Teacher Note

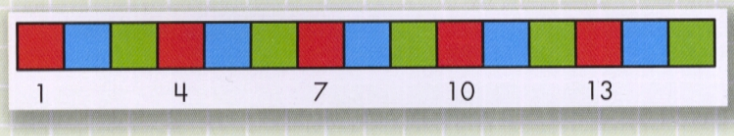
Repeating Patterns and Counting Numbers

Associating the counting numbers with the elements in a repeating pattern provides a context in which students can use familiar repeating patterns to investigate related number sequences.

In the repeating color pattern red-blue-green, red-blue-green, the unit of the pattern has three elements: red is the first element, blue is the second element, and green is the third element. Therefore, when a cube train that is built according to this pattern is numbered in sequence with the counting numbers, starting at 1, each multiple of 3 is associated with green.

The multiples of 3 would be associated in this way with the third element in any repeating pattern with an ABC unit. For example, think about creating a repeating pattern with body movements and counting each movement as you do it. In the body movement pattern, slap knees–clap–tap shoulders, the third element (tap shoulders) always corresponds to a multiple of three.

As students investigate three-element repeating color patterns, they encounter three different "counting by 3" sequences. In the red-blue-green pattern, the greens fall on multiples of 3. The multiples of 3 are an anchor for students in understanding the red-blue-green pattern. After they have established that all greens fall on multiples of 3, they can use this information to figure out the colors associated with other numbers: the numbers associated with the blue cubes are the sequence 2, 5, 8, 11, . . . and the numbers associated with the red squares are the sequence 1, 4, 7, 10, . . .



When students have established that the last element in the pattern (green) always falls on a multiple of 3, and they have generated the number sequences associated with the blue cubes and the red cubes, they use this information to figure out numbers associated with particular cubes in the repeating pattern. For example, to find the color of the 25th cube in the red-blue-green sequence, students' methods may include: 1) putting two 12-cube trains together to get 24 and then seeing that cube 25 would be red; 2) counting by 1s and using the 12-cube train twice, then counting on one more; 3) reasoning that a 12-cube train ends with green, that two 12-cube trains have a total of 24 cubes and that the 24th cube is also green and will be followed by red; or 4) observing that 24 is green because it is a multiple of three (or counting by threes to reach 24) and knowing that red follows green.

Visualizing how the pattern works can become difficult as numbers get larger. Some students try to use the doubling that helped them find the color of the 25th cube (double 12 to get 24 and then add on one more cube) for numbers for which doubling does not work. Kim knows that the 25th cube is red. She reasons, "If you just add another 25 and get 50, it would still be red. So then count on three more cubes, and the 53rd is red, too."

Kim imagines that if she puts together two trains of 25, the 50th cube will also be red. However, 25 is not a multiple of 3, and does not end with a complete unit of red-blue-green. The 25th cube is red. Therefore, she cannot attach another 25-cube train that starts again with red. This breaks the pattern:

Most third graders will not completely sort out when doubling works and when it does not. Work on this idea is a focus in the Grade 4 Patterns, Functions, and Change unit, *Penny Jars and Plant Growth.*

Underlying the number sequences for each color in the pattern is an implicit function. Each green cube falls on a multiple of 3, so we can develop a rule that associates the sequence of green cubes with their numbers: the first green square is associated with 3, the second with 6, the third with 9, and so on. Although students do not make tables or write equations to describe these patterns in Investigation 2, they are developing rules to answer questions (e.g., what color is the 53rd cube in the pattern?) about the underlying functional relationship that is represented in the following table:

Number of green cubes	Position in red-blue-green sequence
1 (first green cube)	3
2 (second green cube)	6
3 (third green cube)	9
4 (fourth green cube)	12
5 (fifth green cube)	15
6 (sixth green cube)	18

Some students may say, "To find the number for the 10th green cube, multiply 10 by 3." This work on developing a rule that relates the two variables. becomes more explicit in Investigation 3. See **Algebra Connections in This Unit,** page 16, for more information about linear functions.