Mathematics

IN THIS UNIT

Fraction Cards and Decimal Squares is the only Grade 4 fractions unit. It builds on the work in the Grade 3 unit, *Finding Fair Shares,* in which students work to understand fractions as equal parts, identify equivalent fractions, and combine fractions with denominators that are the same as well as different.

LOOKING BACK

In Grades 2 and 3, students built an understanding of the meaning of fractions and their relationships. In the context of equal shares, they worked with fractions as parts of a single object ($\frac{1}{3}$ of a brownie), an area ($\frac{1}{3}$ of the surface of a hexagonal pattern block), or a group of things ($\frac{1}{3}$ of the class). Students learned the meanings of the numerator and denominator of a fraction. As they worked with halves, fourths, and eighths, and with thirds and sixths, they gained experience with equivalent fractions, for example, that three sixths and two fourths are both equal to one half of the same whole. Students developed strong visual images of many of these fractions and fraction equivalencies by using pattern blocks and rectangles. They also worked with fractions greater than 1. In Grade 3, students encountered the most familiar decimal fractions, such as 0.5 and 0.25, in the context of money, and became familiar with fraction and decimal equivalents involving halves and fourths.

In this unit, students work with fractions that represent halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths, and decimal fractions in tenths and hundredths.

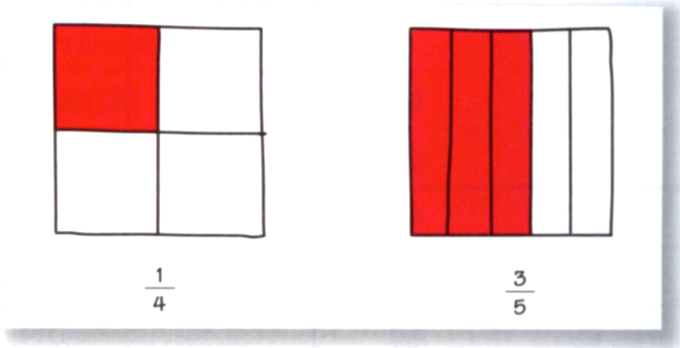
This unit focuses on 3 Mathematical Emphases:

1 Rational Numbers Understanding the meaning of fractions and decimal fractions

Math Focus Points

- Finding fractional parts of a rectangular area
- Finding fractional parts of a group (of objects, people, etc.)
- Interpreting the meaning of the numerator and the denominator of a fraction
- Writing, reading, and applying fraction notation
- Representing fractions greater than 1
- Identifying everyday uses of fractions and decimals
- Reading and writing tenths and hundredths
- Representing tenths and hundredths as parts of an area

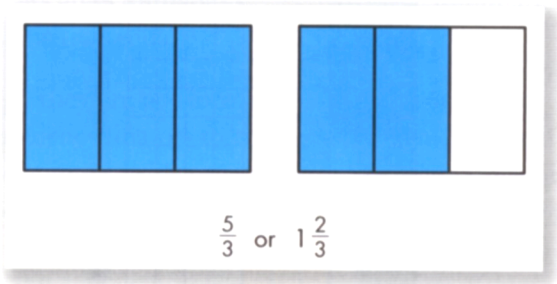
Students continue to work with fractions in the context of area (equal parts of a rectangle) and a group of things (e.g., a fractional part of the class). Through the use of the Fraction Cards, students visualize how fractions are related to 1 as equal parts of a whole and how they are related to one another. These representations are important references for students' understanding of fractions, and help students develop mental images of these numbers.



Fraction Cards

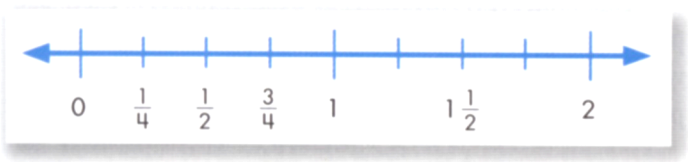
Students continue to focus on the meaning of fractions as equal parts of a whole (thing, area, group). Understanding that fractions refer to equal parts entails both recognizing fractional parts of an area or a group, and being able to divide a whole into fractional parts. It requires an understanding that the denominator of a fraction indicates the number of equal parts into which a whole is divided, and the numerator indicates how many of those parts are being considered.

Students extend their images of equal parts to accommodate fractions that are greater than 1. Students often find the meaning of these fractions, such as $\frac{5}{3}$, more difficult to understand: how can there be five equal pieces if the whole is divided into only three parts? Visualizing fractions that are greater than 1 is an important part of their work in Grade 4. Students learn that these numbers can be written as fractions ($\frac{5}{3}$) or as mixed numbers ($1\frac{2}{3}$).



Students also work with the idea that the same fraction can represent different quantities, depending on the size of the whole. One quarter of the floor of a basketball court is a different size from one quarter of the surface of a sandwich. $\frac{2}{3}$ of my set of marbles is not the same as $\frac{2}{3}$ of all the marbles at the store.

Students extend their use of the number line to represent whole numbers to include numbers less than 1. Although in a practical situation, a fraction represents a part of a particular whole ($\frac{1}{2}$ of the marbles, $\frac{1}{2}$ of the rectangle) and can represent a variety of quantities ($\frac{1}{2}$ of 24 marbles is 12 marbles; $\frac{1}{2}$ of 60 square units is 30 square units), a fraction is also a *number* that always has the same relationship to 1 and to other numbers. Using the number line emphasizes that a fraction is a number. Just as 150 has a certain position on the number line in relation to other numbers, so does a fraction such as $\frac{1}{2}$. As students use and reflect on a variety of representations for fractions, such as rectangles, groups of things, and the number line, they deepen their understanding of the meaning of fractions.



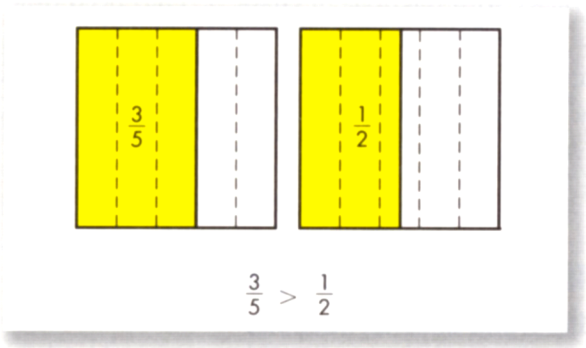
Students are introduced to tenths and hundredths as an extension of the place-value system they have studied for whole numbers. As they represent 0.1 and 0.01 as parts of a rectangle, they see that one tenth is equivalent to ten hundredths and that ten tenths is equivalent to 1. They also relate decimals to familiar fractions and to other decimals; for example, when they represent 0.25, they can see how it is equal to $\frac{1}{4}$ and to $2\frac{1}{2}$ tenths. You might think of students' experience with decimals in Grade 4 as parallel to their experience with fractions in Grade 3. They need time and focus to develop a sound understanding of what these numbers mean (see the **Teacher Note:** Extending Place Value to Tenths and Hundredths, page 157), how they are related to whole numbers, and how they are related to fractions.

2 Rational Numbers Comparing the values of fractions and decimal fractions

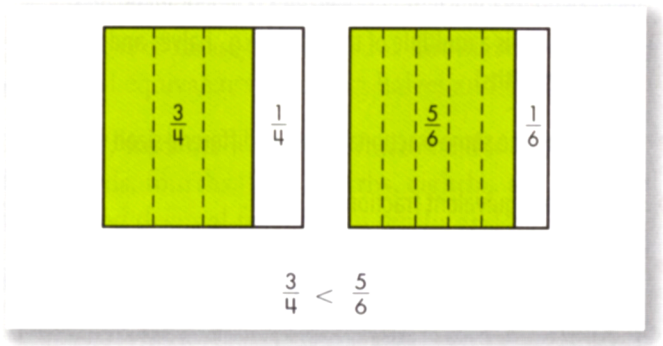
Math Focus Points

- Identifying relationships between unit fractions when one denominator is a multiple of the other (e.g., halves and fourths, thirds and sixths)
- Comparing the same fractional parts of different-sized wholes
- Identifying equivalent fractions
- Ordering fractions and justifying their order through reasoning about fraction equivalencies and relationships
- Representing fractions using a number line
- Comparing fractions to the landmarks 0, $\frac{1}{2}$, 1, and 2
- Ordering decimals and justifying their order through reasoning about representations and the meaning of the numbers
- Identifying decimal and fraction equivalents

As students' repertoire of fractions increases, they draw on their mental images of fractions and on their knowledge of fraction equivalencies and relationships to reason about fraction comparisons and ordering fractions on a number line. They draw on their knowledge of fraction equivalents as part of this reasoning ($\frac{5}{8}$ is more than $\frac{1}{2}$ because $\frac{1}{2} = \frac{4}{8}$ and $\frac{5}{8}$ is greater than $\frac{4}{8}$). They also use the relationship of fractions to landmarks, such as $\frac{1}{2}$ or 1, to determine which of two fractions is greater.



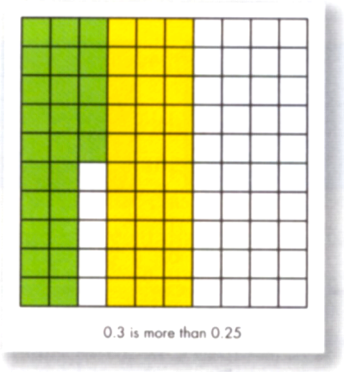
$\frac{3}{5}$ is more than $\frac{1}{2}$ because if you divide something into fifths, it only takes $2\frac{1}{2}$ of those pieces to make $\frac{1}{2}$.



$\frac{3}{4}$ is $\frac{1}{4}$ less than 1. $\frac{5}{6}$ is $\frac{1}{6}$ less than 1. Since $\frac{1}{4}$ is greater than $\frac{1}{6}$, $\frac{3}{4}$ must be less than $\frac{5}{6}$.

In Investigation 2, as students reason about fraction comparisons, they develop and discuss a list of conjectures about fraction relationships (e.g., when the numerator is half the denominator, the fraction is equal to $\frac{1}{2}$; when the numerator is less than half the denominator, the fraction is less than $\frac{1}{2}$).

In their work with decimal fractions, students focus on multiples of 0.1 and multiples of 0.25 as they develop their understanding of the meaning of these numbers. By representing decimal fractions on rectangles divided into tenths and hundredths, they develop visual images of the relationships of these numbers; for example, that 0.9 is more than 0.4 and that 0.25 is more than 0.20.



3 Computation with Rational Numbers Using representations to add rational numbers

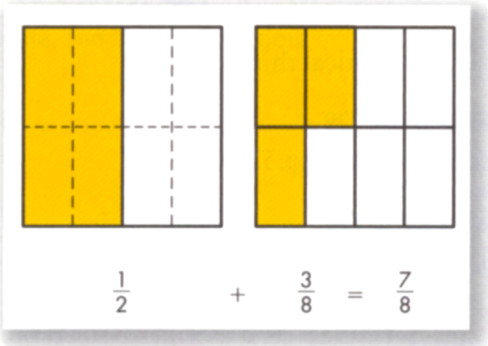
Math Focus Points

- Using representations to add fractions that sum to 1
- Estimating sums of fractions
- Adding fractions with the same and related denominators (e.g., halves, fourths, and eighths; thirds and sixths)
- Estimating sums of decimal numbers
- Adding decimal numbers that are multiples of 0.1 and 0.25 (e.g., 2.3 + 3.25)
- Using representations to combine tenths and hundredths

Students add fractions and decimal fractions by drawing or visualizing representations of these numbers. They use pictures of groups of items or the area of rectangles as well as number lines. This unit does not concentrate on procedures for fraction or decimal computation but on

using good number sense that is based on understandings of the quantities and their relationships. Students draw on their knowledge of fraction equivalencies and other relationships as they add fractions and decimals. For example, by the end of Grade 3, there were some combinations that students already "just knew" from representing fractions with pattern blocks and rectangles. Students continue to develop a repertoire of fraction equivalents and combinations as they find combinations of fractions that equal 1.

Although they are not formally learning about using common denominators to combine fractions, students are developing this idea as they use their knowledge of equivalents to add fractions such as $\frac{1}{2}$ and $\frac{3}{8}$ ("I know that $\frac{1}{2} = \frac{4}{8}$, so $\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$").



As they combine decimal amounts on rectangular grids, each representing one whole, they learn about adding tenths and decimals that have familiar fraction equivalents such as 0.25 and 0.75. Students have a great deal of experience with adding fractions in this unit and supporting their reasoning with visual models, but addition of fractions is not assessed until Grade 5. The major focus of this unit is on solidifying students' understanding of the meaning, order, and equivalencies of fractions and decimals.

Ten-Minute Math activities focus on

- Reading and writing numbers up to 10,000
- Adding multiples of 10 to, and subtracting multiples of 10 from 3- and 4-digit numbers
- Reading and writing decimal fractions and decimal numbers
- Adding tenths and hundredths to, and subtracting them from decimal fractions and decimal numbers
- Describing features of the data
- Interpreting and posing questions about the data

LOOKING FORWARD

In two Grade 5 units, students work with percents, fractions, and decimals as different ways to represent the same quantities. They use equivalencies among these different forms of notation to develop flexibility in understanding and using them. Students order, add, and subtract fractions and order and add decimals by using representations with which they are already familiar (rectangles and number lines). They are also introduced to a new representation (a clock face) for fractions. Students learn about breaking apart fractions and decimals in ways that help them develop addition and subtraction strategies.

Technology Note

Using the *LogoPaths* Software If you are using the *LogoPaths* software this year, give students ongoing access to the computers **outside of math time** during this unit. *LogoPaths* Resource Masters (M1–M6) offer continued work with *Missing Measures* and *Steps* activities. Students can also continue to play *Mazes* and spend time working with the *Free Explore* option of the software. See **Part 5: Technology in *Investigations*** in *Implementing Investigations in Grade 4:* Introducing and Managing the *LogoPaths* Software in Grade 4.