Investigations in Number, Data, and Space®

Math Content by Strand[1]

Geometry

Kindergarten

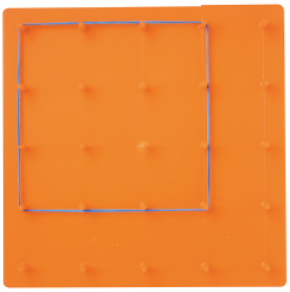
The geometry work in Kindergarten builds on students' firsthand knowledge of shapes to further develop their spatial sense and deepen their understanding of the two-and three-dimensional world in which they live. As students identify the different shapes that make up the world, they are encouraged to use their own words to describe both 2-D and 3-D shapes. In this way, they form images of familiar shapes through associating them with familiar objects.

Students explore the geometric idea that shapes can be combined or subdivided to make other shapes. For example, they investigate how 3-D shapes can be combined to form a particular rectangular prism.

By putting shapes together and taking shapes apart, students deepen their understanding of the attributes

of shapes and how shapes are related.
Students also construct 2-D and 3-D shapes with clay and on Geoboards. As they construct shapes they form mental images of the shapes and think about the attributes of particular shapes.

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.



The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Composing and decomposing 2-D and 3-D shapes
- Describing, identifying, comparing, and sorting 2-D and 3-D shapes

Benchmarks

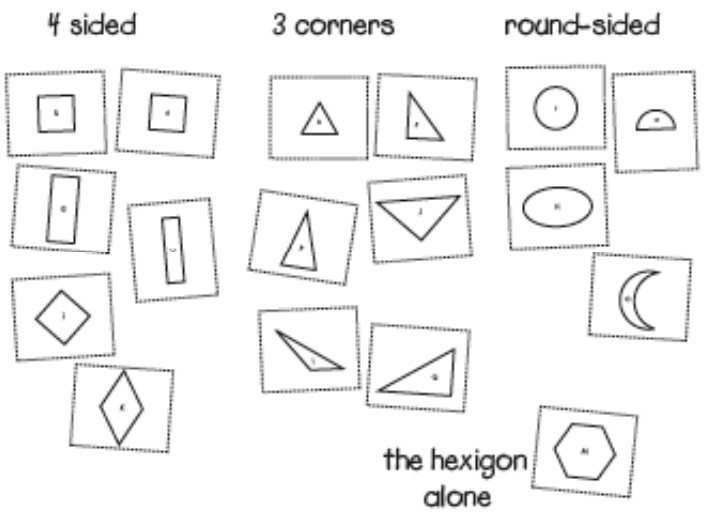
- Describe the overall size, shape, function, and/or features of familiar 2-D and 3-D shapes
- Construct 2-D and 3-D shapes
- Make 2-D and 3-D shapes by combining shapes

Grade 1

The emphasis of geometry work in 1st grade is on careful observation, description and comparison of two-dimensional (2-D) and three-dimensional (3-D) geometric shapes.

Students describe 2-D shapes, sort them and compare them, and they think about questions like the following: What makes a triangle a triangle? How are triangles different from squares?

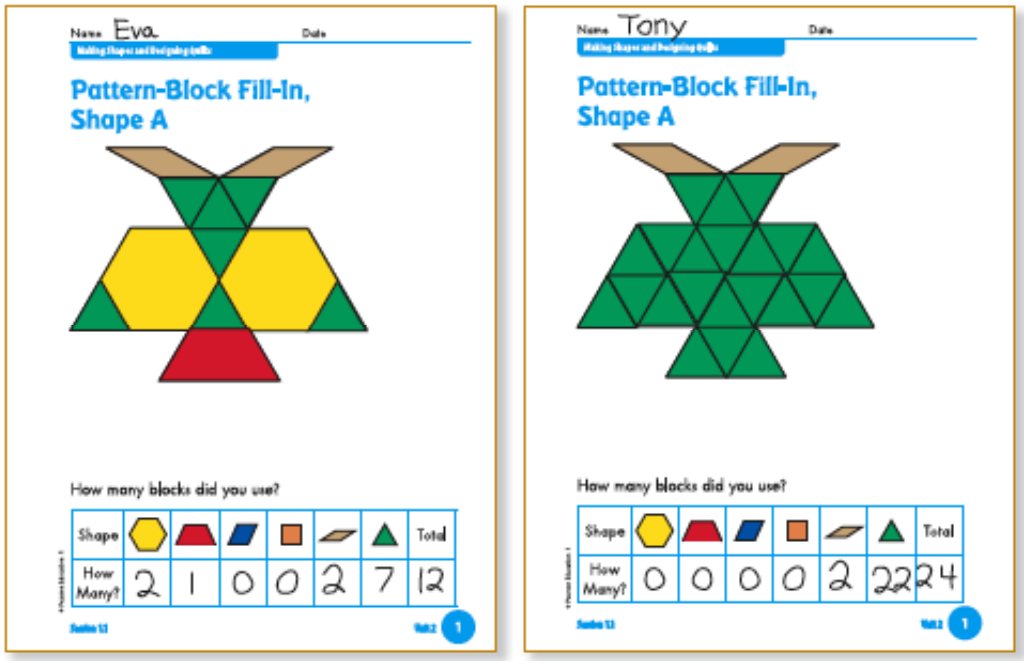
Developing visual images of shapes as well as drawing 2-D shapes are ways that students come to know the important features of shapes. When they sort 2-D shapes, they make groups of shapes that "go together," which requires them to look for similarities and differences among the attributes of different shapes.



One pair's sort of Shape Cards

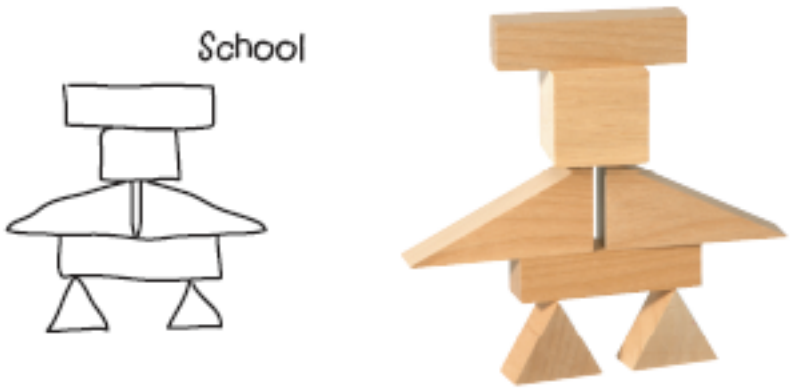
Students look for 3-D shapes in their own environment and they work with 3-D shapes (whose faces are familiar 2-D shapes) such as Geoblocks, manufactured boxes, and boxes made by students.

Students also learn about geometric relationships by composing and decomposing shapes. As they fill in the same shape outline with pattern blocks in different ways, they break apart or combine shapes in order to change how the shape is filled. When using the geoblocks, students notice, for example, that two cubes can be put together to make a rectangular prism and that two triangular prisms can be put together to make a cube.



Eva and Tony's student work of SAB 1, Pattern Block Fill-In, Shape A

Students investigate the relationship between 3-D shapes and 2-D representations of those shapes. By matching 3-D objects to outlines of their faces, to pictures, and to drawings of other students, they identify shapes by looking carefully at some parts of the shape and then visualizing what the whole shape looks like. Moving back and forth between 3-D objects and their 2-D representations helps students describe and compare the characteristics of common 3-D shapes.



A student draws a 2-D representation of his 3-D building.

The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Composing and decomposing 2-D shapes
- Describing, identifying, and comparing 2-D and 3-D shapes
- Exploring the relationships between 2-D and 3-D shapes

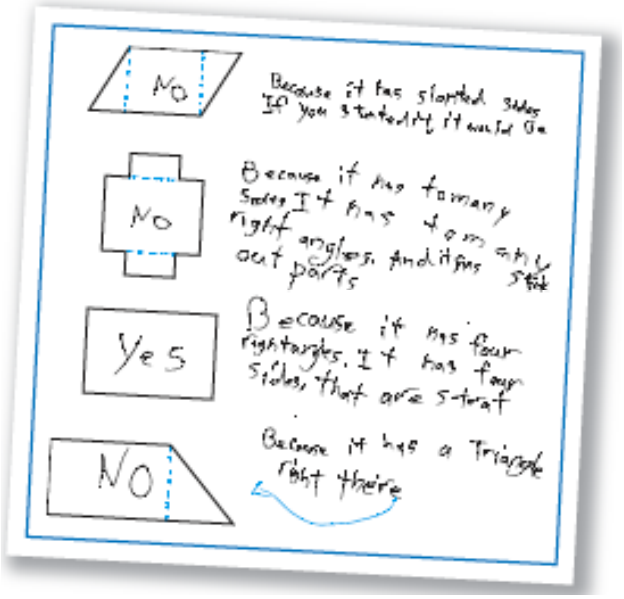
Benchmarks

- Fill a given region in different ways with a variety of shapes
- Use geometric language to describe and identify important features of familiar 2-D shapes
- Identify and describe triangles
- Describe and sort 2-D shapes
- Compose and decompose shapes
- Attend to features of 3-D shapes, such as overall size and shape, the number and shape of faces, and the number of corners
- Match a 2-D representation to a 3-D shape or structure

Grade 2

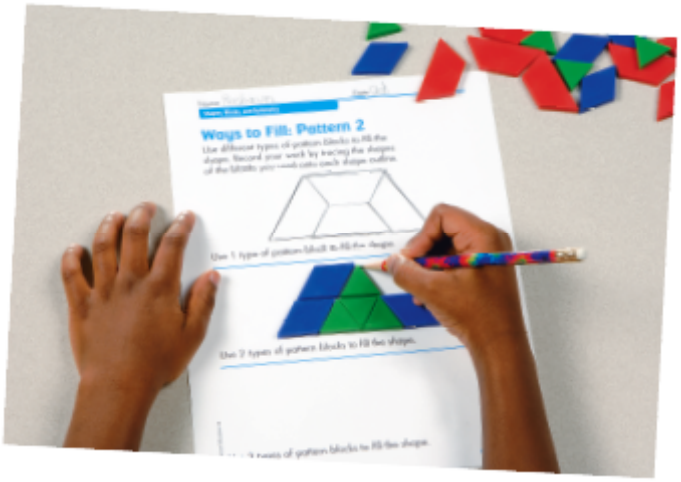
Students describe, sort and compare two-dimensional (2-D) and three-dimensional (3-D) shapes and think about questions like the following: What makes a rectangle a rectangle? How are rectangles different from squares?

Example: Is it a rectangle? Why or why not?



A student explains why shapes are (or are not) rectangles.

Second-grade students study rectangles and rectangular prisms, and consider which properties are important in describing these shapes. They combine and decompose both 2-D and 3-D shapes and explore the relationships between shapes, particularly as they work with pattern blocks and Geoblocks. As they develop knowledge about how shapes are related, they are learning about the important features of shapes.



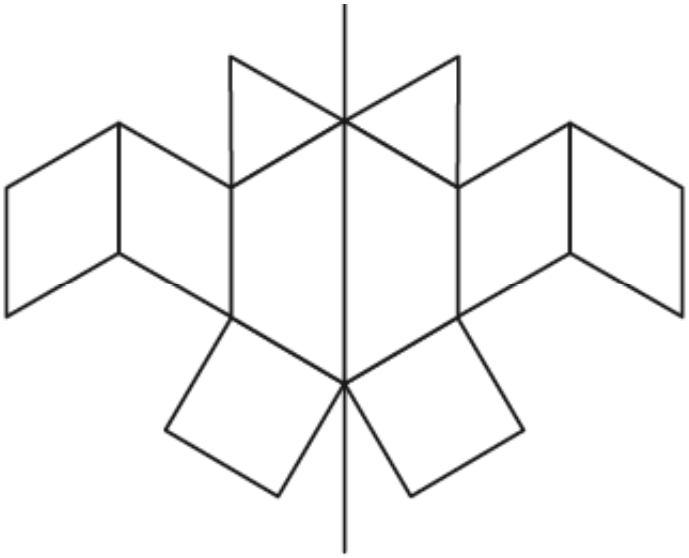
As they use pattern blocks to fill an outline, students find different ways to make the same 2-D shape.

Students find different ways to make a block of a different size.

Students begin their work with arrays, a visual representation that they will continue to use in Grades 3 through 5. As students create rectangles with square tiles, they learn about the structure of an array. In their study of arrays, work in geometry is closely linked with ideas about number. Students develop a variety of numerical strategies, based on the number of tiles in a row and the number of rows, to calculate the area of the rectangle.

A student might respond, "My rectangle has 2 rows with 3 tiles in each row."

Students develop an understanding of mirror symmetry as they identify objects that have mirror symmetry, create patterns and designs, fold and cut paper, and build 3-D structures with mirror symmetry. As they create and investigate symmetrical shapes, they develop language and ideas about what symmetry is and how it behaves.



A design with mirror symmetry

The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Combining and decomposing 2-D and 3-D shapes
- Describing, identifying, comparing, and sorting 2-D and 3-D shapes
- Exploring mirror symmetry

Area Measurement

- Visualizing the structure of arrays

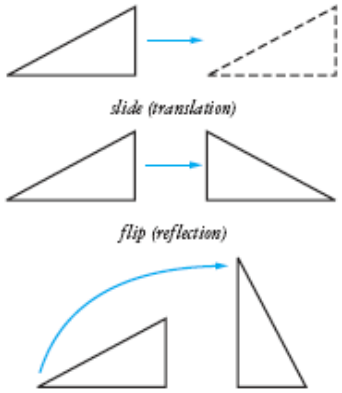
Benchmarks

- Identify the number of sides of a polygon
- Identify the number of rows and the number of squares in each row in an array
- Identify rectangles as four-sided shapes with four right angles
- Identify the number of faces on a rectangular prism and show which faces are congruent
- Make a symmetrical picture based on an image provided

Grade 3

Students study the attributes of two-dimensional (2-D) and three-dimensional (3-D) shapes, and how these attributes determine their classification. For example, a polygon is classified as a triangle or a quadrilateral based on the number of its sides.

Students also investigate the idea that one shape may have more than one name as they consider the properties of squares and rectangles. They describe shapes by whether or not they are *congruent* to other shapes, and use geometric motions—*slides (*translations*), flips* (reflections), and *turns* (rotations)—to determine if shapes are congruent.

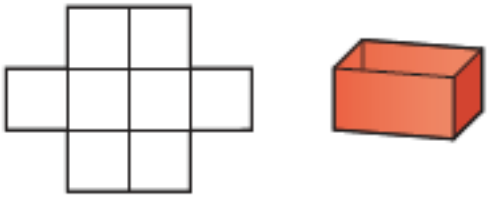


Students describe attributes of common geometric solids (3-D shapes), such as how many edges and faces a solid shape has, or how a pyramid has triangular faces coming to a point.

They learn to distinguish between polyhedra (3-D shapes having only flat surfaces) and nonpolyhedra (3-D shapes that have curved surfaces) and, within the class of polyhedra, between prisms and pyramids.

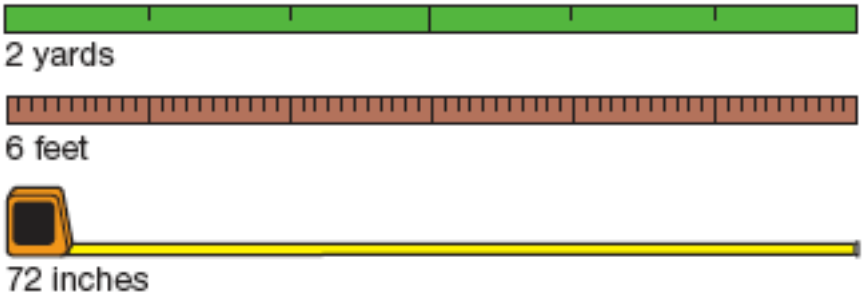
prism *pyramid*

Students learn about how 3-D objects can be represented in 2-D space. For example, they design nets for open boxes that, if constructed in 3-D, would hold a certain number of cubes. They determine the volume of the rectangular prisms that fit into a variety of open boxes.

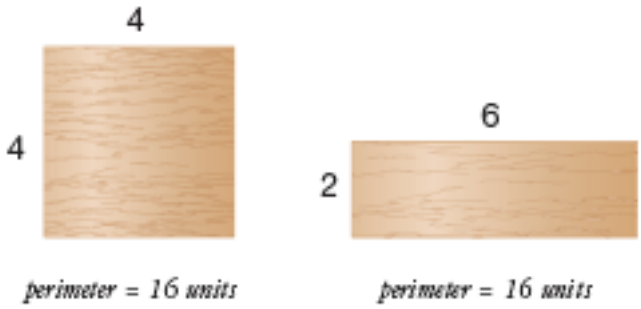


net for open box to hold two cubes

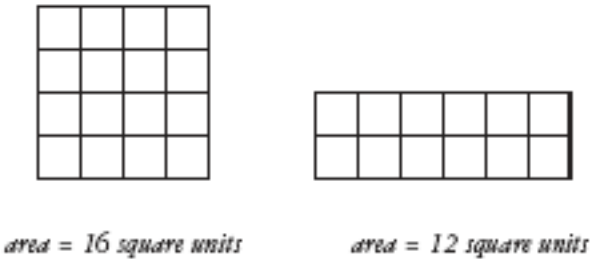
Students' measurement work in Grade 3 includes linear measurement, area, angle measurement, and volume. They measure length and perimeter with both U. S. standard units (inches, feet and yards) and metric units (centimeters and meters). Their work focuses on using measurement tools accurately, and understanding the relationship between measures when the same length is measured with different units.



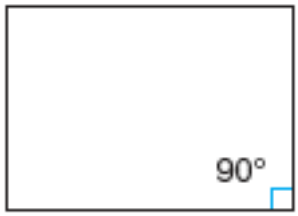
Students learn that the distance around the outside edges of a two-dimensional shape is called the *perimeter*, and consider how different shapes can have the same perimeter.



They identify the amount of 2-D space a given shape covers as its *area,* and learn that area is measured in square units.



They identify the internal angle of a rectangle or square as 90 *degrees*. They use right angles as a benchmark as they consider the sizes of angles of other polygons.



Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Describing properties of 3-D shapes
- Translating between 2-D and 3-D shapes

Linear Measurement

- Measuring length
- Measuring with standard units
- Understanding and finding perimeter

Area Measurement

- Understanding and finding area

Volume

- Structuring rectangular prisms and determining their volume

Benchmarks

- Identify and accurately measure the perimeter of a shape using U.S. standard and metric units
- Identify and find the area of given figures by counting whole and partial square units
- Identify triangles as three-sided closed shapes with three vertices and three angles
- Identify right angles, and recognize whether an angle is larger or smaller than a right angle
- Identify and compare attributes of 3-D solids
- Determine the number of cubes (volume) that will fit in the box made by a given pattern
- Design patterns for boxes that will hold a given number of cubes

Grade 4

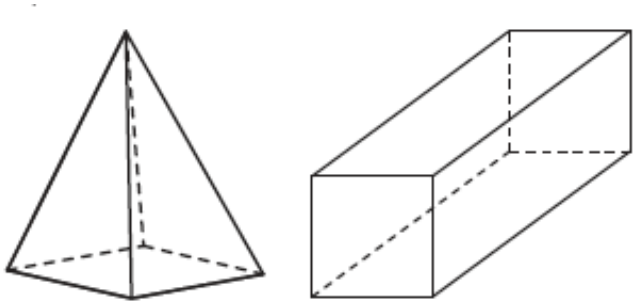
Students expand their understanding of the attributes of two-dimensional (2-D) and three-dimensional (3-D) shapes, and how these attributes determine their classification. Students consider the various attributes of 2-D shapes, such as number of sides, the length of sides, parallel sides, and the size of angles, expanding their knowledge of four-sided figures (quadrilaterals) to include parallelograms, rhombuses, and trapezoids.

Polygons

Not Polygons

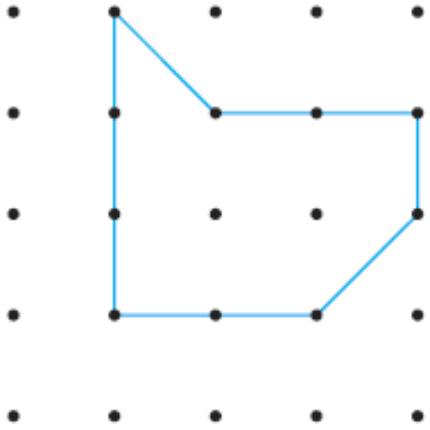
Students also describe attributes and properties of geometric solids (3-D shapes), such as the shape and number of faces, the number and relative lengths of edges, and the number of vertices. They describe classes of shapes, for example, how a pyramid has triangular faces meeting at a point.

They visualize how 3-D shapes can be represented in two dimensions, for example, by silhouettes projected by 3-D objects and structures.



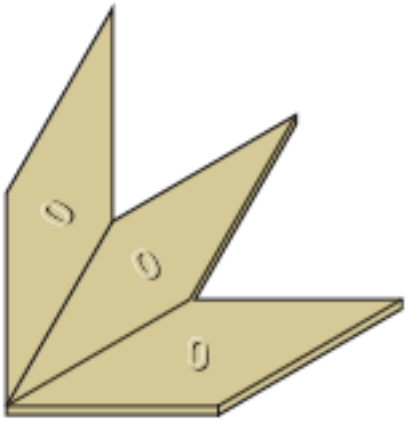
In Grade 4, students continue to build on measurement work from earlier grades, which includes linear measurement, area, angle measurement, and volume. They use both U.S. standard units (inches, feet and yards) and metric units (centimeters and meters) to measure lengths up to 100 feet, and they determine the perimeter of various shapes.

They measure the area of both regular and nonregular polygons in square units by using the understanding that area can be *decomposed*—that is, broken into smaller parts.

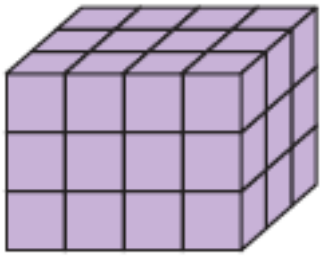


Measuring area on the Geoboard

Students work on determining the size of angles relative to a right angle, or 90 degrees. For instance, if three equal angles form a right angle, then each of the smaller angles must be 1/3 of 90 degrees or 30 degrees.



Finally, students work on understanding volume by structuring and determining the volume of one kind of geometric solid, a rectangular prism, in cubic units. They develop strategies for determining the number of cubes in 3-D arrays of cubes by mentally organizing the cubes--for example as a stack of three rectangular layers, each composed of three rows of four cubes.



Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Describing properties of 3-D shapes
- Translating between 2-D and 3-D shapes

Linear Measurement

- Measuring with standard units

Area Measurement

- Understanding and finding area

Volume

- Structuring rectangular prisms and determining their volume

Benchmarks

- Use appropriate measurement tools to measure distance
- Identify quadrilaterals as any four-sided closed shape
- Know that a right angle measures 90 degrees, and use this as a landmark to find angles of 30, 45, and 60 degrees
- Find the area of polygons using a square unit of measure
- Identify 2-dimensional silhouettes of 3-dimensional solids (e.g. a cone can project a triangular silhouette)
- Draw 2-D representations showing different perspectives of a 3-D object
- Find the volume of cube buildings and rectangular prisms

Grade 5

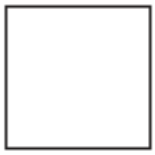
In their work with geometry and measurement in grade 5, students further develop their understanding of the attributes of two-dimensional (2-D) shapes, find the measure of angles of polygons, determine the volume of three-dimensional (3-D) shapes, and work with area and perimeter. Students examine the characteristics of polygons, including a variety of triangles, quadrilaterals, and regular polygons. They consider questions about the classification of geometric figures, for example:

Are all squares rectangles?

Are all rectangles parallelograms?

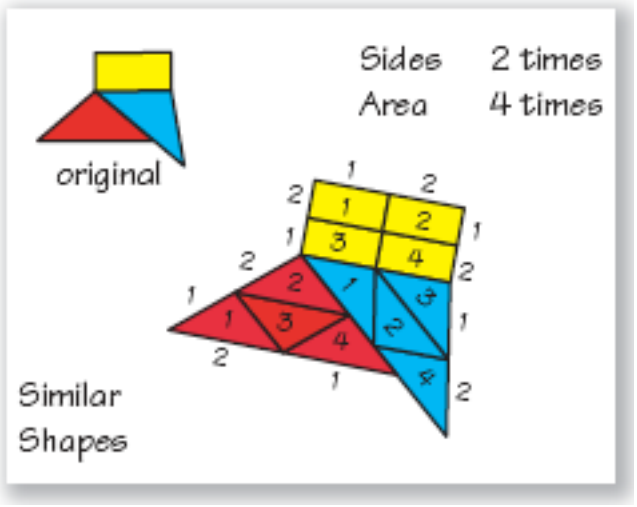
If all squares are rhombuses, then are all rhombuses squares?

1. Samantha says this figure is called a rhombus. Felix says it is called a square. Joshua says it is called a parallelogram.



Can they all be right? How is that possible? Explain.

They investigate angle sizes in a set of polygons and measure angles of 30, 45, 60, 90, 120, and 150 degrees by comparing the angles of these shapes. Students also investigate perimeter and area. They consider how changes to the shape of a rectangle can affect one of the measures and not the other (e.g., two shapes that have the same area don't necessarily have the same perimeter), and examine the relationship between area and perimeter in similar figures.



Students continue to develop their visualization skills and their understanding of the relationship between 2-D pictures and the 3-D objects they represent. Students determine the volume of boxes (rectangular prisms) made from 2-D patterns and create patterns for boxes to hold a certain number of cubes. They develop strategies for determining the number of cubes in 3-D arrays by mentally organizing the cubes—for example as a stack of three rectangular layers, each three by four cubes.

Students deepen their understanding of the relationship between volume and the linear dimensions of length, width, and height. Once students have developed viable strategies for finding the volume of rectangular prisms, they extend their understanding of volume to other solids such as pyramids, cylinders, and cones, measured in cubic units.

Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Creating and describing similar shapes
- Translating between 2-D and 3-D shapes

Linear and Area Measurement

- Finding the perimeter and area of rectangles

Volume

- Structuring rectangular prisms and determining their volume
- Structuring prisms, pyramids, cylinders, and cones and determining their volume

Benchmarks

- Identify different quadrilaterals by attribute, and know that some quadrilaterals can be classified in more than one way
- Use known angle sizes to determine the sizes of other angles (30 degrees, 45 degrees, 60 degrees, 90 degrees, 120 degrees, and 150 degrees)
- Determine the perimeter and area of rectangles
- Identify mathematically similar polygons
- Find the volume of rectangular prisms
- Use standard units to measure volume
- Identify how the dimensions of a box change when the volume is changed
- Explain the relationship between the volumes of prisms and pyramids with the same base and height