Finding Relationships Among Solutions

After completing the activity for 10 toys in Sessions 1.3 and 1.4, the teacher in this first-grade classroom decided to give her students more practice by creating another problem for them to solve.

The teacher chose a context that her students were investigating in another part of the curriculum—pets. Like other How Many of Each? problems, Twelve Pets: Cats and Dogs is a complex situation that requires coordinating and keeping track of three pieces of information: the total number of cats and dogs, the number of cats, and the number of dogs.

In order to solve the problem, students must count and keep track of a set of objects while comparing the number of objects that has been accumulated so far to the required total. Students ask themselves, "Do I have 12? Do I need more? fewer?" At the same time, students need to keep in mind how the two parts combine to reach that total: I have 14 cats and dogs. If I take away 2 cats and dogs so that I only have 12, how many cats will I have? How many dogs? How can I change what I have to get a different combination?

A few of your students may already see some relationships among the solutions they have found. They may find ways to change one solution to get others, perhaps by varying addends systematically (e.g., 1 cat and 11 dogs, 2 cats and 10 dogs, and so on). Some may use their knowledge of relationships among number combinations: I have 6 cats and 6 dogs. I know 3 and 3 is 6, so if I take 3 cats and make them dogs, I get 3 cats and 9 dogs.

As you observe students working on this problem, you can learn a great deal about how they are thinking about number combinations and the relationships among them. You will also see how they use what they know about number combinations to solve a complex problem.

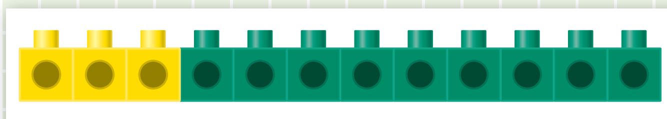
The examples that follow are drawn from a class in which many students found several solutions to the problem. Three approaches are described.

- Finding Several "Unrelated" Solutions
- Breaking the 12 into Parts and Recombining the Parts
- A Systematic Approach

Finding Several "Unrelated" Solutions

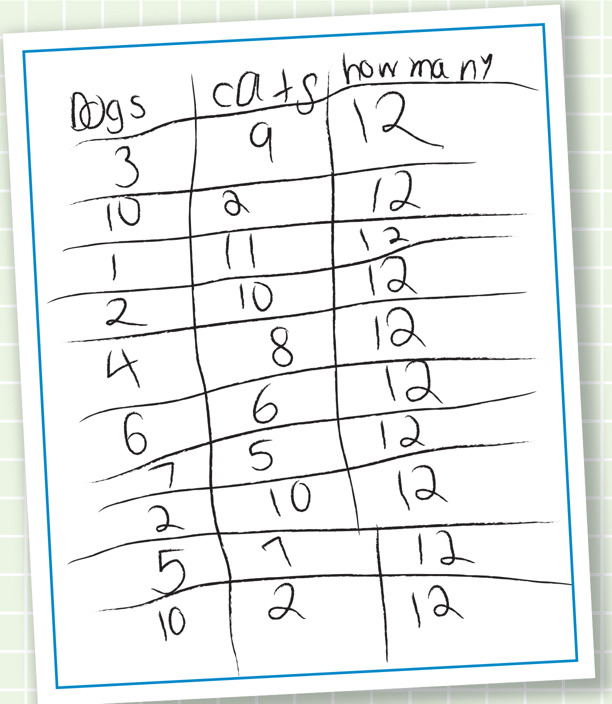
Soon after the class begins working, the teacher visits Nicky. Many of these students have taken an approach similar to Nicky's. They found several solutions by counting and perhaps by using their knowledge of number combinations, but did not seem to notice relationships among the number combinations.

Nicky explains that she has 3 cats, so she built a train of 3 yellow cubes to represent cats. Now she is adding green cubes for dogs. After adding 5 green cubes, Nicky quickly counts the whole cube train and gets 8. She adds 3 more green cubes. This time, she counts more slowly, and gets 11. She adds another green cube, says "12," and then carefully recounts the whole stack to check. She records 3 for cats, and counts the green cubes in the train to determine the number of dogs. She announces that there are 9 and records this.



Nicky explains that she is going to use 2 green cubes to show her grandmother's 2 dogs. She connects 2 green cubes and begins adding yellow cubes, stopping occasionally to count the total number of cubes.

When the teacher returns later in the session, Nicky has recorded 10 solutions, including 2 repeated solutions. She explains that she started out with the actual numbers of pets belonging to people she knew, then decided to choose a number and find how many more she needed to make 12. When the teacher asks whether there are more solutions, Nicky seems uncertain and does not seem to have a way to answer the teacher's question.



Nicky's Work

The problem is at an appropriate level of challenge for Nicky. She is counting up to 12 accurately, is able to keep all the parts of the problem in mind, and has strategies for checking her work. Nonetheless, the teacher decides that Nicky should work on a How Many of Each? problem with a slightly smaller total, 9, for homework. Although Nicky relies on counting strategies to find combinations of 12, the teacher knows that she is familiar with some combinations of smaller numbers. With a smaller total, Nicky may be able to use her knowledge of number combinations to find solutions and may begin noticing some relationships among the solutions she has found.

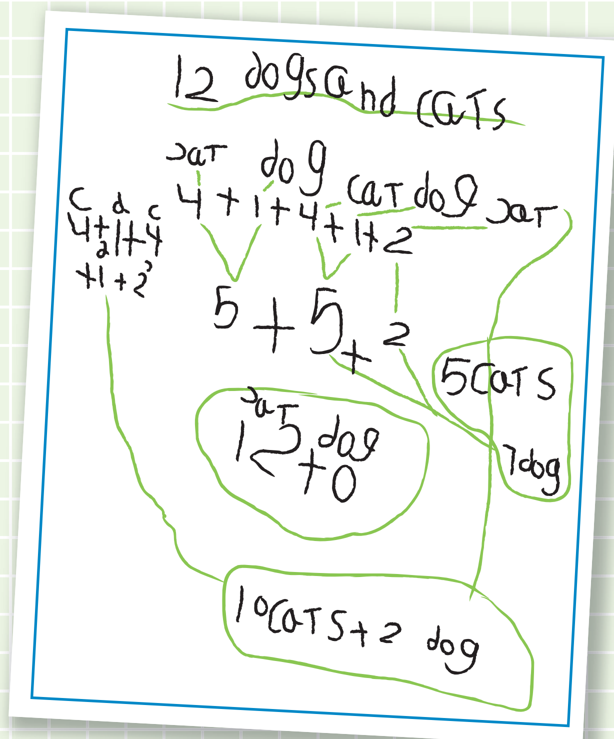
Two students, Lyle and Toshi, are beginning to recognize relationships among number combinations. Their work is described in the following paragraph.

Breaking the 12 into Parts and Recombining the Parts

When the teacher arrives, Lyle is puzzling over what he has recorded. He explains that he found solutions by first breaking 12 into 4 + 1 to make 5, another 4 + 1 to make 5, and then 2. He started by writing these 2 ways of breaking up 12 in two lines on his paper:

$$4 + 1 + 4 + 1 + 2$$
$$5 + 5 + 2$$

By adding all the numbers up again, he got 12 cats and 0 dogs. By recombining the numbers in his first expression (4 + 4 + 2, and 1 + 1), he got 10 cats and 2 dogs. By recombining a 5 and the 2 in his second expression, he got 5 cats and 7 dogs. He tried making every alternate number in the first line a cat (see the upper left of his paper), but ended up with 10 cats and 2 dogs again. Now, he is trying to find another way to combine the numbers to make something different.



Lyle's Work

The teacher knows that Lyle is familiar with many number combinations. However, organizing work carefully and working systematically is hard for him, as it is for most first graders. She briefly considers asking him to recopy his work and find other ways to combine the addends in the expression 4 + 1 + 4 + 1 + 2, but decides instead to encourage him to find more solutions by building on what he knows about number combinations. She hands Lyle a clean sheet of paper and asks him to think of new ways to break 12 into parts.

Teacher: So, when you started out, you found one way to show 12. You did 4 and 1 and 4 and 1, and then 2 more. Is there another way you could break 12 into parts? Other numbers you could add to make 12?

Lyle stares at his paper a moment or two, and then says, "Oh! I could do 2s." He records 2 + 2 + 2 + 2 + 2, notes that he is up to 10, and then adds + 2. He then circles the first three of the 2s, and says he could do 6 + 6. Then, he says, he could try just two of the 2s.

The teacher leaves to visit another student, but intends to return soon to see whether Lyle has found other ways to combine the 2s. She will suggest that he compare his solutions with Stacy's who broke 12 into four 3s and found all the ways to combine the 3s into two addends. She also makes a mental note to encourage Lyle to organize his work more carefully when he is working on simpler problems.

A Systematic Approach

Near the end of class, Toshi announces that he has found all the solutions.

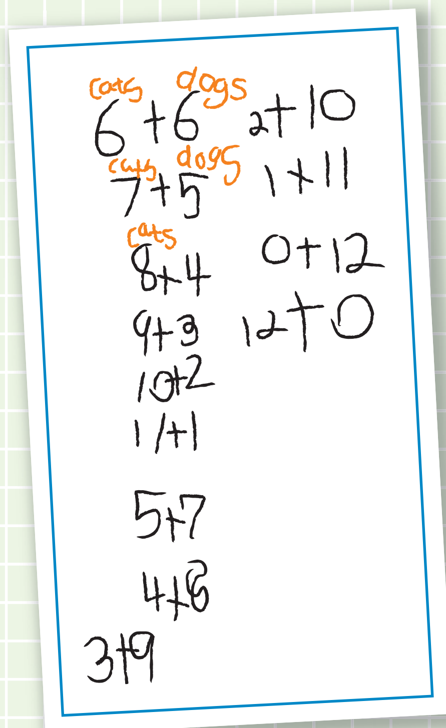
Teacher: How do you know you have them all?

Toshi: I started with 6 cats and 6 dogs because I know 6 and 6 is 12. You can take one cat and make it a dog, and it's 7 cats and 5 dogs, then it goes 8, 9, 10, 11. Then I did it the other way around, and did the zeros at the end.

The teacher points to one of the combinations, 8 and 4, and asks Toshi how he knows it equals 12. He counts "9, 10, 11, 12," keeping track of each number on his fingers.

She points to another combination, 3 and 9. He begins counting on from 3, but loses track of his count and ends up at 11. The teacher asks him to count again, more slowly. This time, he ends up at 12. The teacher asks about several of the other combinations Toshi has recorded. Some he "just knows," some he quickly determines by counting up from one of the numbers, and others—in particular, those that begin with a smaller addend—are more difficult for him.

Although Toshi is not fluent with all of the number combinations he has recorded, he is able to reason about relationships among number combinations, and has a good understanding of why his strategy works: If you increase the number of cats by 1 and decrease the number of dogs by 1, you still have 12. To give Toshi more practice with number combinations and a chance to think more about ways to make 12, the teacher asks him (for homework) to solve the problem with 3 kinds of animals and a total of 12 in all.



Toshi's Work