Investigations in Number, Data, and Space®

Math Content by Strand[1]

Early Algebra: Numbers and Operations

Introduction[2]

Algebra is a multifaceted area of mathematics content that has been described and classified in different ways. Across many of the classification schemes, four areas foundational to the study of algebra stand out: (1) generalizing and formalizing patterns; (2) representing and analyzing the structure of number and operations; (3) using symbolic notation to express functions and relations; and (4) representing and analyzing change.

In the *Investigations* curriculum, these areas of early algebra are addressed in two major ways: (1) work within the counting, number, and operations units focusing on generalizations that arise in the course of students' study of number and operations and (2) a coherent strand, consisting of one unit in each grade, K-5, that focuses on patterns, functions, and change. This essay will focus on the former. For information on the latter, see http://investigationsnew.dev.terc.edu/library/curric_math/patternsfunctions_2ed.pdf.

Making General Claims About Numbers and Operations

Each *Investigations* unit on counting, numbers, and operations includes a focus on generalizing about numbers and operations. Even in beginning work with numbers and operations in Kindergarten and Grade 1, students are already noticing regularities about numbers and operations. For example, in the K-1 game *Double Compare*, each student in the pair turns over two number cards. The student with the greater sum says "me." In this early work, before students know their single-digit addition combinations, most students are counting all or counting on to determine the sum. But consider how students are reasoning in the following brief episode:

> Bridget and Siva are playing *Double Compare*. Bridget draws a 5 and a 2; Siva draws a 5 and a 3 and immediately says "me," indicating that he has the greater sum. Siva usually counts both amounts by ones to get the sum, so the teacher asks him, "How did you know you have more?" Siva responds, "Because I have a 3 and she had a 2 and 3 is bigger." Bridget is nodding vigorously and adds, "The 5s don't count."

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

[2] Introduction excerpted from the Teacher Note, Foundations of Algebra in the Elementary Grades, *Implementing Investigations in Grade Kindergarten, 1, 2, 3, 4, and 5*

How are the students in this episode figuring out who has the greater sum? Why does Siva only compare 3 with 2, and what does Bridget mean when she says the "5s don't count"? Implicit in these students' work is a general claim about adding numbers that many young students use: If you are comparing two addition expressions, and one of the addends in the first expression is the same as one of the addends in the second, then you need only compare the other two addends to determine which expression has the greater sum. This is a mouthful to put into words, and students might not be able to articulate this idea completely; nevertheless, they are reasoning based on this idea. In later years, this idea can be represented with symbolic notation:

For any numbers a, b, c, and d when $a = c$ and $b < d$, then $a + b < c + d$.

$a = c$	$b < d$	$a + b < c + d$
$5 = 5$	$2 < 3$	$5 + 2 < 5 + 3$

Part of the teaching work in the elementary grades is to help students articulate, represent, investigate, and justify general claims that arise naturally in the course of their work with numbers and operations. In each of the number and operations units in Grades K-5, the Algebra Connections essay highlights several of the general ideas about properties and relationships relevant to the work in that curriculum unit, with examples of how students think about and represent them. For an example from the Kindergarten curriculum, see http://investigationsnew.dev.terc.edu/library/curric_gl/sample_gk_u2_ae.pdf. Investigation and discussion of some of these generalizations are built into unit sessions; for others, Algebra Notes alert the teacher to activities or discussions in which these ideas are likely to arise and could be pursued.

In the course of articulating, representing, and justifying their ideas about such general claims, students in the elementary grades are beginning to engage in proving—a central part of mathematics. They consider the questions: Does this generalization apply to *all* numbers (in the domain under consideration, usually whole numbers)? Why does it work? How do you know? In two of the number and operation units in each grade, 2-5, you will find a Teacher Note that focuses on proof and justification. These Teacher Notes provide examples of the ways that students at that grade level engage in proving and how their proofs, based on representations, are related to the proofs a mathematician might carry out.

For most adults, notation such as use of variables, operations, and equal signs is the chief identifying feature of algebra. The notation, however, expresses rules about how operations work; rules that students can reason out for themselves. This *reasoning*—about how numbers can be taken apart and put together under different operations—*not* the notation is the primary work of elementary students in algebra.

Examples of the general claims about number and operations that are highlighted in the curriculum are described below.

Kindergarten

Over the course of the year, Kindergarten students encounter a number of general ideas as they count and begin their work on addition and subtraction. For example, students develop ideas about how numbers describe the size of a set and that the number of objects in a set is fixed no matter how it is arranged and counted, and different sets may have the same number of objects. As kindergarteners repeatedly count a set made up of things in two different colors (e.g., a set of 8 rods; 5 of them red and 3 of them yellow), they begin to make the following generalization:

- When counting a set of objects, it does not matter in what order one counts them; the result is the same no matter how many objects are in the set.

Students' observations about the constancy of the total, regardless of the order in which they count a set of objects, lays the foundation for what they will in later years call the commutative property of addition.

This property comes up again as students play *Toss the Chips*. As students toss 6 chips to see how many land showing red and how many land showing yellow, they notice that there is something the same about one student's 2 red and 4 yellow and another student's 4 red and 2 yellow. Both arrangements separate the set into one part with 4 objects and the other part with 2. As students notice other arrangements of "opposites" such as 1 red and 5 yellow and 5 red and 1 yellow, the teacher comments, "I wonder if we will see opposites when we play *Toss the Chips* again." This sets the stage for raising the question about whether this property is special about the particular numbers the students have used or if it will happen with other numbers, too.

Kindergarten students have many opportunities to work with problems and contexts that involve combining and decomposing quantities: six tiles can be arranged as a row of 4 and a row of 2, a row of 5 and a row of 1, two rows of 3, etc; when 6 two-color counters are tossed, they may land showing 4 red and 2 yellow, 5 red and 1 yellow, 3 red and 3 yellow, etc. This work helps them begin to make the following generalization:

- The same number can be decomposed in different ways (i.e., Six can be a group of 4 and a group of 2. Six can also be a group of 5 and a group of 1.)

As students come to recognize that these numerical relationships remain constant across different contexts, they develop an understanding of the operation of addition. That is, what had once been very different actions for them—arranging a set of tiles, tossing a set of chips, and collecting counters—is now subsumed under one operation. Furthermore, through their engagement with various additive contexts such as story problems in which quantities are combined, students begin to generalize about how the operation behaves. For example, consider a problem where Yoshio *"found 3 balls by the swings"* and then found *"2 more by the slide."* The teacher's question, "Do you think Yoshio had more than 3 balls or fewer than 3 balls at the end of the story?" draws students' attention to an important feature of the addition of counting numbers: when quantities are added, the result is greater than either of the addends.

This is in contrast to some other contexts students encounter. For example, consider a problem in which "*Mia had 5 grapes. Then she ate 1 of the grapes."* Before asking students to find the

number of grapes Mia has left, the teacher asks, "Does Mia have more or fewer than 5 grapes at the end of the story?" In this problem, which involves separating or removing—a context students will later associate with subtraction—the result is less.

Thus, the generalizations Kindergarten students are approaching might be stated as follows:

- When adding (the counting numbers starting with 1), the sum is greater than any of the addends. When subtracting, the difference is less than the amount from which you're subtracting.

Grade 1

Throughout the course of Grade 1, students encounter a number of general ideas as they work with counting, numbers, and operations. For example, as students work on activities such as the "How Many of Each?" problems, they recognize that 5 blocks and 4 marble and 4 blocks and 5 marbles both result in a total of 9 objects. They encounter this idea again when they play games like *Counters in a Cup* and notice that 2 beans outside the cup and 7 beans under the cup result in the same total as 7 beans outside the cup and 2 beans under the cup. These activities lead to a beginning understanding of what in later years they will call the commutative property of addition.

- Two numbers added in either order yield the same sum: 2 + 7 = 7 + 2

As students play *Counters in a Cup* and discover that the number of the counters in the cup can be determined by either counting up or counting back (or by adding up or subtracting back), they encounter the inverse relationship between addition and subtraction. Students also encounter the inverse relationship between addition and subtraction as they work on related story problems. For example:

> *Vic and Libby were in charge of collecting pencils during cleanup time. Vic found 7 pencils and Libby found 3. How many pencils did they collect?*
>
> *Libby and Vic put the 10 pencils in a pencil basket. Then Diego came by and took 3 of them for the kids at his table. How many pencils were left in the basket?*

In the first problem, 7 and 3 are joined to make 10; in the second, 3 is removed from 10, leaving 7. Students are asked, Does the first problem help solve the second? As students do the important work of examining the relationship between these two problems, some students may say, "Seven and three come together to make 10. If 3 goes away, 7 is left and that's the answer." In other words, they are employing the following generalization:

- Addition and subtraction are related. If adding two numbers gives a certain sum, then subtracting one of the addends from the sum results in the other addend: 7 + 3 = 10; 10 – 7 = 3; 10 – 3 = 7

Students apply the commutative property and their understanding of the inverse relationship between addition and subtraction, as they develop strategies for solving addition and subtraction

problems. They also apply the inverse relationship between addition and subtraction and their understanding that the same number can be decomposed in different ways when they create equivalent expressions in order to solve a problem (e.g., 6 + 4 = 5 + 5 and 8 + 5 = 10 + 3) or when they use addition combinations they know to solve more difficult problems (e.g., since 5 + 5 = 10, 5 + 6 must equal 10 + 1, or 11).

Other generalizations highlighted in first grade include:

- If one number is greater than another, and the same number is added to each, the first sum will be larger than the second: 3 + 5 > 2 + 5

- If 1 is added to an addend, the sum increases by 1. Or more generally, if any number is added to (or subtracted from) an addend, the sum increases (or decreases) by that number: 5 + 5 = 10, so 5 + 6 = 11; 5 + 5 = 10, so 5 + 4 = 9

- If an amount is added to one addend and subtracted from another addend, the sum remains the same: 6 + 6 = 12; 7 + 5 = 12

- Subtraction "undoes" addition, as in 22 + 8 – 8 = 22.

- Any missing addend problem can be solved by subtraction. Conversely, any subtraction problem can be solved as a missing addend:
  10 + <u>6</u> = 16; 16 – <u>6</u> = 10 or 16 – 10 = <u>6</u>

Grade 2

Throughout the course of Grade 2, students encounter a number of general ideas as they work with counting, numbers, and operations. In Kindergarten and first grade, students noticed that when adding two numbers the sum remained the same even if the order in which they were added changed. In Grade 2, the question of whether order matters when adding is extended to problems involving three or more addends. As students find the sum of 4 + 3 + 6 and move around cube towers to show that nothing is being added or taken away when the order is changed to 4 + 6 + 3, they are applying the commutative property of addition more than once. As they solve number strings with multiple addends and change the order and groupings to make the problems easier to solve, they are beginning to develop an understanding of what in later years they will call the associative property of addition.

- No matter how many addends there are, the grouping and the order in which they are added can be changed and the sum stays the same: 15 + 8 + 3 + 7 + 12 + 5 = (15 + 5) + (8 + 12) + (3 + 7)

Students apply the commutative and associative properties of addition as they develop strategies for solving addition problems. They use the inverse relationship between addition and subtraction when they use subtraction to "undo" addition as they create expressions to equal Today's number (e.g., 4 + 30 – 30 = 4; 4 + 20 – 20 = 4). As in first grade, students also encounter the inverse relationship between addition and subtraction as they work on related story problems. For example:

Kira picked 6 flowers in the morning and 12 flowers in the afternoon. How many flowers did she pick that day?

There were 18 grapes on a plate and then Jake ate 12 of them. How many grapes were left on the plate?

After solving and discussing these problems, one second grader said, "When two numbers come together and one goes away, the one that's left is the answer. In other words, Sawyer employed the following generalization:

- Addition and subtraction are related. If adding two numbers gives a certain sum, then subtracting one of the addends from the sum results in the other addend: 6 + 12 = 18; 18 – 12 = 6; 18 – 6 = 12

Students also apply the inverse relationship between addition and subtraction when they solve problems with an unknown change and recognize that these problems can be solved using either addition or subtraction. Consider the following problem:

I had 18 pennies in my pocket. My mom gave me some more pennies. Then I had 21 pennies. How many pennies did my mom give me?

In one second grade class, about half the students saw this problem as subtraction (21 – 18 = ___), the other half as a missing addend 18 + ____ = 21).

In second grade, students work on articulating and applying generalizations about subtraction. Discussions about subtraction can focus students' attention on how subtraction behaves differently than addition. This is important because students frequently lose sight of the fact that generalizations apply to a *particular operation*. For example, once they recognize that order doesn't matter in addition (e.g., 3 + 4 = 4 + 3), they might assume that order doesn't matter in subtraction, though, of course this is not the case: 4 - 3 ≠ 3 – 4. Second grade students examine the question of order in subtraction in the context of expressions for *Today's Number*. For example, when looking at combinations of two addends to make the number 7, the teacher states, "Remember when we discovered that, with addition, you can change the order of the numbers and get the same answer? …Would that also be true for subtraction? Can we take one of the ways we found to make 7, say 9 – 2, turn it around, and get the same answer? Using tools and representations, such as cubes and number lines, students consider this question. This is a complex idea that students will revisit in later grades when they begin work with negative numbers. As long as the students recognize that 9 – 2 does not equal 2 – 9, the answer to 2 – 9 (or a similar problem) does not need to be resolved.

Another way in which subtraction differs from addition has to do with the effect of changing numbers in a subtraction expression. In an addition expression, such as 4 + 10, increasing either addend increases the sum; likewise, decreasing either addend decreases the sum. However, in a subtraction expression such as 14 – 10, decreasing 14 decreases the difference, but decreasing 10 *increases the difference.* For example, in solving 14 – 9, a second grader might reason, "I know

how to subtract 10: 14 – 10 = 4. To do 14 – 9, I took 1 less away, so I'm left with 1 more. 4 + 1 = 5." Although this student is working to solve a particular problem, the explanation includes an important generalization:

- If 1 *less* is subtracted from a number, the result is 1 *more;* if 1 *more* is subtracted from a number, the difference is 1 *less*.

Other generalizations highlighted in second grade include:

- Two numbers added in either order yield the same sum: 8 + 5 = 5 + 8
- If 1 is added to an addend, the sum increases by 1. Or more generally, if any number is added to (or subtracted from) an addend, the sum increases (or decreases) by that number: 12 + 12 = 24, so 12 + 13 = 25; 12 + 12 = 24, so 12 + 11 = 23
- Subtraction "undoes" addition, as in 22 + 8 – 8 = 22.
- When two even numbers are added, the sum is even. When two odd numbers are added, the sum is even. When an even number and an odd number are added, the sum is odd: 4 + 4 = 8; 3 + 5 = 8; 4 + 5 = 9

Grade 3

Throughout the course of Grade 3, students encounter a number of general ideas as they work with numbers and operations. Students apply the commutative and associative properties of addition as they develop strategies for solving addition problems. For example, as third graders encounter problems that involve adding larger, multidigit numbers, they recognize that they can decompose and recombine the addends in a variety of ways and commonly develop strategies in which the parts are put in a different order or different groupings to more easily find the sum.

Grade 3 students call upon their understanding of the inverse relationship between addition and subtraction as they practice subtraction combinations related to the additions combinations to 10 + 10 (e.g., if 7 + 5 = 12, then 12 – 5 = 7), express the difference between two numbers as a missing addend, determine the difference between a given and 100 or 1,000 by adding up or subtracting back, and create equivalent expressions in order to solve a problem (e.g., 48 + 72 = 50 + 70).

Addition and subtraction generalizations highlighted in Grade 3 include:

- Two numbers added in either order yield the same sum: 8 + 5 = 5 + 8
- No matter how many addends there are, the grouping and the order in which they are added can be changed and the sum stays the same: 73 + 26 = (70 + 3) + (20 + 6) = (70 + 20) + (3 + 6)

- Addition and subtraction are related. If combining two addends gives a certain sum, then subtracting either addend from that sum results in the other addend: 57 + 43 = 100; 100 – 43 = 57; 100 – 57 = 43

- If the same amount is subtracted from a quantity that has been increased by a second amount, the difference will be greater by that second amount e.g.,
  If 126 – 75 = 51, then (126 + 100) – 75 = (51 + 100) or 226 – 75 = 151

- If more is subtracted from a number, the difference is less. The difference is exactly the amount less as the extra amount that was subtracted: 57 – 25 = 32; 57 – (25 + 2) = 32 - 2

In their work on multiplication, third graders develop a sense of how this operation works by using a variety of representations such as skip counting, story contexts, arrays, and drawings of multiplicative situations. As students construct arrays for different numbers in the *Arranging Chairs* activity, they notice that some of the factor pairs on their lists are reversals; for example 18 chairs can be arranged into a 3 x 6 array and into a 6 x 3 array. This leads to a developing understanding of the commutative property of multiplication.

When given a multiplication combination that they do not know, students are encouraged to build up the answer by beginning with a part of the problem that they do know. The student who uses 5 x 4 and 4 x 4 to find the product of 9 x 4 [9 x 4 = (5 x 4) + (4 x 4) = 20 + 16 = 36] is using the distributive property, as is the student who starts with 10 x 4 and then subtracts one 4 [9 x 4 = (10 x 4) – (1 x 4) = 40 – 4 = 36]. The student who knows the product of 4 x 9 and "turns that around" to find the product of 9 x 4 is calling upon the commutative property of multiplication. The student who starts with 9 x 2 and then doubles that product to find the product of 9 x 4 [9 x 4 = 9 x (2 x 2) = (9 x 2) x 2 = 18 x 2 = 36] is using the associative property of multiplication. Students are not expected to know the names of or fully understand these properties. Students use them implicitly as they learn factor combinations and perform calculations to solve problems. This work lays the foundation for understanding these three properties of multiplication in the future.

When third graders solve division problems using a multiplication strategy, they are employing the inverse relationship between multiplication and division. For example, consider the following problem:

> *There are 28 desks in the classroom. The teacher puts them in groups of four. How many groups of desks are in the classroom?*

Some third graders may solve this problem by counting by 4s to 28. Others may use reasoning based upon multiplication combinations they know. For example, "I know that five groups of four would be 20 chairs. Two more groups would be 8 more chairs, so there are seven groups in all."

Generalizations about multiplication and division highlighted in Grade 3 include:

- Two numbers multiplied in either order yield the same product: 6 x 7 = 7 x 6
- Multiplication problems can be solved by breaking apart a factor to begin with a part of the problem that is known: 9 x 4 = (5 x 4) + (4 x 4); 9 x 4 = (10 x 4) – (1 x 4); 9 x 4 = (9 x 2) x 2
- Multiplication and division are related. If multiplying two numbers gives a certain product, then dividing that product by one of the original factors results in the other factor: 6 x 7 = 42; 42 ÷ 6 = 7; 42 ÷ 7 = 6

Grade 4

Throughout the course of Grade 4, students encounter a number of general ideas as they work with numbers and operations. Students apply the commutative and associative properties of addition as they refine their strategies for solving addition problems with multidigit numbers. They apply the inverse relationship between addition and subtraction as they solve subtraction problems in which addition strategies are useful, express the difference between two numbers as a missing addend, and add up or subtract back to determine the difference between a given number and 1,000.

Algebraic ideas underlie the work fourth graders do as they investigate and articulate generalizations that arise when they create equivalent expressions in order to solve a problem. For example:

- If the same amount is added to (or subtracted from) both numbers in a subtraction problem, the difference does not change: 124 – 89 = 125 – 90 = 35

Part of the work of fourth grade is helping fourth graders verbalize the ideas that lay the foundation for algebra. For example, as students apply the commutative and distributive properties and the inverse relationship between multiplication and division in solving multiplication problems, they are challenged to use story problems, arrays, and arithmetic expressions to provide arguments explaining how they know a given statement is always true. When such representations are shared, students are asked to explain the connections among the representations, contexts, and expressions used and to think about how these representations can be used to justify a general claim.

Fourth graders also use arrays, story contexts and other representations as they consider the following questions about multiplication:

- If a number is a factor of a second number, are all the factors of the first number also factors of the second number?
- If one factor in a multiplication expression is halved and another factor is doubled, what is the effect on the product?

When fourth graders solve division problems using a multiplication strategy, they are employing the inverse relationship between multiplication and division. For example, consider the following problem:

> *There are 168 fourth graders and for field day we need to have teams of 14. How many teams can we make?*

Some fourth graders may solve this problem using the distributive property of multiplication and reason, "14 x 10 equals 140. Two more 14s is 28. That is 168. That means 10 + 2 or 12 teams." Other students may use different groupings of 14 until they reach 168.

Generalizations about multiplication and division highlighted in Grade 4 include:

- Two numbers multiplied in either order yield the same product: 32 x 20 = 20 x 32
- Multiplication problems can be solved by breaking apart a factor to begin with a part of the problem that is known: 18 x 9 = (10 x 9) + (8 x 9); 18 x 9 = (20 x 9) – (2 x 9)
- Numbers can be broken into parts to be multiplied, but each part of each number must be multiplied by each part of the other number: 38 x 12 = (30 + 8) x (10 + 2) = (30 x 10) + (30 x 2) + (8 x 10) + (8 x 2)
- A factor of a number is a factor of multiples of that number: 3 is a factor of 15; 30 is a multiple of 15, so 3 is a factor of 30.
- Multiplication and division are related. If multiplying two numbers gives a certain product, then dividing that product by one of the original factors results in the other factor: 12 x 14 = 168; 168 ÷14 = 12; 168 ÷ 12 = 14
- If one of the factors in a multiplication problem is doubled (or tripled) and the other is halved (or divided by 3), the product remains the same: 164 x 4 = 328 x 2

Grade 5

Throughout the course of Grade 5, students encounter a number of general ideas as they work with numbers and operations. They create equivalent problems in addition and subtraction. They apply the inverse relationship between addition and subtraction as they solve subtraction problems that involve comparison or missing addends, add up or subtract back to determine the distance of a given number is from 10,000, and play games such as *Close to 7,500.*

As fifth graders perform calculations, they frequently make claims about numerical relationships. Part of the work of fifth grade involves helping students to strengthen their ability to verbalize these claims and consider such questions as: Does this claim hold for all numbers? How can we know? Finding ways to answer these questions will provide the basis for making sense of formal proof when it is introduced in later grades. For example, as fifth graders work on solving the problem 1,232 – 196, students may change the 196 to 200 since 200 is "easier to work with." But how should they adjust 1,232 so that they have an equivalent problem? They might think first

about an earlier agreed upon generalization regarding the creation of equivalent expressions for *addition*:

- If an amount is added to one addend and subtracted from another addend, the sum remains the same: 1,232 + 196 = 1,228 + 200

Will this also work for subtraction? Students use representations and story contexts to determine how to create equivalent problems in subtraction and conclude that they cannot do this the same way that they did for addition. The representations and story contexts they create lead to the following generalization:

- If the same amount is added to (or subtracted from) both numbers in a subtraction problem, the difference does not change: (e.g., 1,232 – 196 = 1,236 – 200).

Students also verbalize and consider whether their claims hold true for all numbers in their work with multiplication and division. For example, as students apply the commutative and distributive properties and the inverse relationship between multiplication and division in solving problems, they are challenged to use story problems, unmarked arrays, and other representations to provide arguments about whether a general claim is always true (at least for other whole numbers). When such representations are shared, students are asked to explain the connections among the representations, contexts, and arithmetic expressions used and how these could also work for other numbers.

As fifth graders discuss their methods for creating equivalent expressions for multiplication, they continue work begun in fourth grade on particular generalizations about the operation. For example, they consider the following question:

- If one factor in a multiplication expression is halved (or divided by 3) and another factor is doubled (or tripled), what is the effect on the product?

Students in Grade 5 use arrays, diagrams, story contexts, and other representations to justify their conjecture that the product will remain the same for all pairs of numbers when one factor is doubled and the other is halved.

Like third and fourth graders, fifth grade students often solve division problems using a multiplication strategy. When they do so, they are employing the inverse relationship between multiplication and division and recognizing that division problems can be solved by relating them to missing factor problems (e.g., 462 ÷ 21 = ____ and ____ x 21 = 462).

Generalizations about multiplication and division highlighted in Grade 5 include:

- Two numbers multiplied in either order yield the same product: 32 x 20 = 20 x 32

- Numbers can be broken into parts to be multiplied, but each part of each number must be multiplied by each part of the other number: 69 x 36 = (60 + 9) x (30 + 6) = (60 x 30) + (60 x 6) + (9 x 30) + (9 x 6)

- Multiplication and division are related. If multiplying two numbers gives a certain product, then dividing that product by one of the original factors results in the other factor: 12 x 14 = 168; 168 ÷ 14 = 12; 168 ÷ 12 = 14

- A factor of a number is a factor of multiples of that number: 3 is a factor of 15; 30 is a multiple of 15, so 3 is a factor of 30.

- If one of the factors in a multiplication problem is doubled (or tripled) and the other is halved (or divided by 3), the product remains the same: 164 x 4 = 328 x 2