Investigations in Number, Data, and Space®

Math Content by Strand[1]

Patterns, Functions, and Change

Kindergarten

Kindergarten students construct, describe, extend, and determine what comes next in repeating patterns. To identify and construct repeating patterns, students must be able to identify the attributes of the objects in the pattern. Therefore students first work on sorting objects by their attributes, before they begin constructing their own patterns. Students encounter patterns with two (AB, AAB, ABB) or three (ABC) elements. As students construct and describe many different patterns, they become more familiar with the structure of patterns, are able to identify what comes next in a pattern, and can begin to think about how two patterns are similar and different.

Example: What is the same about these cube trains? What is different?

After having many opportunities to construct their own patterns and extend patterns made by others, students begin to analyze the structure of a repeating pattern by identifying the *unit* of the pattern—the part of the pattern that repeats over and over.

Example: What is the repeating unit of this pattern?

Emphases

Data Analysis

- Sorting and Classifying

Repeating Patterns

- Constructing, describing, and extending repeating patterns
- Identifying the unit of a repeating pattern

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

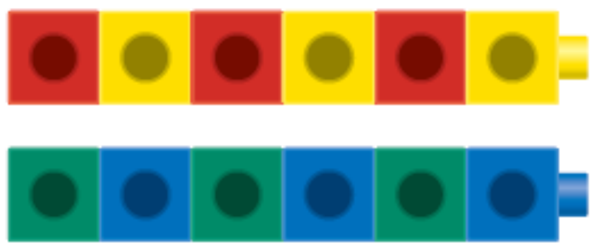
Benchmarks

- Copy, construct, and extend simple patterns, such as AB and ABC
- Begin to identify the unit of a repeating pattern

Grade 1

Students begin their work on patterns in first grade by creating, describing, extending, and making predictions about repeating patterns. By building or acting out these patterns and thinking through how the pattern continues, students analyze the regularities of the pattern to determine what comes next or what will come several steps ahead in the pattern. Students analyze the structure of a repeating pattern by identifying the *unit* of the pattern—the part of the pattern that repeats over and over. By focusing on the unit of the repeating pattern, students shift their focus from seeing that "red follows yellow and yellow follows red" to how the repeating pattern is constructed of an iterated red-yellow unit. This focus allows students to analyze more complex patterns.

Students also compare patterns and begin to notice how patterns are the same. For example, a red, yellow, red, yellow pattern and a green, blue, green, blue pattern have the same structure.



Students then work with number sequences associated with repeating patterns. Associating the counting numbers with this pattern allows new kinds of questions about the pattern, such as the following: "What color will the 17th square be?" "Is the 20th square black?" Numbering the elements of a repeating pattern provides another way to describe that pattern.

Students also consider situations that have a constant increase. They investigate three different contexts— collecting pennies in a jar, making Staircase Towers from connecting cubes, and making repeating patterns with pattern blocks. In each situation, a sequence of numbers is generated by the situation.

Example: I have one penny in a jar, and each day I add three more pennies.

Comparison across contexts helps students focus on how the same start number and the same amount of constant increase can create the same number sequence in different situations.

Emphases

Repeating Patterns

- Constructing, describing, and extending repeating patterns
- Identifying the unit of a repeating pattern

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

Benchmarks

- Construct, describe, and extend a repeating pattern with the structure AB, ABC, AAB, or ABB
- Identify the unit of a repeating pattern for patterns with the structure AB or ABC
- Describe how various AB or ABC patterns are alike (e.g., how is a red-blue pattern like a yellow-green pattern?)
- Determine what comes several steps beyond the visible part of an AB, ABC, AAB, or ABB repeating pattern
- Construct, extend, and describe a pattern that has a constant increase for the sequences 1, 3, 5, …; 2, 4, 6, …; 1, 4, 7, …; 2, 5, 8, …; and 3, 6, 9, … through counting and building

Grade 2

Second-grade students explore situations with constant ratios in two contexts: building cube buildings with the same number of "rooms" on each "floor" and covering a certain number of one pattern block shape with another pattern block shape. In both of these contexts, students build and record how one variable changes in relation to the other.

For example: it takes two of the red trapezoids to cover one of the yellow hexagons, 4 trapezoids to cover 2 hexagons and so forth. They use what they are learning to determine quantities later in the series.

Example: How many trapezoids will you need to cover 10 hexagons?

2 trapezoids cover 1 hexagon, 4 trapezoids cover 2 hexagons, and so on.

Tables are introduced and used as a central representation. Organizing data in a table can help students uncover a rule that governs how one quantity changes in relation to another. For example, for every increase of 1 for one quantity, the other quantity increases by 2, (e.g., for each additional trapezoid, from the pattern block set, 2 trapezoids are needed to cover it). Students compare tables that show different relationships, both within the same context and between the two contexts, and notice how different situations can have the same underlying relationship between quantities.

Number of Hexagons	Number of Trapezoids
1	2
2	4
3	6
4	8
5	10

Students also work with number sequences associated with repeating patterns that reveal important characteristics of the pattern and provide an avenue into studying the number sequences themselves.

Example: If the pattern keeps going in the same way, what color will the 12th cube be?

As students explore two-element and three-element repeating patterns, they encounter the odd number sequence, the even number sequence, and three different "counting by 3" sequences. An important part of second-grade students' work on pattern is considering how and why different situations generate the same number sequence.

Emphases

Linear Relationships

- Describing and representing ratios

Using Tables and Graphs

- Using tables to represent change

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

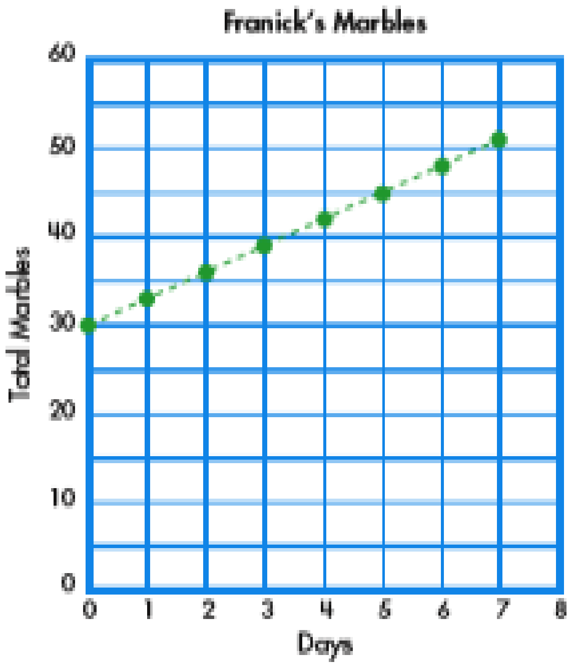
Benchmarks

- Explain what the numbers in a table represent in a constant ratio situation (involving ratios of 1:2, 1:3, 1:4, 1:5, and 1:6)
- Complete and extend a table to match a situation involving a constant ratio
- Extend a repeating pattern and determine what element of the pattern will be in a particular position (e.g., the 16th position) if the pattern keeps going

Grade 3

Students study situations of change as they examine temperature change over time in different places around the world, analyze number sequences generated by repeating patterns, and consider a fantasy situation of constant change in which children receive a certain number of *Magic Marbles* each day. They make, read, and compare line graphs that show a relationship between two variables in situations of change over time. Students learn how to find the two values represented by a point on a coordinate graph by referring to the scales on the horizontal and vertical axes. Students focus on seeing a graph as a whole, thinking about the overall shape of a graph, and discussing what that overall shape shows about the change in the situation it represents. A class temperature graph is created over the course of the year and discussed regularly. Students learn to read and interpret temperatures using standard units.

Students also use tables as a representation that shows how one variable changes in relation to another variable. Emphasis is on how the numbers in the table relate to the situation they represent and to graphs of the same situation.



Day	Franick
Beginning	30
Day 1	33
Day 2	36
Day 3	39
Day 4	42
Day 5	45
Day 6	48
Day 7	51

Table and graph from Grade 3 Unit 6

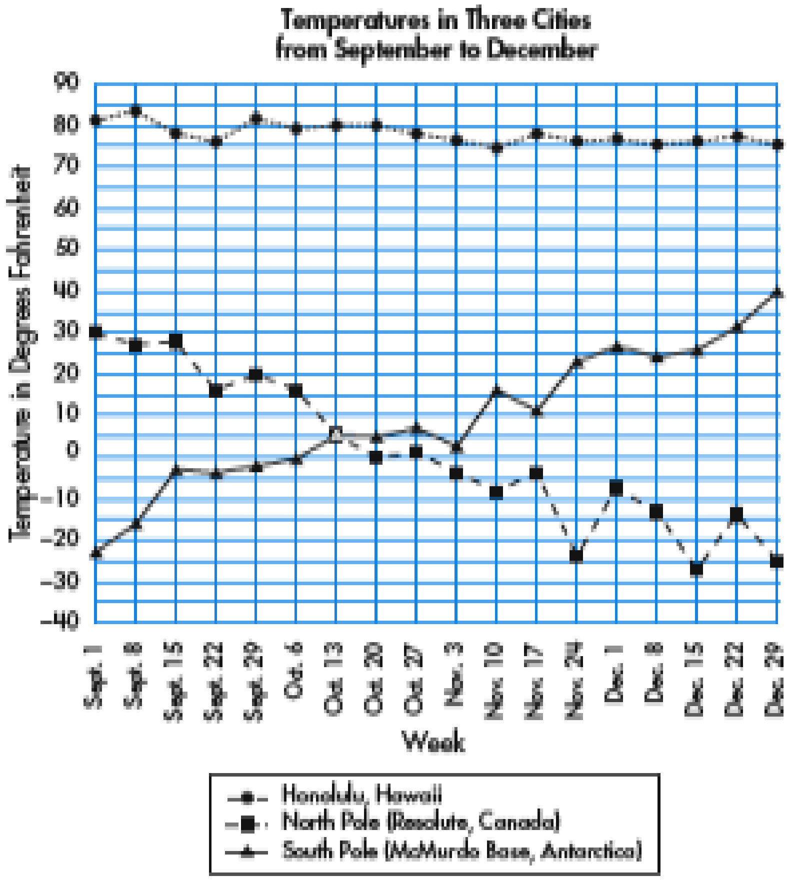


Table and graph from Grade 3 Unit 6 and *Line graph from Grade 3 Unit 6*

Students use both tables and graphs to examine and compare situations with a constant rate of change. They examine the relationship between columns of the table and consider why the points on graphs representing such situations fall in a straight line. By examining the tables and graphs, students consider any initial amount and the constant rate of change to develop general rules that express the relationship between two variables in these contexts.

Emphases

Using Tables and Graphs

- Using graphs to represent change
- Using tables to represent change

Linear Change

- Describing and representing a constant rate of change

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

Measuring Temperature

- Understanding temperature and measuring with standard units

Benchmarks

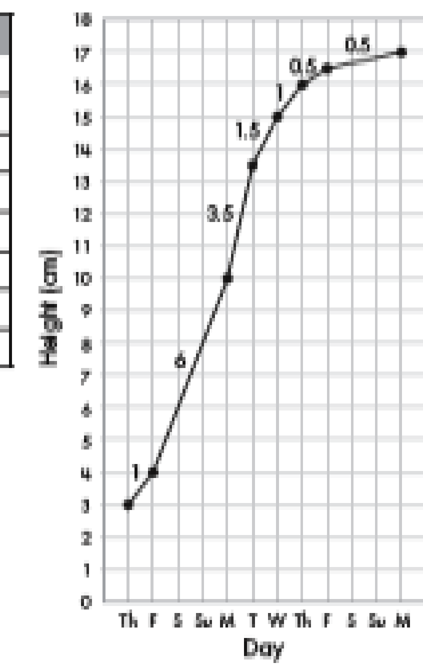
- Interpret graphs of change over time, including both the meaning of points on the graph and how the graph shows that values are increasing, decreasing, or staying the same
- Interpret temperature values (e.g., relate temperatures to seasons, to what outdoor clothing would be needed)
- Create a table of values for a situation with a constant rate of change and explain the values in the table in terms of the situation
- Compare related situations of constant change by interpreting the graphs, tables, and sequences that represent those situation

Grade 4

In Grade 4, students use graphs and tables to represent change. One focus of their work is how a line graph shows the *rate of change*, as they consider questions such as the following:
"How does this graph show the parts of the story that are about *speed* and the parts of the story that are about *changes in speed*?"

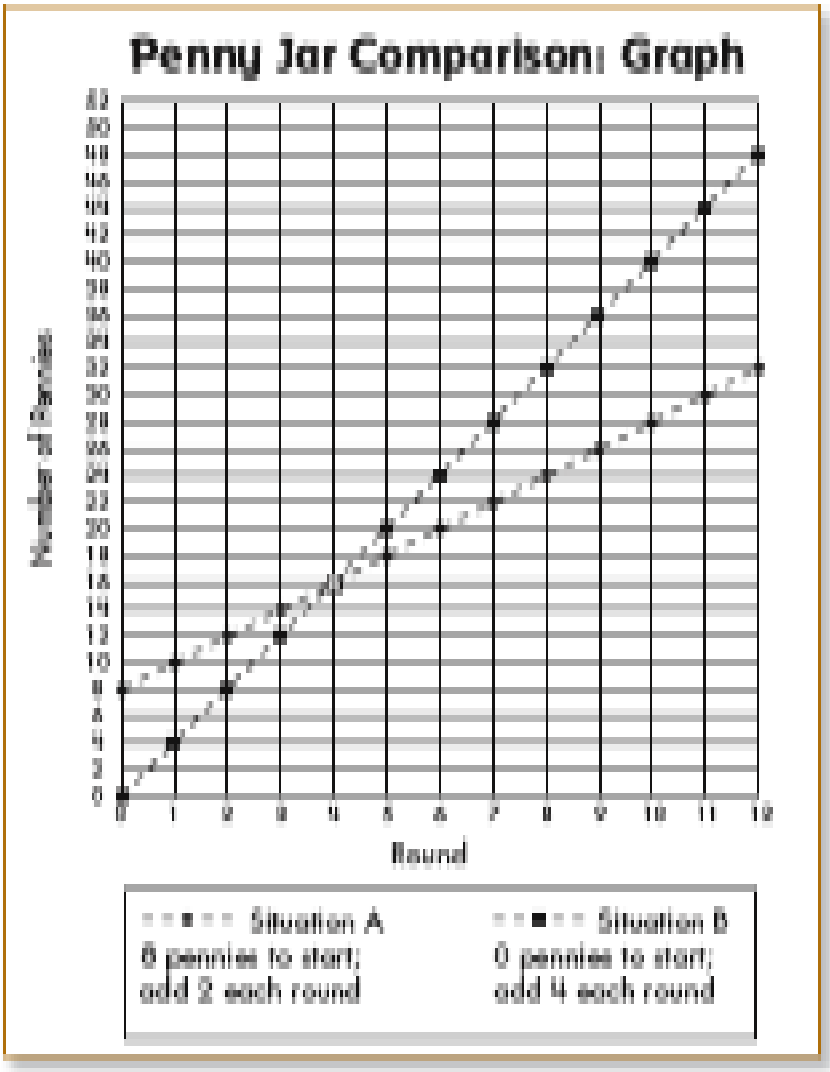
"What was the rate of growth for this plant? When was it growing more slowly or more quickly?"

Day	Height
Thursday	3 cm
Friday	4 cm
Monday	10 cm
Tuesday	13.5 cm
Wednesday	15 cm
Thursday	16 cm
Friday	16.5 cm
Monday	17.5 cm



Students create tables and graphs for situations with a constant rate of change and use them to compare related situations.

For example: Penny Jar A has 8 pennies in the jar to start and 2 pennies are added in each round; Penny Jar B has 0 pennies in the jar to start and 4 pennies are added in each round. Will the number of pennies in Penny Jar B, which starts with fewer pennies, ever "catch up" to the number of pennies in Penny Jar A?



By analyzing tables and graphs, students consider how the starting amount and the rate of change define the relationship between the two quantities (e.g., number of rounds, total number of pennies), and develop rules that govern that relationship. At first students articulate these rules in words (as they did in grade 3), but they also are introduced to the use of symbolic notation and equations to represent their rules. They use these rules to determine the value of one variable when the value of the other is known.

How many pennies are in Penny Jar A after 10 rounds?

Emphases

Using Tables and Graphs

- Using tables to represent change
- Using tables to represent change

Linear Relationships

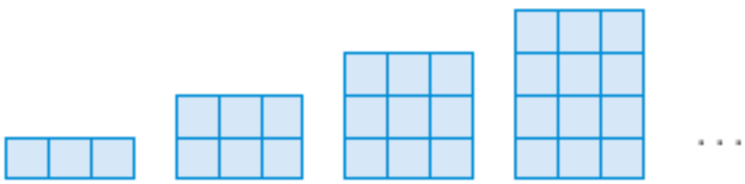
- Describing and representing a constant rate of change

Benchmarks

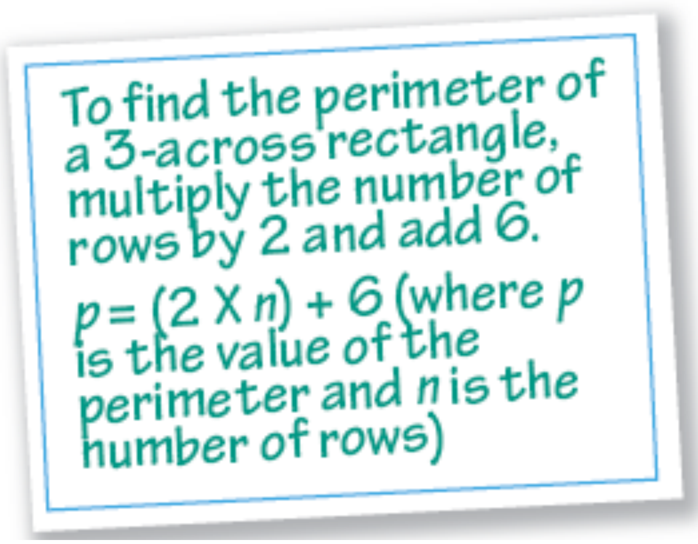
- Connect tables and graphs of change over time to each other and to the situations they represent.
- Make a graph on a coordinate grid from a table of values.
- Describe how a graph shows change: where the rate of change is increasing, decreasing, or remaining constant, and how differences in steepness represent differences in the rate of change.
- Take into account the starting amount and the amount of change in describing and comparing situations of constant change.
- In a situation of constant change, write rules (using words or arithmetic expressions) to determine the value of one quantity, given the value of the other.

Grade 5

In Grade 5, students continue their work from Grades 3 and 4 by examining, representing, and describing situations in which the rate of change is constant. Students create tables and graphs to represent the relationship between two variables in a variety of contexts. They also articulate general rules for each situation. For example, consider the perimeters of the following set of rectangles made from rows of tiles with three tiles in each row:



If the value of one variable (the number of rows of three tiles) is known, the corresponding value of the other variable (the perimeter of the rectangle) can be calculated. Students express these rules in words and then in symbolic notation. For example:



For the first time in Grade 5, students create graphs for situations in which the rate of change is itself changing–for example, the change in the area of a square as a side increases by a constant increment–and consider why the shape of the graph is not a straight line as it is for situations with a constant rate of change.

Throughout their work, students move among tables, graphs, and equations and between those representations and the situation they represent. Their work with symbolic notation is closely related to the context in which they are working. By moving back and forth between the contexts, their own ways of describing general rules in words, and symbolic notation, students learn how this notation can carry mathematical meaning.

Emphases

Using Tables and Graphs

- Using graphs to represent change
- Using tables to represent change

Linear Change

- Describing and representing a constant rate of change

Nonlinear Change

- Describing and representing situations in which the rate of change is not constant

Benchmarks

- Connect tables and graphs to represent the relationship between two variables
- Use tables and graphs to compare two situations with constant rates of change
- Use symbolic notation to represent the value of one variable in terms of another variable in situations with constant rates of change