Investigations in Number, Data, and Space®

RESEARCH

Computational Algorithms and Procedures

Learning, with understanding, about computation and algorithms to solve problems is a natural and fertile site for learning about the detailed nature of numbers and operations and their various models and representations. [Hyman Bass, "Computational Fluency, Algorithms, and Mathematical Proficiency: One Mathematician's Perspective," Teaching Children Mathematics, Feb. 2003, p. 326].

In the revised version, Investigations will continue to have a major emphasis on developing computational fluency for whole-number operations through work that supports students to understand the meaning of the operations and the structure of the Base-10 number system. This work includes becoming fluent with "basic facts," developing efficient, accurate, and flexible methods for solving computation problems, making and justifying generalizations about the operations, and analyzing and comparing a variety of algorithms. Although each student may settle on one primary method for solving problems in each operation, students are expected to study more than one algorithm for each operation. Students study a variety of algorithms for a number of reasons, including:

• Justifying different algorithms or procedures provides insight into characteristics of the operations. Comparing algorithms also illuminates underlying properties.

• Access to different algorithms and procedures leads to flexibility in solving problems. One method may be better suited to a particular problem; one method can be used to check another.

In students' study of operations in the Investigations curriculum, there is a time when they build strategies they are comfortable with, that make sense to them, and that they can gradually apply to harder problems. They use what have been called more "transparent" algorithms and methods, in which the properties of the operations and the place value of the numbers are not hidden by shortcut notation.

An algorithm that will be used to program a machine must be efficient to achieve computational speed, but does not have to be transparent. If humans will learn and use the algorithm, however, transparency and ease of use are important [Ibid., p. 324].

For example, in these two methods of solving a multiplication problem, the values of all the partial products and the application of the distributive property can be seen clearly.

Method 1	Method 2	
59	59 x 72 = (60 x 72) - (1 x 72)	
x 72		
18	4200	60 x 70
100	+ 120	60 x 2
630	4320	
3500	- 72	1 x 72
4248	4248	

Later, after students have worked with such methods and understand them well, they practice the ones they use routinely so they can use them efficiently.

Still later, when they are firmly grounded in understanding the operation and solving problems accurately, efficiently, and flexibly, they study and compare algorithms, including the U.S. traditional algorithms for addition, subtraction, and multiplication, in order to learn about the mathematical relationships underlying them. In particular, students' algorithms for solving addition and multiplication problems are usually quite similar to the historically taught algorithms in the United States; with a solid foundation in more transparent methods, students are ready to learn how the shorthand notation of these algorithms relates to the procedures they are carrying out but notating differently.

Students can come to appreciate the economy and generality of these traditional algorithms, which are an important achievement of mathematics, and use them along with other algorithms and methods. These algorithms are part of the knowledge in the world that students encounter, in their families, and elsewhere. They should be familiar, not mysterious. As with other algorithms and methods they use, students should understand how these algorithms work and what the notation represents.
