SESSION 2.3

Multiple Turn Over

Math Focus Points

- Determining whether one number is a factor or multiple of another
- Identifying and learning multiplication combinations not yet known fluently
- Using known multiplication combinations to determine the products of more difficult combinations

Vocabulary

multiple
factor

Today's Plan		Materials
1 ACTIVITY **Finding Factors and Multiples**	20 MIN INDIVIDUALS PAIRS	• Chart: "Factors and Multiples"* • Chart: "Factors and Multiples: Yes or No?"* • Calculators (optional)
2 ACTIVITY **Introducing *Multiple Turn Over***	15 MIN CLASS	• M45*; M46–M49*; M50* • T26
3 ACTIVITY **Playing *Multiple Turn Over***	25 MIN GROUPS PAIRS	• *Student Activity Book,* p. 19 • M46–M49; M50 • Calculators (optional)
4 SESSION FOLLOW-UP **Daily Practice and Homework**		• *Student Activity Book,* pp. 20–22 • *Student Math Handbook,* pp. 22, 24, 25; G9 • M41*

*See *Materials to Prepare*, p. 55.

Ten-Minute Math

Quick Images: Seeing Numbers Show *Quick Images: Seeing Numbers* (T24), Images 3 and 4 (one at a time). For each pattern, ask students to write several different equations to find the total number of dots. For the first two viewings, give students 3 seconds to look at the pattern; the third time, leave the image displayed. Have two or three students explain how they saw the images (including any revisions they made) and their equations, showing how their numbers match the patterns.

ACTIVITY

20 MIN INDIVIDUALS PAIRS

Finding Factors and Multiples

Before introducing the game *Multiple Turn Over,* use this activity to reinforce the meaning of the terms *factor* and *multiple* and the difference between them.

Start by showing students the "Factors and Multiples" chart you prepared. Throughout the activity, add words and drawings to record students' ideas about factors and multiples. Post this chart where students can refer back to it throughout the Investigation.

Sometimes the words *factor* and *multiple* can be confusing. Let's talk about those terms while we make a class chart to help us remember the difference between them.

Try to think back to third grade, when you skip counted on a 100 chart. In skip counting, which numbers are factors and which numbers are multiples?

Students may recall that a factor is the number you are skip counting by (2s, 6s, 10s, etc.), and the multiples of that number are the numbers you land on while skip counting (6, 12, 18, etc.). Add an example of skip counting to the chart.

Factors and Multiples

Factor	Multiple
the number you count by when you skip count	the numbers you land on
2	2, 4, 6, 8, 10

Where do you land if you skip count by 5 four times? Six times? How would you write these as multiplication equations?

When students come up with the equations, write them on the board.

$$4 \times 5 = 20$$

$$6 \times 5 = 30$$

How do these equations show that 20 and 30 are multiples of 5?

Algebra Note

❶ **Using Landmark Combinations** When deciding whether 3 is a factor of 51, some students may use $10 \times 3 = 30$ as a landmark combination and then reason as follows:

$10 \times 3 = 30$

$51 - 30 = 21$

$7 \times 3 = 21$

So, $17 \times 3 = 51$.

Teaching Note

❷ **Using Calculators to Find Factors** Students may use calculators to find factors of some large 2-digit or 3-digit numbers. If your class is not familiar with the way a calculator could be used for this, spend 10 minutes exploring this with them. Students may realize that they can try out different factors by skip counting on the calculator (e.g., 6 + 6 + 6 + 6 . . .) or that they can use division (*"72 ÷ 6 = 12, so I know that 6 and 12 are factors of 72"*). Read Part 5 of *Implementing Investigations in Grade 4,* Technology in *Investigations:* Calculators and Computers.

Listen for the understanding that 20 and 30 are multiples of 5 because they are the product found by multiplying 5 by a whole number.

How can you use the word *factor* for some of the numbers in the equations?

Students should understand that 4 and 5 are factors of 20 because their product is 20. Similarly, 5 and 6 are factors of 30. Add a labeled equation to the "Factors and Multiples" chart.

4	×	5	=	20
factor		factor		multiple

Next, turn students' attention to the three-column chart you prepared, "Factors and Multiples: Yes or No?" Write 3 in the "Factor" column and 51 in the "Multiple" column.

Factor	Multiple	Yes or No?
3	51	

Here's a problem for you: Is 3 a factor of 51? Turn to someone sitting near you and work together for a few minutes to answer that question.

Bring the class back together, and ask students to offer ways to solve the problem.❶

Discuss various strategies, which might include the following:

- Think of the problem as a missing factor problem, $3 \times ? = 51$. Some students might first narrow their choices and try only numbers greater than 10 (because $3 \times 10 = 30$) and less than 20 (because $3 \times 20 = 60$).
- Think of the problem as division, $51 \div 3 = ?$ If the answer is a whole number, then 3 is a factor of 51.
- Skip count by adding 3s until 51 is reached. Some students will repeatedly add 3. Others may use a calculator and enter 3 + 3 and then press the equal key repeatedly.❷

Some students might use a calculator to determine whether one number is a factor of another.

Give students several more problems to try. Solicit responses to the new problems, and ask students to explain their reasoning.

Students might say:

"6 is not a factor of 86. I added 6 a lot of times on the calculator and didn't land on 86."

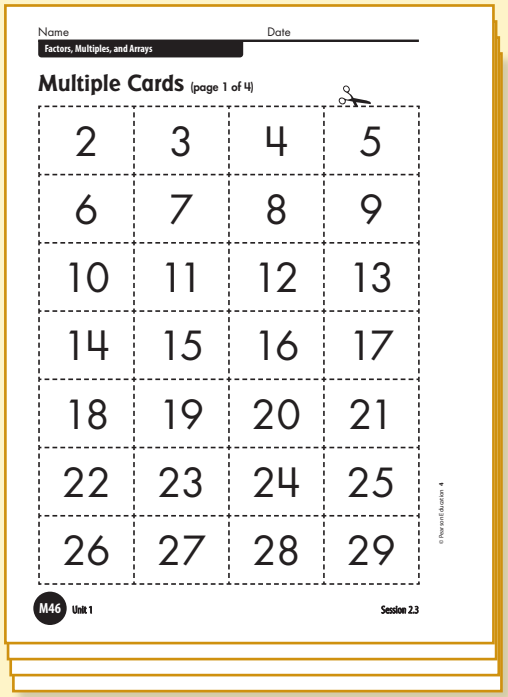
"4 is not a factor of 58. I know because 4 × 15 = 60, and 58 is only 2 less."

"9 is a factor of 108. I just know that 9 × 12 is 108."

ONGOING ASSESSMENT: Observing Students at Work

Students determine whether a number is a multiple of a given factor.

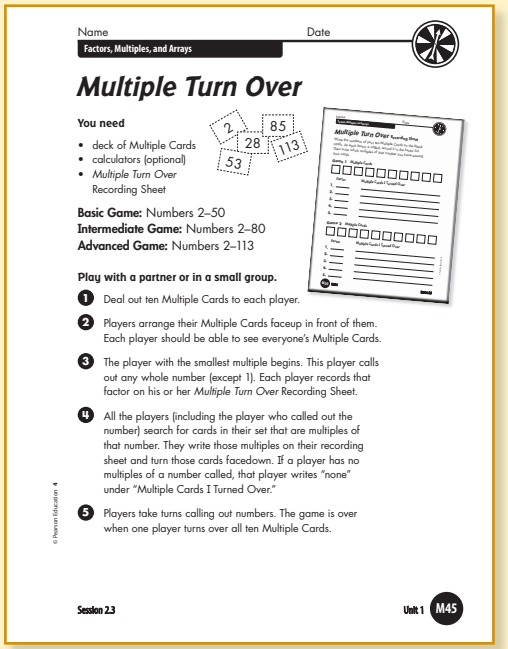
- **Are students using the terms factor and multiply correctly?**
- **How do students determine if one number is a factor of another?** Do they skip count? Use a calculator? Use what they know?


Name Date
Factors, Multiples, and Arrays
Multiple Cards (page 1 of 4)

2	3	4	5
6	7	8	9
10	11	12	13
14	15	16	17
18	19	20	21
22	23	24	25
26	27	28	29

M46 Unit 1 Session 2.3

▲ Resource Masters, M46–M49


Name Date
Factors, Multiples, and Arrays
Multiple Turn Over

You need
- deck of Multiple Cards
- calculators (optional)
- *Multiple Turn Over* Recording Sheet

Basic Game: Numbers 2–50
Intermediate Game: Numbers 2–80
Advanced Game: Numbers 2–113

Play with a partner or in a small group.
1. Deal out ten Multiple Cards to each player.
2. Players arrange their Multiple Cards faceup in front of them. Each player should be able to see everyone's Multiple Cards.
3. The player with the smallest multiple begins. This player calls out any whole number (except 1). Each player records that factor on his or her *Multiple Turn Over* Recording Sheet.
4. All the players (including the player who called out the number) search for cards in their set that are multiples of that number. They write those multiples on their recording sheet and turn those cards facedown. If a player has no multiples of a number called, that player writes "none" under "Multiple Cards I Turned Over."
5. Players take turns calling out numbers. The game is over when one player turns over all ten Multiple Cards.

Session 2.3 Unit 1 M45

▲ Resource Masters, M45

As students work, notice the strategies they use. For example, if you see students skip counting on a calculator and starting at 0, suggest that they start with a multiplication combination that will get them as close as possible to the target number. Ask questions such as the following:

- I see that you're skip counting to see whether 9 is a factor of 108. You started at 0. Do you know a multiplication combination with 9 that will get you closer to 108? Do you know 10 × 9?
- You said that 10 × 9 is 90. That's like skip counting by 9 ten times. So start at 90, and see whether you can skip count to get to 108. How many more times did you have to skip count?

2 ACTIVITY

15 MIN CLASS

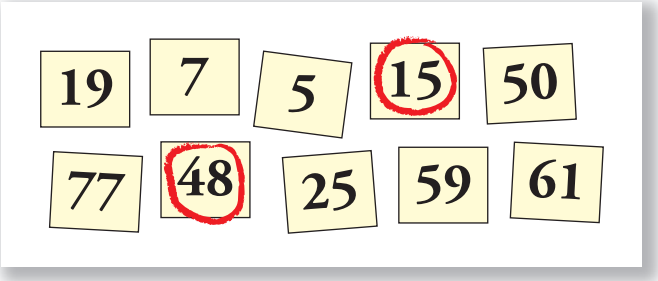
Introducing *Multiple Turn Over*

Multiple Turn Over is a game in which students are encouraged to use numerical reasoning based on what they know about factors and multiples, such as the facts that all even numbers are multiples of 2 and multiples of 5 must end in either 5 or 0.

Play a few rounds with the whole class, introducing the rules as explained on *Multiple Turn Over* (M45). For this introduction, limit the Multiple Cards to those in the intermediate set, 2–80. Randomly choose ten cards from that set, and draw them on the board (or display transparent cards). Instruct students to randomly choose ten cards from their own decks and place these cards face up in front of them.

Begin by naming a factor. Start with a small number such as 3. (Do not start with 2 because too many numbers are its multiples.) Players pick out all the multiples of 3 from among the numbers in front of them. They turn these cards facedown.

Circle the multiples of 3 among the Multiple Cards you have drawn on the board. Explain the thinking that has led you to select those cards.



The factor that I named was 3. I saw that I had the Multiple Card 15, and I selected it because 5 × 3 = 15. I also had the Multiple Card 48. I wasn't sure about this number, but I knew that 10 × 3 = 30. Because 6 × 3 = 18, I knew that six more 3s would get me from 30 to 48.

As you play, demonstrate how to keep track of each round on the transparent *Multiple Turn Over* Recording Sheet (T26). After you have recorded 15 and 48, erase the numbers on the cards you have drawn on the board to indicate that they are "turned over."

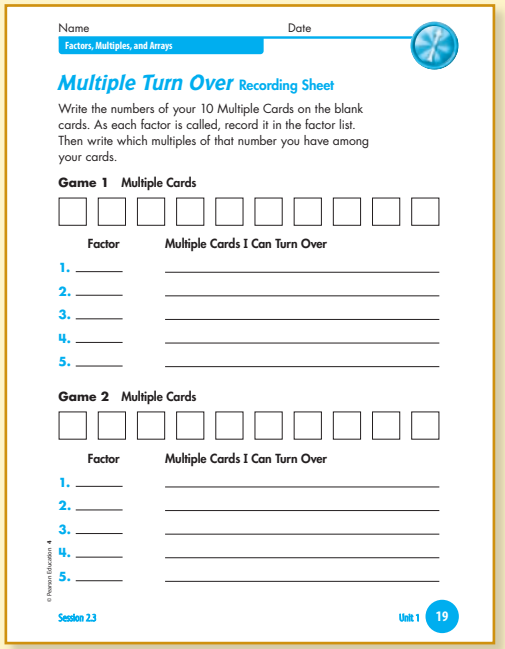
A volunteer then suggests another factor. All players pick out all the multiples of that number from among their remaining Multiple Cards and turn them over. Continue to model your own thinking as you select Multiple Cards from those left on the board, and ask for a couple of volunteers to model their thinking as well.

ACTIVITY 3 Playing *Multiple Turn Over*

25 MIN PAIRS GROUPS

Students play *Multiple Turn Over* in pairs or small groups for the remainder of the session. They use the recording sheet on *Student Activity Book* page 19. Have calculators available for use as desired, and make available extra recording sheets (M50) as needed. Let students know that they will also have time to play the game in the Math Workshops in the next two sessions.❸ ❹

Multiple Turn Over *gives students practice with identifying multiples of a given number.*



Name Date

Factors, Multiples, and Arrays

Multiple Turn Over Recording Sheet

Write the numbers of your 10 Multiple Cards on the blank cards. As each factor is called, record it in the factor list. Then write which multiples of that number you have among your cards.

Game 1 Multiple Cards

Factor — Multiple Cards I Can Turn Over

1. _____
2. _____
3. _____
4. _____
5. _____

Game 2 Multiple Cards

Factor — Multiple Cards I Can Turn Over

1. _____
2. _____
3. _____
4. _____
5. _____

Session 2.3 Unit 1 19

▲ Student Activity Book, p. 19; Resource Masters, M50; T26

Teaching Note

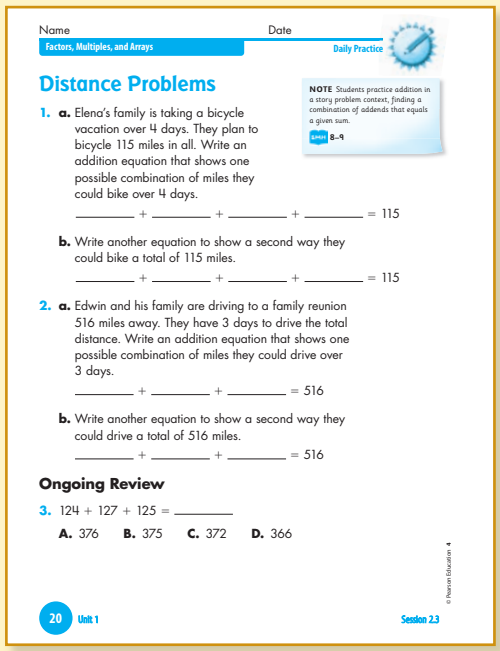
❸ **Winning Strategies** As students become accomplished at the game *Multiple Turn Over,* they develop strategies to help them become the first player to turn over all ten Multiple Cards. They become increasingly fluent with the multiples of each number. They learn to call factors of their own numbers that are *not* factors of the other player's numbers. They notice that they must name the number itself in order to turn over a prime number.

Professional Development

❹ **Dialogue Box:** Identifying Factors and Multiples in *Multiple Turn Over,* p. 133

Professional Development

Part 5: Technology in *Investigations:* Calculators and Computers in *Implementing Investigations in Grade 4:* Using Calculators with the Curriculum



Name Date

Factors, Multiples, and Arrays — Daily Practice

Distance Problems

NOTE Students practice addition in a story problem context, finding a combination of addends that equals a given sum.

Goals 8–9

1. **a.** Elena's family is taking a bicycle vacation over 4 days. They plan to bicycle 115 miles in all. Write an addition equation that shows one possible combination of miles they could bike over 4 days.

_____ + _____ + _____ + _____ = 115

b. Write another equation to show a second way they could bike a total of 115 miles.

_____ + _____ + _____ + _____ = 115

2. **a.** Edwin and his family are driving to a family reunion 516 miles away. They have 3 days to drive the total distance. Write an addition equation that shows one possible combination of miles they could drive over 3 days.

_____ + _____ + _____ = 516

b. Write another equation to show a second way they could drive a total of 516 miles.

_____ + _____ + _____ = 516

Ongoing Review

3. 124 + 127 + 125 = _____

A. 376 **B.** 375 **C.** 372 **D.** 366

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▲ Student Activty Book, p. 20

ONGOING ASSESSMENT: Observing Students at Work

Students solve problems in which they determine which numbers are multiples of a given number.

- **What characteristics of multiples do students use?** For example, do students know that even numbers are multiples of 2, or that multiples of 5 end in 5 or 0?
- **Do students use knowledge of multiplication combinations?** Do they recognize most products of multiplication combinations up to 12 × 12?
- **How do students determine factors of more difficult multiples?** Do they skip count? Do they reason from multiplication combinations they know?
- **Are students also considering the Multiple Cards in the other player's hand when they choose which factors to name?**

DIFFERENTIATION: Supporting the Range of Learners

There are three levels of *Multiple Turn Over.* Use your observations of the work students did with Multiplication Cards in Session 2.2, page 67, to help them determine the level at which they should begin.

Basic level (numbers 2–50) Those students still learning the multiplication facts with products to 50 should play the basic level, which students played in Grade 3.

Intermediate level (numbers 2–80) Most students will probably start with the intermediate level. This allows them to review the multiples to 50 they worked on in Grade 3, while using this knowledge to determine the factors of larger numbers.

Advanced level (numbers 2–113) Some students may be ready to work with all of the numbers in the deck and should start with the advanced level.

Intervention Some students will use the calculator to skip count to determine whether a number is a multiple of a particular factor. If you notice students skip counting from 0, ask questions to encourage them to start with a multiplication combination that will get them as close as possible to the target number. For example:

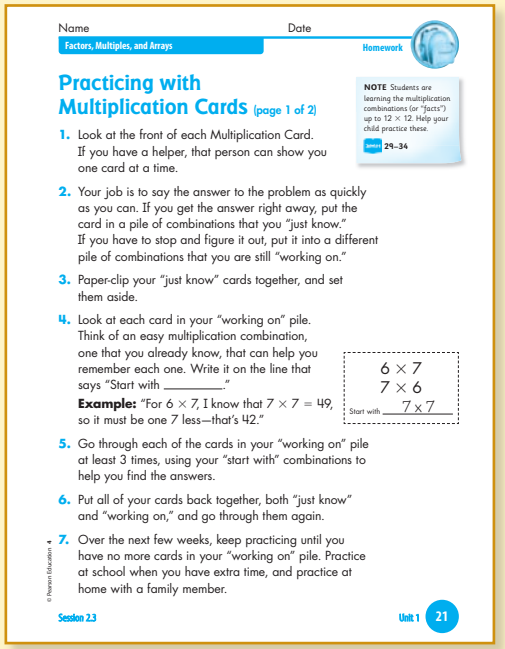
- I see that you're skip counting on the calculator to see whether 6 is a factor of 72. You started at 0, with 6 + 6. Is there a multiplication combination you know, with 6 as one of the factors, that will get you closer to 72? Do you know 6 × 6, or 10 × 6? Can you start with the product of one of those combinations?
- You said that 6 × 6 is 36, and that's like skip counting by 6 six times. If you start at 36, how many more times will you have to count by 6 to get to 72?

SESSION FOLLOW-UP 4
Daily Practice and Homework

Daily Practice: For ongoing review, have students complete *Student Activity Book* page 20.

Homework: For practice at home with the multiplication combinations to 12 × 12, have students fill in a copy of Blank Multiplication Cards (M41) with combinations from their "working on" sets. Suggest that they cut apart these cards at home and practice them with a friend or family member. *Student Activity Book* page 21 provides a reminder of the routine, and on page 22, students record how they worked on the combinations they have chosen.

Student Math Handbook: Students and families may use *Student Math Handbook* pages 22, 24, 25 and G9 for reference and review. See pages 134–139 in the back of this unit.



Name Date

Factors, Multiples, and Arrays — Homework

Practicing with Multiplication Cards (page 1 of 2)

NOTE Students are learning the multiplication combinations (or "facts") up to 12 × 12. Help your child practice these.

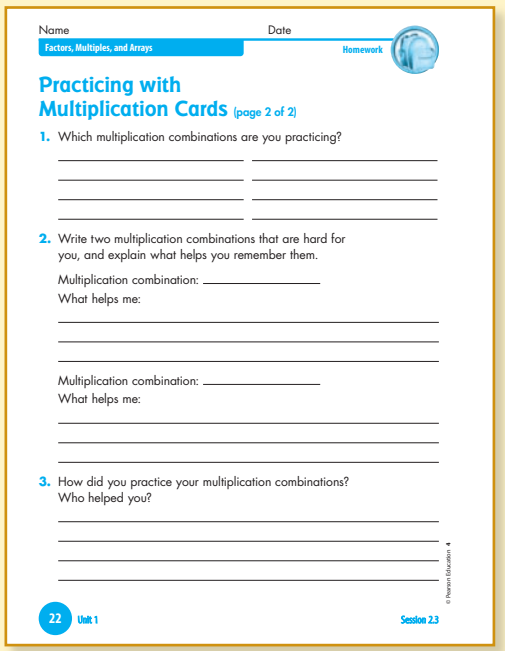
Games 29–34

1. Look at the front of each Multiplication Card. If you have a helper, that person can show you one card at a time.
2. Your job is to say the answer to the problem as quickly as you can. If you get the answer right away, put the card in a pile of combinations that you "just know." If you have to stop and figure it out, put it into a different pile of combinations that you are still "working on."
3. Paper-clip your "just know" cards together, and set them aside.
4. Look at each card in your "working on" pile. Think of an easy multiplication combination, one that you already know, that can help you remember each one. Write it on the line that says "Start with _____."
   Example: "For 6 × 7, I know that 7 × 7 = 49, so it must be one 7 less—that's 42."

   6 × 7
   7 × 6
   Start with 7 × 7
5. Go through each of the cards in your "working on" pile at least 3 times, using your "start with" combinations to help you find the answers.
6. Put all of your cards back together, both "just know" and "working on," and go through them again.
7. Over the next few weeks, keep practicing until you have no more cards in your "working on" pile. Practice at school when you have extra time, and practice at home with a family member.

Session 2.3 — Unit 1 21

▲ Student Activity Book, p. 21



Name Date

Factors, Multiples, and Arrays — Homework

Practicing with Multiplication Cards (page 2 of 2)

1. Which multiplication combinations are you practicing?

2. Write two multiplication combinations that are hard for you, and explain what helps you remember them.
   Multiplication combination: _____
   What helps me:

   Multiplication combination: _____
   What helps me:

3. How did you practice your multiplication combinations? Who helped you?

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▲ Student Activity Book, p. 22