Algebra Connections

IN THIS UNIT

In this unit, your students will have opportunities to engage with ideas that lay a foundation for algebra. Five-year-olds can and do think algebraically. Part of the work of Kindergarten is helping students learn to verbalize those thoughts, both as a way of engaging with generalizations about numbers and operations and as a foundation for meaningful use of algebraic notation in the future.

Many times over the course of a year, kindergarteners will work on the idea that a number describes the size of a set. The number of objects in a set is fixed no matter how it is arranged and counted, and different sets may have the same number of objects. Explorations of these issues offer opportunities for algebraic discussions. For example, consider the following vignette in which students are counting a set of red and yellow rods.

Emma has arranged the rods, placing the five red rods in a row above the three yellow rods. Jason counts them and the class agrees that there are eight.

Teacher: I noticed that when Jason counted, he counted the red ones first and the yellow ones second. Is there anybody who can count the yellow ones first and the red ones second? [The teacher rearranges the rods so that the reds are below the yellows. Mia volunteers to count them.]

Teacher: How many do you think she'll end up with if she starts with the yellow? [Many students say "the same thing."] The same thing? Carmen, why do you think she's going to end up with the same thing if we count them in a different order?

Carmen: Because you'll still have all the other ones unless you just take one away.

Teacher: Okay, we're still going to have all the other ones unless we take one away. Are we going to take one away? [Carmen responds "No."] No, what are we doing that's different?

Carmen: You're doing them in a different order.

Teacher: We're doing it in a different order. I have a really important question for you. Do you think it *matters* if we change the order? [Some students say "Yes," some say "No."] What will happen if we count the yellows first?

Rebecca: Nothing.

Teacher: Rebecca says, "Nothing." Nothing's going to happen; it's not going to change it. Rebecca, how come nothing will happen?

Rebecca: 'Cause it's eight still there.

Teacher: Okay, it's eight of them still.

Mitchell: Yeah. When you change it around and you have the yellow ones, it's still the same number.

Teacher: Okay, you say that when you change them around—tell me if I have this right—if you change it around, you have yellow ones still.

Mitchell: And it's still the same number.

Teacher: And it's still going to be the same number. Should we test it and see? [The class says yes. Mia touches each rod as she counts them, starting with the yellow and then counting the red rods.] What happened?

Hugo: Same number. . . . It's always going to be the same number.

Teacher: Is this getting the same number no matter what order you count something special about the rods, or would it work with anything?

Emma: If you take the same number, it would be the same.

Teacher: Do you think it's *always* going to be the same number? Even if we change the order around?

Lionel: It doesn't matter what number it is; it's still going to be the same number.

The students in this class are discussing whether order matters when counting. They explain that whether one counts the red rods first and then the yellow rods, or the yellow rods first and then the red rods, the total number of rods remains the same. While most of the discussion focuses on the eight rods laid out before them, Lionel generalizes to other numbers. He says, "It doesn't matter what number it is; it's still going to be the same number."

This idea—does order matter when counting?—will appear again in other forms throughout the grades. As students come to understand addition, the question will change to whether order matters when adding. Does changing the order of addends change the result? They will be able to rely on a similar image of red and yellow rods and know that the total number of rods is independent of the position of each set.

Years from now, students may represent their answer by using algebraic notation, $a + b = b + a$, and refer to it as the commutative property of addition. They may also ask similar questions about the other operations and conclude that addition and multiplication are commutative and that subtraction and division are not.

But most kindergarteners are not yet thinking in terms of operations; they are developing their understanding of numbers through counting. They are coming to recognize numbers as representing quantity; it describes the size of a set of objects independent of what those objects are. A set of eight fingers, eight pencils, eight children, and eight noses all share the attribute of "eightness."

Students will work on the question of order when counting in a variety of other contexts over the course of the year. For example, "When we counted the number of students in class today, and we started with Hugo, there were 24 children. How many children do you think there will be if we start with Rebecca? Why do you think so?" As students discuss such questions, they are developing a sense that the number of objects in a given set is fixed, no matter how they are counted.

The example presented illustrates the kind of "early algebraic reasoning" that is accessible to kindergarteners. This early algebra work involves students in reasoning, generalizing, representing, and communicating. They explore questions that may begin with a particular problem—does it matter whether we first count five red rods and then three yellow rods or first three yellow rods and then five red rods?—but extend to a whole class of problems. When counting a set of objects, it does not matter what order one counts them; the result is the same, no matter how many objects are in the set.