Teacher Note

About the How Many of Each? Problems in This Unit

In Unit 1, students solved several How Many of Each? Problems. The goal was for students to be able to do the following:

- make sense of problems of this type, which involve coordinating three pieces of information (i.e., the number of blocks, the number of marbles, and the total number of toys)
- find at least one solution
- develop ways to show such solutions on paper
- solve problems that have more than one correct solution

By the end of Unit 1 and at the beginning of this unit, most students can find more than one solution to a How Many of Each? problem. However, many students work almost randomly, not noticing relationships among solutions and not using one solution to find another. Although they keep a record of their work, many students do not notice when they arrive at the same solution more than once. Some may even tell you that they have found all possible solutions—not because they have ways to check, but because they have so many solutions that they think there could not possibly be more.

The focus of the How Many of Each? work in this unit is on finding all of the solutions for a given context (or as many as students can) and thinking about how to show that students have them all. Note that many first graders will be unable to find and show that they have found all of the possible 2-addend combinations of a given number. Yet, all students are challenged in this way because it encourages them to find relationships among solutions and to think and work more systematically.

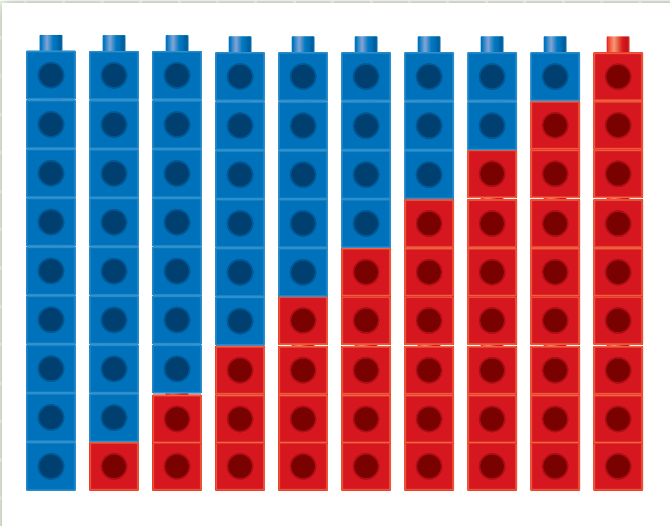
Using One Solution to Find Another

The challenge of finding as many 2-addend combinations of a number as possible often moves students into thinking about whether the solutions they have found provide clues for finding others. For example, as they look for more combinations, some students discover that if 8 marbles and 1 block is a solution, so is 1 marble and 8 blocks; if 4 marbles and 5 blocks is a solution, so is 5 marbles and 4 blocks. By reversing the numbers in one solution, they find a second.

Another way students derive new combinations is to take one marble in a solution they have and turn it into one block. That is, if 8 marbles and 1 block is a solution, then so is 7 marbles and 2 blocks. They can also take one block and make it one marble. If 2 marbles and 7 blocks is a solution, so is 3 marbles and 6 blocks. See **Teacher Note:** Finding Relationships Among Solutions, page 179, for more information.

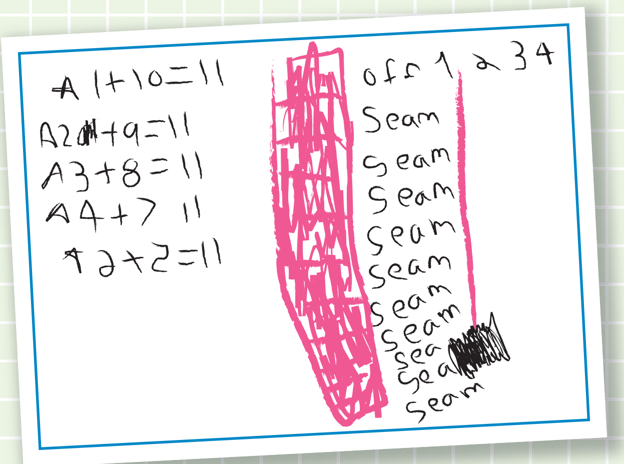
Showing You Have Them All

For many students, figuring out whether they have all of the possible combinations requires finding a systematic way of recording or showing solutions, a way that will "catch" all possibilities. In one class, a child presented the following cube structure to show that she had found all of the possible combinations of 9 blocks and marbles.



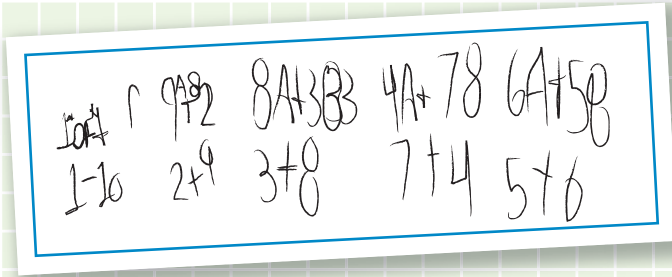
Each column represents 1 solution. The first is 0 blocks and 9 marbles; the next is 1 block and 8 marbles; the next is 2 blocks and 7 marbles; up to 9 blocks and 0 marbles. It is impossible to have any more than 9 blocks or 9 marbles, and all the possibilities less than 9 are included.

Other students may organize this information in a list or attempt to describe their process. Consider these examples of student work for a How Many of Each? problem about 11 apples and bananas.

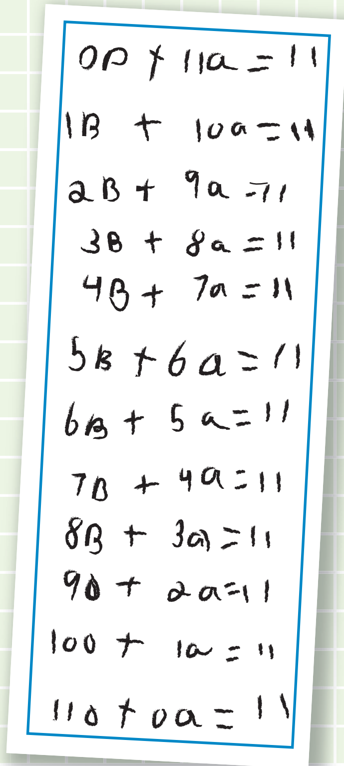


Tamika's Work

Tamika says, "Take 1 off, then 2, then 3 and keep doing the same ("seam")." The explanation that underlies all of the student work is the same: You cannot go higher than 11, and all possible values less than 11 are included.



Sample Student Work



Sample Student Work

Although some students may think that solutions with 0 blocks or 0 marbles are not correct because the problem says "*some* are marbles and *some* are blocks," students who include 0 blocks and 9 marbles and 9 blocks and 0 marbles are taking this problem a step beyond its context and phrasing. They are determined to find *all* of the combinations of 9.