Marbles and Blocks: *How Many of Each?*

In this case, Laura McCann's first-grade students are working on solving a How Many of Each? *problem involving blocks and marbles. Realizing that modeling this type of problem has proven to be difficult for some of her students in the past, Ms. McCann works to make the problems more accessible for some of her learners.*

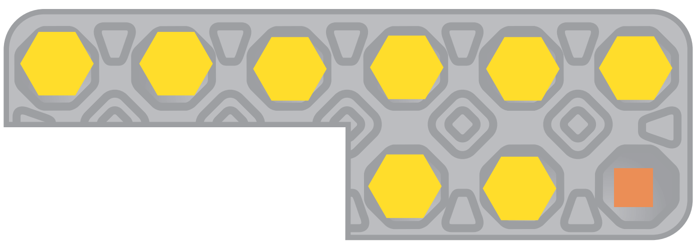
As my class shifts from geometry back to numbers and operations, I hear some enthusiasm from the students. We begin with the *How Many of Each?* problem concerning 9 toys. I remind the students that some of the toys are blocks, and some of the toys are marbles.

As I observe the class during the introductory discussion, I notice that some students seem ready to begin and appear to remember these problems from our last number unit. Others seem a little foggy but start to nod with understanding as the conversation continues. But I also spy a group of students who seem unsure of this assignment. I remember that these are the same students who had some difficulty with these problems the last time around. In thinking back to some of the mistakes they had previously made when solving these problems—like adding 9 blocks and 9 marbles to get 18 instead of finding a combination of blocks and marbles with a total of 9—I realize that these students need more structured support for solving this type of problem. I have an idea of where I can begin with them today.

While the class begins working, I notice that Courtney and Jasmine are tapping their pencils and looking around the room. I decide to bring over some egg cartons and pattern blocks to help them work through the problem. While the girls tell me what they know about the problem, I quickly resize the egg cartons so each carton has only 9 cups. From their explanation of the problem, I can tell that the girls seem to understand that there are some blocks and some marbles, but they cannot be more specific about how to determine how many of each object there could be.

I begin by asking them how they can use the pattern blocks as visual models. The girls quickly decide that hexagons will represent the marbles because they have a roundish shape, and the squares will be the blocks because they resemble blocks. Next, I introduce the egg cartons and point out that these trays will hold exactly 9 toys. I explain that their job will be to fill each cup with one toy. After they have filled the tray, they must count how many blocks they used and then how many marbles.

Courtney immediately fills the tray with 9 hexagons and says, "9." I commend her for following directions but then explain that in the context of this problem there are definitely some of each, so there must be at least 1 block in the mix. She takes out 1 hexagon and refills the cup with a square. She then counts the totals of each, and I ask her to show these amounts on her paper.



Jasmine, who is sitting next to her, seems to understand how to use this new tool with a little less scaffolding and has already come up with a possibility.

I leave the girls and begin to circulate around the room to check in on the other students. I am quickly approached by two boys, Edward and Peter, who tell me they are finished and have found every solution. I ask them to record their strategy on the back of their paper and emphasize that I want them to explain how they know they have found every solution. I look at Edward's paper and see that he has two columns—one column labeled "Blocks" and one labeled "Marbles"—and that he started with 1 block and 8 marbles. I notice that each time he subtracted one from the marbles column he added it to the blocks column.

Edward's Strategy:

Blocks	Marbles
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1

I decide to check back in with Jasmine and Courtney. I notice that Jasmine is now finding solutions without using the egg carton. It seems that the initial structuring helped her to visualize the task, and she is now able to work without it. I see that Courtney is still using the egg carton to help her. She has chosen to fill the eggcups in a pattern—hexagon, square, hexagon, square, hexagon, square, hexagon, square, hexagon, so I know that she is not yet ready to think about approaching this problem systematically in the way that Edward and Peter can. However, she is working on the important idea of breaking up a quantity into two parts in different ways.

Finally, it is time to share. I call on Courtney first as I know she has only a few to share, and I would like to give her an opportunity early on. I call on several other students early on for this same reason. I call on students with every, or almost every, solution later on so that I can observe them scan their sheets to see if they can pick out a solution we have not previously recorded. When we have all of the solutions, I ask the students to share how they know we have all of the solutions. I know that Courtney, and perhaps Jasmine, have not thought about this question. Their level of reasoning is still emerging, and this conversation could be beyond their reach, but I feel that it is important for them to be part of it just the same.

Jonathan raises his hand to share that he knows we have them all because he remembers from our previous work on these problems that every number leading up to the total will be used once by each item. He has written at the bottom of his paper:

~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~

Each number is slashed out as a sort of checklist that he used for the number of marbles and the number of blocks. When I ask why he didn't include the numbers 9 or 0, he confidently replied that because the problem said there were some of each, there has to be at least one marble or block and therefore 9 and 0 cannot be part of the solution.

Jane shares next. Her conjecture is that she knows we have all the solutions because we have eight solutions recorded, and that the number of solutions will be one less than the total number of items. I take a minute to work this through in my head; is she right? I tell her that I am intrigued by this thinking and am wondering if some of my mathematicians will check out her theory with other numbers. It will be a worthwhile discussion to come back to, but I am running out of time and cannot digress right now. It is Scott who ultimately shares the approach that I saw Edward use earlier. Scott also adds one to each column while subtracting one from the other column.

As Scott explains his strategy, I see some students nodding in recognition, and I realize that they are the ones who have solved the problem the same way. In an attempt to help some of my other students understand this approach, I grab a box of counters and a box of Geoblocks. I line up 8 counters in a row and tell the students that the counters represent the marbles. I then place 1 Geoblock down to represent the blocks. Next, I demonstrate how by removing one "marble" at a time and replacing it with a block, we can see Sam's strategy happening.

I hope that these accommodations have added depth to our work on this type of problem. I know I will not have to wait long to find out!

Knowing that visualizing the structure of whole and parts in How Many of Each? *problems is difficult for some of her students, Ms. McCann finds ways to make the problems more accessible. She provides some of her learners with tools so they can see the action happening in the problem. During sharing time, Ms. McCann encourages all of her students to participate in the solution sharing and demonstrates some of the more complex strategies for the class.*

Questions for Discussion

1. **How did Ms. McCann make the *How Many of Each?* problems more accessible for Courtney and Jasmine? Have you had students in your classroom who have struggled with these problems? How did you help your learners?**
2. **What are the important mathematical ideas that came up during sharing?**
3. **How does Ms. McCann help her range of learners both participate and learn from each other during the class discussion? What strategies do you use to do this in your classroom?**