Dialogue Box

Do We Have *All* the Combinations?

The students in this class have had several experiences with How Many of Each? problems. Prior to this discussion, the class had been trying to find all of the ways to make 9 toys if some were marbles and some were blocks. The students generated a list of 10 possible combinations, but were unable to reach a consensus on whether this list represented all of the ways to make 9. The teacher decided to end the discussion and continue it today.

5 blocks + 4 marbles
2 marbles + 7 blocks
9 marbles + 0 blocks
3 marbles + 6 blocks
6 marbles + 3 blocks
8 marbles + 1 block
1 marble + 8 blocks
0 marbles + 9 blocks
5 marbles + 4 blocks
7 marbles + 2 blocks
Today we found 10 ways. Is that all?

Teacher: The other day, we talked about finding all of the different ways to make 9. Here is the list that we came up with so far. While you were working on How Many of Each? problems, I saw Isabel use an interesting strategy, and I'm going to ask her to share it with everyone.

She gives Isabel a stick of 9 cubes to use for her demonstration. Isabel breaks one cube off.

Teacher: What does that show?

Isabel: 1 marble and 8 blocks.

The teacher records 1 marble + 8 blocks on a new piece of chart paper.

Teacher: How come you broke only 1 off?

Isabel: I don't know . . . because then you could go 2 [breaks off 2 cubes]. And then 3, and then 4, 5, 6, 7, 8.

Teacher: So you had a plan. You wanted to start with 1 on purpose. Who can tell us about Isabel's plan?

Marta: She thought she could go 1 and 8 so she could find all the ways and then 2 and 7.

Teacher: If Isabel's plan were to do 1 and 8 [breaks 1 cube off the tower of 9 to demonstrate], who can show me what our next answer would be?

Neil: 8 and 1.

Teacher: OK, I think you might be thinking of another strategy. Can you hold onto that idea until we finish talking about Isabel's strategy? [Neil agrees and the teacher reiterates Isabel's plan.] So what would be the next answer?

Neil: 2 and 7.

Teacher: And then what?

Paula: Um . . . 3 and 6.

Marta: We're counting up and down. Cool!

1 marble + 8 blocks
2 marbles + 7 blocks
3 marbles + 6 blocks

Teacher: How do you know that these are making 9?

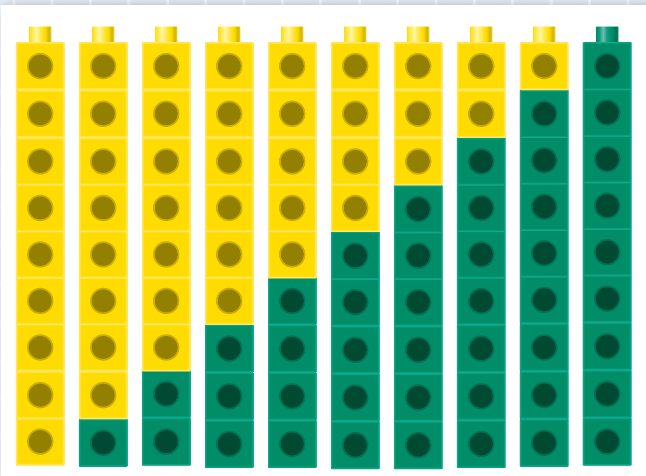
Marta: Because there are 9 cubes.

Teacher: So, is there anything important about the tower of cubes we're using? Why did you want to make sure that there were 9 cubes?

Jacob: Well, it has to have a certain amount. It has to have 9.

Deshawn: [signals that he agrees] It has to have 9 for this problem, but other days we used different numbers so we would need a different size tower.

Stacy: I think I can prove that we have all the ways. [She uses green and yellow cubes to make the figure below on the floor in front of her.]



Diego: Look! It makes a ladder.

Vic: Oh, it's like a staircase!

Shaquana: I can see the opposites [holds up the 2nd and the 9th towers].

Teacher: How does that prove that that's all of the ways?

Stacy: Because I took away all of the ways. Because . . . I can't really explain it, but I know it is.

Teacher: OK, so maybe if you work a little more with it, you'll be able to explain. We have one other idea to talk about. Neil said he would do 8 and 1 next. So we would have 1 marble and 8 blocks and 8 marbles and 1 block. And then what would you do next?

Neil: 7 marbles and 2 blocks and 2 marbles and 7 blocks.

Teacher: And what would come next?

Paula: 6 marbles and 3 blocks and 3 marbles and 6 blocks.

Although many students are not yet able to work systematically to generate all the 2-addend combinations of a number, these students have discovered and discussed 2 ways that they might organize their work to think about whether they have all the possible combinations. Note that even after Stacy has built towers that she is sure show every combination, it is difficult for her to explain how they show that those are all of the ways. Understanding the structure of a problem or strategy often precedes being able to verbalize an explanation of it.