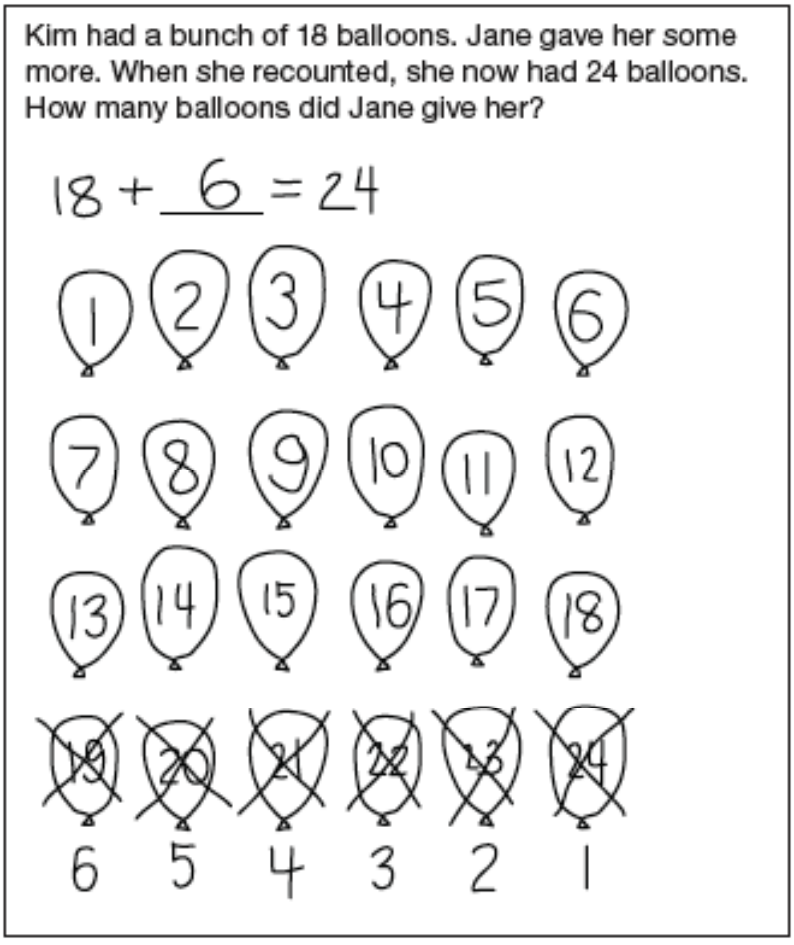
46 – 32 = 14
46 – 2 = 44
44 – 10 = 34
34 – 10 = 24
24 – 10 = 14

Adding up

46 – 32 = 14
32 + 10 = 42
42 + 4 = 46
10 + 4 = 14

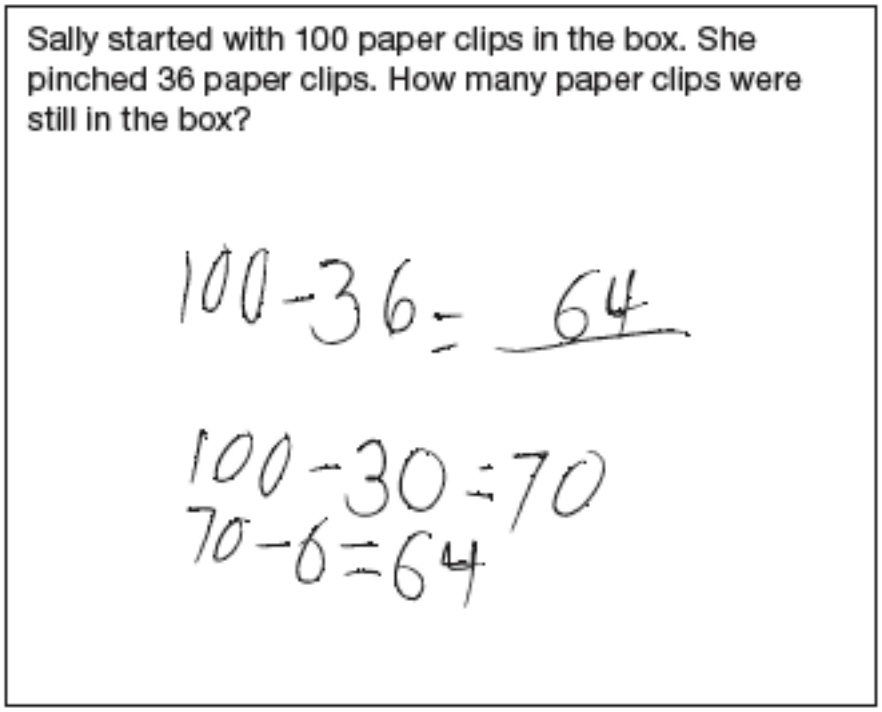
Knowing the single-digit addition combinations helps students as they work to develop efficient strategies for adding and subtracting. Students are expected to be fluent with addition combinations up to 10 + 10 by the end of the year. Students use the relationship between addition and subtraction to solve subtraction problems and to develop fluency with the subtraction expressions related to the addition combinations to 10 + 10.

Students use mathematical tools and representations to model and solve problems to clarify and communicate their thinking. They are encouraged to show their mathematics work on paper in ways that make sense to them; many use some combination of pictures, words, numbers and mathematical symbols and notation.



A student uses pictures, numbers, and notation to show his solution.

Students are expected to use standard notation to write equations to represent addition or subtraction problems. They are also expected to have methods for clearly showing their work, including: sticker notation, numbers, equations, the number line and 100 Chart, or some combinations of these.



A student writes an equation and uses a number line to show her work.

The Algebra Connections pages of each of the four curriculum units that focus on addition and subtraction show how students are applying the commutative and associative properties of addition as they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 5+ 9 = 4 + 10 or 5 + 9 = 5 + 10 - 1). In addition, these pages highlight the work that students do in proving generalizations about adding odd and even numbers.

Emphases

Whole Number Operations

- Using manipulatives, drawings, tools, and notation to show strategies and solutions
- Making sense of and developing strategies to solve addition and subtraction problems with totals to 100
- Understanding the properties of addition and subtraction
- Adding even and odd numbers

Computational Fluency

- Knowing addition combinations to 10+10

Benchmarks (compiled from Units 1, 3, and 8)

- Determine the difference between two numbers (up to 45)
- Interpret addition and subtraction story problems (read a story problem and determine what needs to be figured out)
- Have at least one strategy for solving addition and subtraction (as removal) story problems
- Demonstrate fluency with addition combinations to 10 + 10
- Understand what it means to double a quantity
- Use known combinations to add several numbers in any order
- Interpret and solve subtraction (removal) and unknown change story problems with totals up to 45
- Count on or break numbers apart to add two or more numbers up to a total of 45
- Write an equation that represents an addition or subtraction situation
- Determine the difference between a number and any multiple of 10 up to 100
- Add multiples of 5, up to 100
- Subtract two-digit numbers
- Reason about partners, teams, and leftovers to make and justify generalizations about what happens when even and odd numbers are added
- Add two 2-digit numbers accurately and efficiently

Data Analysis

Students' work on data begins with sorting activities in which they sort objects by their attributes, describing what distinguishes one group from another. This early work in classification provides experience in considering only certain attributes of an object while ignoring others. Students then apply these ideas to categorical data. They *classify* data with many different values, for example the responses to the question, "What is your favorite weekend activity?" by grouping the data into categories (outdoor and indoor activities; or things you do by yourself, things you do with one friend, and things you do with a group). By grouping the data in different ways, students can use the same data to answer different questions.

Example: What is your favorite activity to do on the weekend?

Indoor	Outdoor
reading	soccer
playing board games	walking my dog
	playing basketball

Alone	With More Than 1 Person
reading	soccer
walking my dog	playing basketball
	playing board games

Students use a variety of representations: Venn diagrams, towers of cubes, line plots, and their own representations. By comparing a variety of representations of the same data, they learn how different representations can make different aspects of the data set more visible. Students are introduced to line plots and other frequency distributions in which each piece of data is represented by one symbol (e.g., an X, a square, or a stick-on note). In using this kind of representation, students have to think through the meaning of two ways numbers are used in describing the data: Some numbers indicate the value of a piece of data (I have *8 pockets*); other numbers indicate *how often* a particular data value occurs (*7 children* have 8 pockets). Students describe data by considering the number of pieces of data that occur at each value, the mode and the highest and lowest values.

Through experiencing an entire data investigation from start to finish, students encounter many of the same issues encountered by statisticians as they decide how to collect, keep track of, organize, represent, describe, and interpret their data. They develop their own survey questions about "favorite things", and collect and organize the survey data. They also collect data from different grades about the number of teeth lost and represent and compare these data to their own class data.

Emphases

Data Analysis

- Sorting and Classifying Data
- Representing Data
- Describing Data
- Designing and Carrying Out a Data Investigation

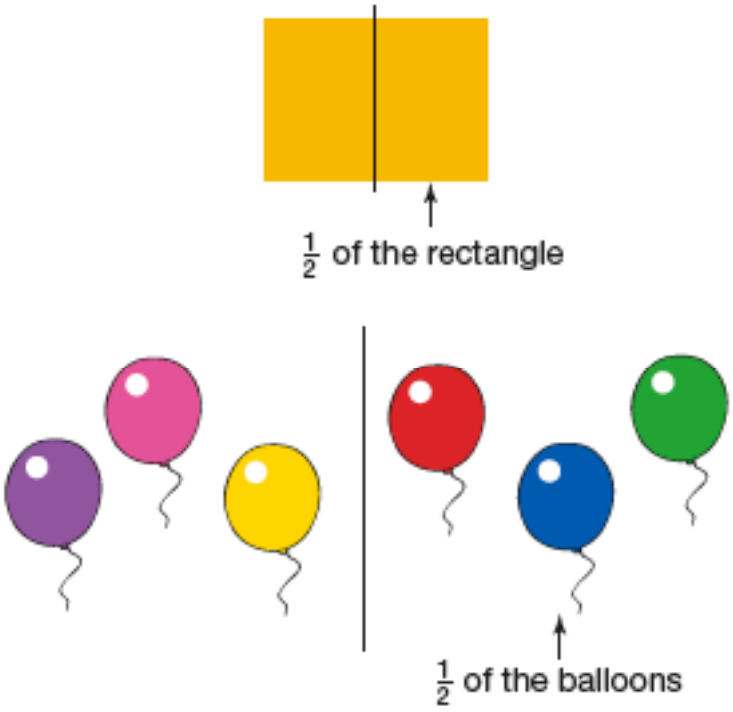
Benchmarks

- Use a Venn diagram to sort data by two attributes
- Identify categories for a set of categorical data and organize the data into the chosen categories
- Order and represent a set of numerical data
- Describe a numerical data set, including the highest and lowest values and the mode

- Read and interpret a variety of representations of numerical and categorical data
- Compare two sets of numerical data

Fractions

Second graders develop an understanding of what fractions are and how they can be used to name quantities. They learn that fractions are quantities that are equal parts of a whole whether the whole is a single object or a set of objects. Students work with 1/2, 1/3, 1/4, 2/3, 2/4, and 3/4 of single objects such as blocks, rectangles, squares and flags. They work with 1/2, 1/3, and ¼ of sets of objects such as balloons, sandwiches and other objects shared among a group of people.



Students learn how fractions are expressed in words—one half, two thirds—and represented using numbers—for example, 1/2, 2/3. They learn that the denominator represents the number of equal parts in the whole and that the numerator represents the number of the equal parts being considered, though they are not expected to use the words *denominator* and *numerator* when describing fractions. Students also learn the notation for mixed numbers through dividing sets. For example, if two girls share three sandwiches, each girl gets 1 1/2 sandwiches.

Emphases

What Fractions Are

- Understanding fractions as equal parts of a whole
- Using terms and notation

Benchmarks

- Identify 1/2, 1/3, and 1/4 of a region
- Find 1/2 of a set of objects
- Recognize that a fraction divides the whole into *equal* parts

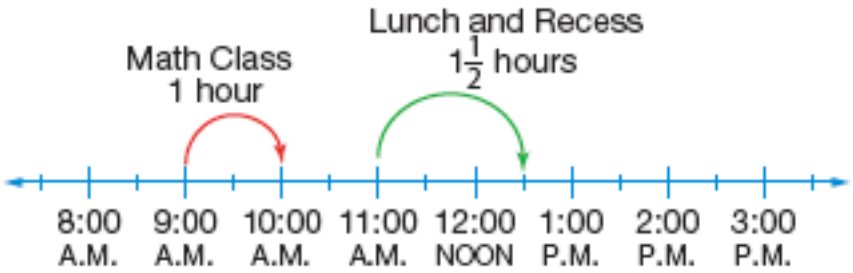
Measurement

In Grade 2, students continue to develop their understanding of length and how it is measured. They first compare lengths of objects by indirect and direct comparison and then use linear units to measure objects and compare measurements. Students learn about iterating a unit and about the relationship between sizes of units and the results of measuring: the smaller the unit, the greater the count for the same length.

By discussing their methods for measuring, students learn that agreeing on a common unit is critical for communicating measurement information to others and comparing results. This leads to work with standard measures: inches, feet, and centimeters.

As students move from using non-standard units (e.g., cubes) to measure objects to using standard tools of measurement such as rulers and yardsticks, the emphasis is on making sure that their use of a measuring tool is connected to making sense of length as an attribute of objects. Students begin their work with standard measurement tools by constructing their own inch rulers, which helps foster not only an understanding of the conventional units, but also the process of measuring with a tool and the principles that underlie the design and use of the tool. Students become accustomed to both systems of measurement: metric and U.S. Standard.

Students practice naming, notating, and telling time on digital and analog clocks. They also work with the idea that time can be represented as a horizontal sequence. Students work with timelines, associating events with a particular time. Students determine intervals of time with an emphasis on starting and ending times on the hour or half hour.



Emphases

Linear Measurement

- Understanding length
- Using linear units
- Measuring with standard units

Time

- Representing time and calculating duration

Benchmarks

- Identify sources of measurement error
- Recognize that the same count of different-sized units yields different lengths
- Recognize that, when measuring the same length, larger units yield smaller counts
- Measure objects using inches and centimeters
- Use a ruler to measure lengths longer than one foot
- Solve problems involving the beginning time of an event, ending time of an event, and duration of the event; given two of these, find the third for events beginning and ending on the hour or half-hour
- Use a timeline to record and determine duration to the hour or half-hour

Patterns, Functions, and Change

Second-grade students explore situations with constant ratios in two contexts: building cube buildings with the same number of "rooms" on each "floor" and covering a certain number of one pattern block shape with another pattern block shape. In both of these contexts, students build and record how one variable changes in relation to the other.

For example: it takes two of the red trapezoids to cover one of the yellow hexagons, 4 trapezoids to cover 2 hexagons and so forth. They use what they are learning to determine quantities later in the series.

Example: How many trapezoids will you need to cover 10 hexagons?

2 trapezoids cover 1 hexagon, 4 trapezoids cover 2 hexagons, and so on.

Tables are introduced and used as a central representation. Organizing data in a table can help students uncover a rule that governs how one quantity changes in relation to another. For example, for every increase of 1 for one quantity, the other quantity increases by 2, (e.g., for each additional trapezoid, from the pattern block set, 2 trapezoids are needed to cover it). Students compare tables that show different relationships, both within the same context and between the two contexts, and notice how different situations can have the same underlying relationship between quantities.

Number of Hexagons	Number of Trapezoids
1	2
2	4
3	6
4	8
5	10

Students also work with number sequences associated with repeating patterns that reveal important characteristics of the pattern and provide an avenue into studying the number sequences themselves.

Example: If the pattern keeps going in the same way, what color will the 12th cube be?

As students explore two-element and three-element repeating patterns, they encounter the odd number sequence, the even number sequence, and three different "counting by 3" sequences. An important part of second-grade students' work on pattern is considering how and why different situations generate the same number sequence.

Emphases

Linear Relationships

- Describing and representing ratios

Using Tables and Graphs

- Using tables to represent change

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

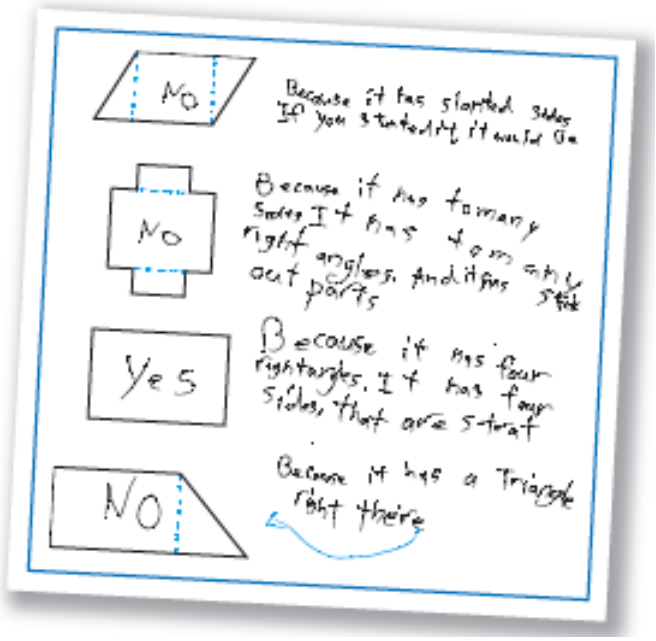
Benchmarks

- Explain what the numbers in a table represent in a constant ratio situation (involving ratios of 1:2, 1:3, 1:4, 1:5, and 1:6)
- Complete and extend a table to match a situation involving a constant ratio
- Extend a repeating pattern and determine what element of the pattern will be in a particular position (e.g., the 16^{th} position) if the pattern keeps going

Geometry

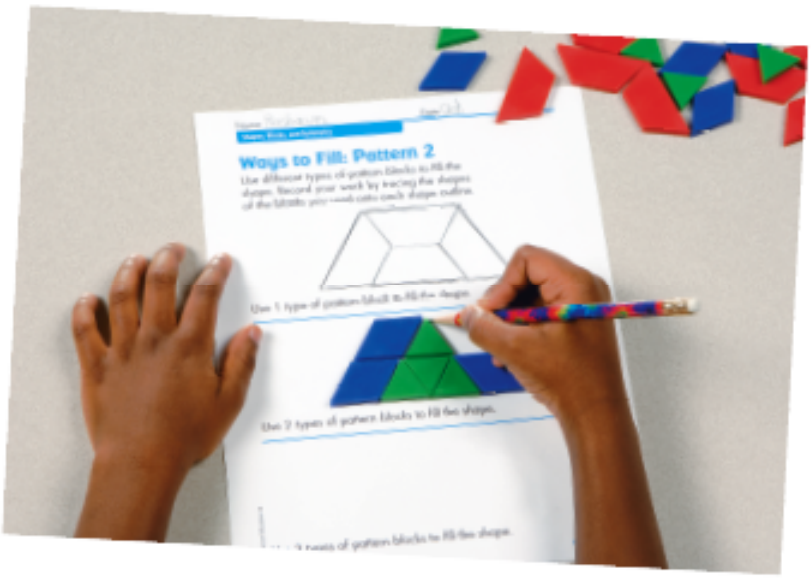
Students describe, sort and compare two-dimensional (2-D) and three-dimensional (3-D) shapes and think about questions like the following: What makes a rectangle a rectangle? How are rectangles different from squares?

Example: Is it a rectangle? Why or why not?



A student explains why shapes are (or are not) rectangles.

Second-grade students study rectangles and rectangular prisms, and consider which properties are important in describing these shapes. They combine and decompose both 2-D and 3-D shapes and explore the relationships between shapes, particularly as they work with pattern blocks and Geoblocks. As they develop knowledge about how shapes are related, they are learning about the important features of shapes.



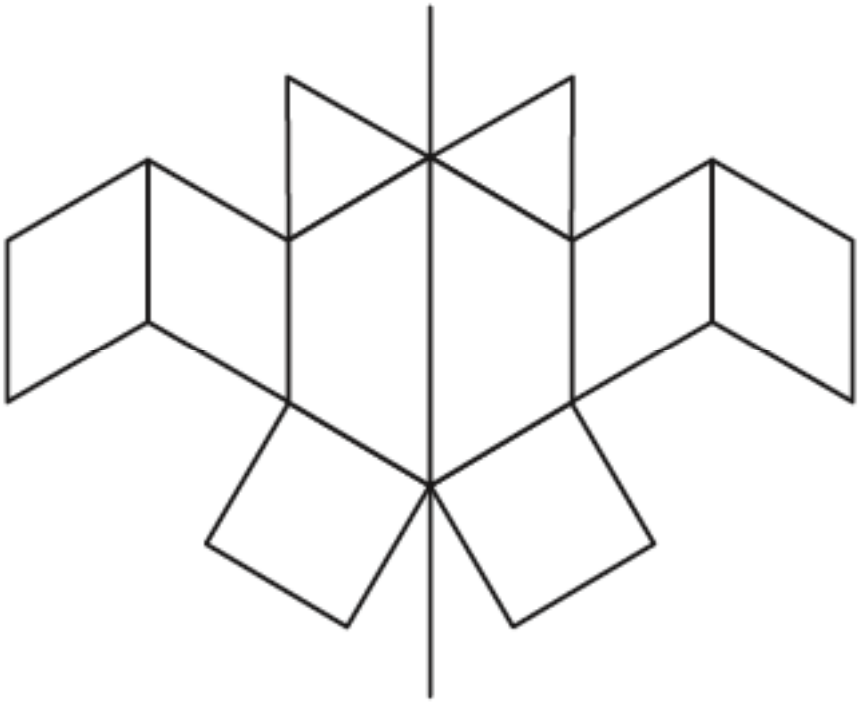
As they use pattern blocks to fill an outline, students find different ways to make the same 2-D shape.

Students find different ways to make a block of a different size.

Students begin their work with arrays, a visual representation that they will continue to use in Grades 3 through 5. As students create rectangles with square tiles, they learn about the structure of an array. In their study of arrays, work in geometry is closely linked with ideas about number. Students develop a variety of numerical strategies, based on the number of tiles in a row and the number of rows, to calculate the area of the rectangle.

A student might respond, "My rectangle has 2 rows with 3 tiles in each row."

Students develop an understanding of mirror symmetry as they identify objects that have mirror symmetry, create patterns and designs, fold and cut paper, and build 3-D structures with mirror symmetry. As they create and investigate symmetrical shapes, they develop language and ideas about what symmetry is and how it behaves.



The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Combining and decomposing 2-D and 3-D shapes
- Describing, identifying, comparing, and sorting 2-D and 3-D shapes
- Exploring mirror symmetry

Area Measurement

- Visualizing the structure of arrays

Benchmarks

- Identify the number of sides of a polygon
- Identify the number of rows and the number of squares in each row in an array
- Identify rectangles as four-sided shapes with four right angles
- Identify the number of faces on a rectangular prism and show which faces are congruent
- Make a symmetrical picture based on an image provided