Investigations in Number, Data, and Space®

Math Content by Strand[1]

Number and Operations with Whole Numbers
Multiplication and Division

Grade 3

In Grade 3, students investigate the properties of multiplication and division, including the inverse relationship between these two operations, and develop strategies for solving multiplication and division problems. Their work focuses on developing the idea that multiplication involves some *number of equal-sized groups*, and that division also involves equal groups.

Students are introduced to arrays—rectangular arrangements of objects in rows and columns—to help them develop visual images that support their understanding of multiplication.

3 by 4 array

They use these rectangular arrays to represent the relationship between a product and its factors. Students determine, describe, and compare sets of multiples, noticing their characteristics and relationships, and use these to investigate important ideas about how multiplication works.

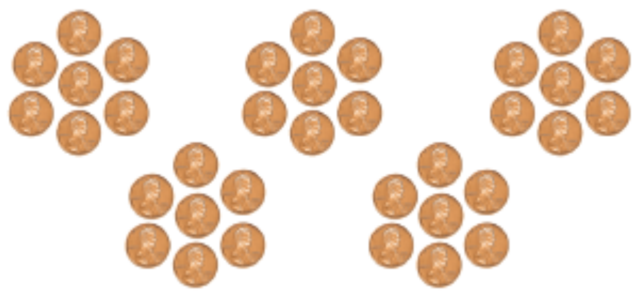
[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

They learn the multiplication combinations with products up to 50.

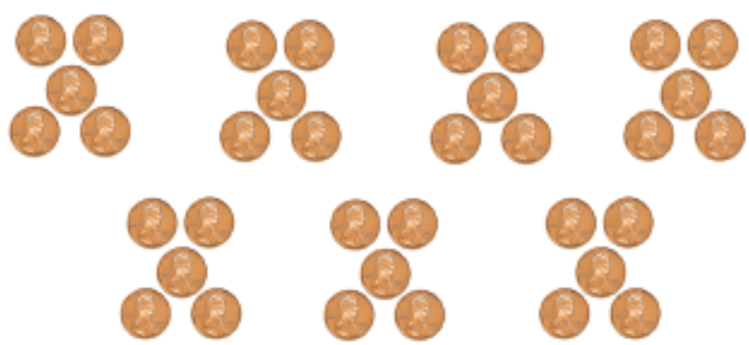
4 x 5 = 5 x 4 = Start with __________	7 x 6 = 6 x 7 = Start with __________

Students solve division situations that involve *sharing*, ("Divide 35 pennies among 5 people equally. How many pennies are in each share?") and those that involve *grouping* ("How many groups of 5 pennies can I make if I have 35 pennies?").

Sharing: Divide 35 pennies among 5 people equally. How many pennies are in each share?



Grouping: How many groups of 5 pennies can I make if I have 35 pennies?



Students use their knowledge of the relationship between division and multiplication by reasoning in ways like the following: "I know that five 5s is 25, and two more 5s make 35, so I have 7 groups of 5." Students are also introduced to two forms of division notation — 35 ÷ 5 and $5\overline{)35}$ — and learn how to interpret these numbers and symbols in terms of the meaning and actions of division.

The Algebra Connections page in the curriculum unit that focuses on multiplication and division shows how students are applying the commutative and distributive properties of multiplication as they solve problems. It also highlights students' application of the inverse relationship between multiplication and division.

Emphases

Whole Number Operations

- Understanding the meaning of multiplication
- Reasoning about numbers and their factors and multiples
- Understanding and working with an array model of multiplication
- Developing strategies for division based on understanding the inverse relationship between multiplication and division

Computational Fluency

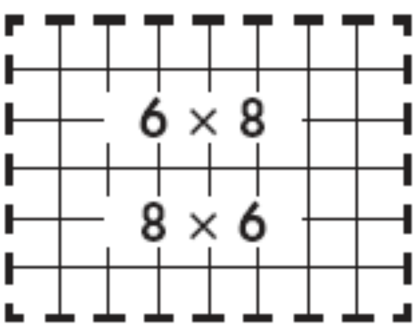
- Learning the multiplication combinations with products to 50 fluently

Benchmarks

- Demonstrate an understanding of multiplication and division as involving groups of equal groups
- Solve multiplication combinations and related division problems using skip counting or known multiplication combinations
- Interpret and use multiplication and division notation
- Demonstrate fluency with the multiplication combinations with products up to 50 (by the end of Grade 3)

Grade 4

In Grade 4, three of the four curriculum units on number and operations with whole numbers focus on multiplication and division. This major component of students' work centers on reasoning about numbers and their factors and multiples, using models, representations, and story contexts to help them visualize and solve multiplication and division problems; and understanding the relationship between multiplication and division.



Combinations We're Working On	Start With
6 × 8	5 × 8 (and add one more 8) 3 × 8 (and double it) 6 × 4 (and double it)

Students learn the multiplication combinations (facts) to 12 x 12 so that they can use these fluently to solve both multiplication and division problems. They develop strategies for solving multiplication and division problems based on looking at the problem as a whole, thinking about the relationships of the numbers in the problem, and choosing an approach they can carry out easily and accurately, often breaking the numbers apart or changing the numbers in some way. Visualizing how multiplication works is critical in applying the distributive property to solve problems and in keeping track of parts of the problem. Learning to multiply by multiples of 10 is also a key component of this work.

Examples of Multiplication Strategies

Breaking numbers apart by addition

48 x 42 =

40 x 40 = 1,600

40 x 2 = 80

8 x 40 = 320

8 x 2 = 16

1,600 + 80 + 320 + 16 = 2,016

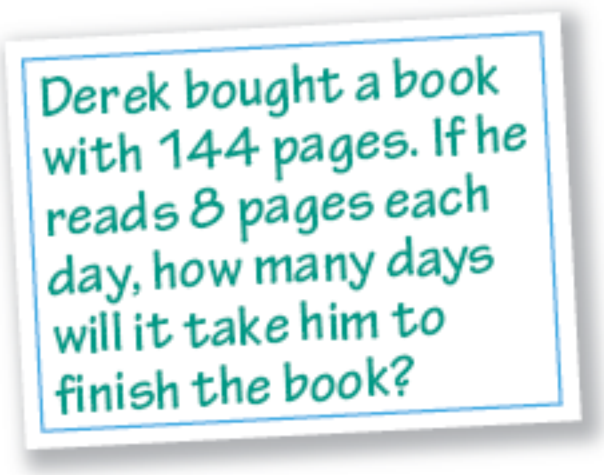
48 x 42 =

48 x 40 = 1,920

48 x 2 = 96

1,920 + 96 = 2,016

Students interpret and solve division problems, both in story contexts and numerical contexts. They work with both grouping and sharing situations, and consider how to make sense of a remainder within the context of the problem. They use the inverse relationship between multiplication and division to solve division problems, including those related to the multiplication combinations to 12 x 12 (the division "facts"), and problems in which 3-digit numbers are divided by 1-digit and small 2-digit divisors.



The Algebra Connections pages in the three curriculum units that focus on multiplication and division show how students are applying the commutative and distributive properties of multiplication, as well as the inverse relationship between multiplication and division, as they solve problems. These pages also highlight particular generalizations about multiplication that students work on in Grade 4: If a number is a factor of a second number, are all the factors of the first number also factors of the second number? If one factor in a multiplication expression is halved and another factor is doubled, what is the effect on the product?

Emphases

Whole Number Operations

- Understanding and working with an array model of multiplication
- Reasoning about numbers and their factors
- Understanding and using the relationship between multiplication and division to solve division problems
- Understanding division as making groups of the divisor

Computational Fluency

- Fluency with the multiplication combinations to 12 x 12
- Solving multiplication problems with 2-digit numbers

Benchmarks (compiled from Units 1, 3, and 8)

- Use known multiplication combinations to find the product of any multiplication combination to 12 x 12
- Use arrays, pictures or models of groups, and story contexts to represent multiplication situations
- Find the factors of 2-digit numbers
- Multiply 2-digit numbers by one-digit and small 2-digit numbers (e.g. 12, 15, 20), using strategies that involve breaking the numbers apart
- Solve division problems (2- and small 3-digit numbers divided by 1-digit numbers) including some that result in a remainder
- Use story problems, pictures, or concrete models to represent division situations
- Multiply by 10 and multiples of 10
- Demonstrate fluency with multiplication combinations to 12 x 12

- Multiply 2-digit numbers efficiently
- Solve division problems with 1- and small 2-digit divisors by using at least one strategy efficiently

Grade 5

In Grade 5, students consolidate their understanding of the computational strategies they use for multiplication. All students should be able to carry out strategies that involve breaking one or both factors apart, multiplying each part of one factor by each part of the other factor, then combining the partial products. They also practice notating their solutions clearly. They use representations and story contexts to connect these strategies, which are based on the distributive property of multiplication, to the meaning of multiplication. As part of their study of multiplication, students analyze and compare multiplication algorithms, including the U.S. algorithm for multiplication.

Examples of Multiplication Strategies

Breaking numbers apart by addition

148 x 42 =

40 x 100 = 4,000

40 x 40 = 1,600

40 x 8 = 320

2 x 100 = 200

2 x 40 = 80

2 x 8 = 16

4,000 + 1,600 + 320 + 200 + 80 + 16 = 6,216

148 x 42 =

100 x 42 = 4,200

48 x 40 = 1,920

48 x 2 = 96

4,200 + 1,920 + 96 = 6,216

Changing one number to create an easier problem

148 x 42 =

150 x 42 = 6,300 (100 x 42 + 1/2 of 100 x 42)

2 x 42 = 84

6,300 – 84 = 6,216

Students continue to learn ways to solve division problems fluently, focusing on the relationship between multiplication and division. They solve division problems by relating them to missing factor problems (e.g., 462 ÷ 21 = ____ and ____ x 21 = 462), by building up groups of the divisor, and by using multiples of 10 to solve problems more efficiently. As students refine their computation strategies for division, they find ways to
use what they already know and understand well (familiar factor pairs, multiples of 10s, relationships between numbers, etc.) to break apart the harder problems into easier problems. They also work on notating their solutions clearly and concisely.

___ × 21 = 1,275

6 × 21 = 126
60 × 21 = 1,260   1,275 − 1,260 = 15

Answer: 60 R15

1,275 ÷ 21 =

630 ÷ 21 = 30   1,275 − 630 = 645
630 ÷ 21 = 30   645 − 630 = 15
1,260 ÷ 21 = 60

Answer: 60 R15

21)1,275

10 × 21 = 210
20 × 21 = 420
30 × 21 = 630
60 × 21 = 1,260

60 R15
21)1,275

21)1,275

− 420   20 × 21 = 420
855
− 630   30 × 21 = 630
225
− 210   10 × 21 = 210
15

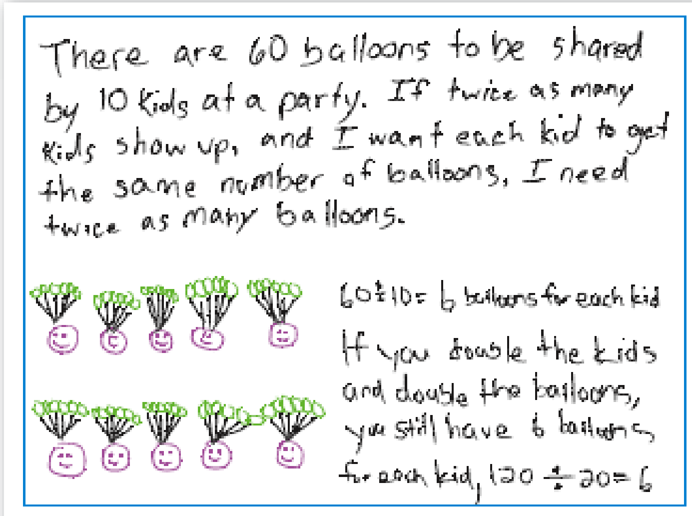
20 + 30 + 10 = 60

Answer: 60 R15

Examples of clear and concise notation

Students also study underlying properties of numbers and operations and make and justify general claims based on these properties. They study the relationship between a number and its factors, which supports mental computation strategies for multiplication and division with whole numbers. For example, students consider multiplication expressions related by place value (e.g., 3 x 6 = 18; 3 x 60 = 3 x 6 x 10 = 180), and equivalent multiplication expressions (e.g., 24 x 18 = 12 x 36 or 24 x 18 = 72 x 6). This work includes finding longer and longer multiplication expressions for a number and considering the prime factorization of a number.

Students also investigate equivalent expressions in multiplication and division. For example, they investigate why doubling one factor and halving the other factor (or tripling and thirding, etc.) in a multiplication expression of the form *a x b* maintains the same product. They also examine how and why the ratio between dividend and divisor must be maintained to generate equivalent division expressions. In this work, students develop mathematical arguments based on representations of the operations.



Sample student work

The Algebra Connections pages in the two curriculum units that focus on multiplication and division show how students are applying the commutative and distributive properties of multiplication, as well as the inverse relationship between multiplication and division, as they solve problems. These pages also highlight particular generalizations about multiplication that students work on in Grade 5 as they create equivalent expressions for multiplication: If one factor in a multiplication expression is halved (or thirded) and another factor is doubled (or tripled), what is the effect on the product?

Emphases

Whole Number Operations

- Reasoning about numbers and their factors
- Understanding and using the relationship between multiplication and division to solve division problems
- Representing the meaning of multiplication and division
- Reasoning about equivalent expressions in multiplication and division

Computational Fluency

- Solving multiplication problems with 2-digit numbers
- Solving multiplication problems with 2- and 3-digit numbers
- Solving division problems with 2-digit divisors

Benchmarks

- Find the factors of a number
- Solve multiplication problems efficiently
- Solve division problems with 1-digit and 2-digit divisors
- Explain why doubling one factor in a multiplication expression (a x b) and dividing the other by 2 results in an equivalent expression
- Solve division problems efficiently