



## Developing Fluency with the Facts<sup>1</sup>

"The *Investigations* curriculum, like [the National Council of Teachers of Mathematics], supports the importance of students learning the basic combinations fluently through a focus on reasoning about number relationships: 'Fluency with whole-number computation depends, in large part, on fluency with the basic number combinations—the single digit addition and multiplication pairs and their counterparts for subtraction and division. Fluency with basic number combinations develops from well-understood meanings for the four operations and from a focus on thinking strategies . . .' (Principles and Standards for School Mathematics, pp. 152-153)

In other words, students learn these combinations best by using strategies, not simply by rote memorization. Relying on memory alone is not sufficient, as many of us know from our own schooling. If you forget—as we all do at times—you are left with nothing. If, on the other hand, your learning is based on an understanding of numbers and their relationships, you have a way to rethink and restructure your knowledge." (From the Grade 1 Unit Twos, Fives, and Tens; p. 146)

**Learning the Facts.** Facts are presented to students as an important mathematical tool, and fluency with them is considered part of the larger work of achieving computational fluency. The work with and practice of such facts takes place in the day-to-day teaching of the curriculum: variations of Classroom Routines and Ten-Minute Math activities with constraints that focus on particular combinations (e.g. use combinations of 10, or square numbers, to make Today's Number); games and activities that focus on specific sets of facts (e.g. Plus 9 or 10 BINGO and games that use Array Cards); homework and Practice Pages; and, for addition, subtraction, and multiplication, the sorting of cards – designed for explicit practice with the facts -- into envelopes of "Facts I Know" and "Facts I'm Still Working On". Each of these cards includes a line for a "Clue" or a known problem to "Start With", that helps students think about and use what they know to learn combinations they find difficult to remember (e.g.  $7+7=14$  might be a clue on the  $7+8$  card;  $3 \times 8=24$  on the  $6 \times 8$  card).

The focus of the work with these cards, and with the facts in general, is to help students take ownership of the work; to make the work meaningful and engaging; and to provide opportunities to learn about important mathematical properties and relationships. For example, how does knowing  $8+8$  help you solve  $9+8$ ? help you solve  $16-8$ ? Or, if you know your  $\times 10$  facts, how can that help you with the  $\times 5$  facts? with the  $\times 9$  facts? Note that much of the work on subtraction and division facts relies on fluency with their counterparts in addition and multiplication, and an understanding of the relationship between addition and subtraction, and multiplication and division.

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<sup>1</sup> This document applies to the 2nd edition of *Investigations* (2008, 2012). See <http://investigations.terc.edu/CCSS/> for changes when implementing *Investigations and the Common Core Standards*.

**Assessing the Facts: How?** In K-2, assessments of the facts generally involve the teacher meeting with individual or small groups of students to do an activity (e.g. How Many Am I Hiding?), play a game (e.g. Tens Go Fish), or quickly work through a set of Addition Combination Cards (e.g. the Near Doubles). In 3-5, all of the facts are assessed by giving students a sheet of about 30 problems and giving them 3 minutes to solve as many as they can. The purpose of the time limit is so that students can determine which facts they know fluently and which they need to continue to work on. In *Investigations*, “Fluency means that combinations are quickly accessible mentally, either because they are immediately known or because the calculation that is used is so effortless as to be essentially automatic.” (From the Grade 1 Unit Twos, Fives, and Tens; p. 146)

**Assessing the Facts: When?** The table below shows when students are expected to be fluent with which facts in the 2<sup>nd</sup> edition of *Investigations* and in the *Investigations and the Common Core* materials.

Grade	<i>Investigations</i> 2012	<i>Investigations</i> and the CCSS
K	—	“Fluently add and subtract within 5.” (K.OA.5)
1	Make 10 Combinations	Demonstrate “fluency for addition and subtraction within 10.” (1.OA.6)
2	Addition Combinations to 10+10	“Fluently add and subtract within 20 using mental strategies. Know from memory all sums of two one-digit numbers.” (2.OA.2)
3	Subtraction Combinations  Multiplication Combinations with Products to 50	“Fluently multiply and divide within 100, using strategies ... Know from memory all products of two one-digit numbers. (3.OA.7)
4	Multiplication Combinations to 12 x 12	—
5	Division Combinations	—

For more on how the facts fit into the larger work of helping students develop computational fluency with addition and subtraction, and multiplication and division, see the documents about these strands on [www.investigations.terc.edu/curriculumMath](http://www.investigations.terc.edu/curriculumMath).