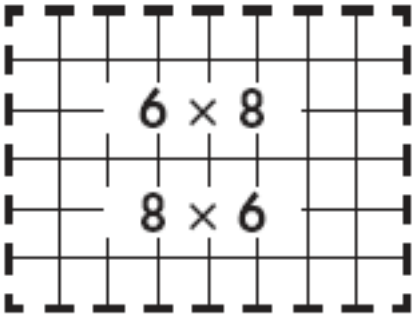
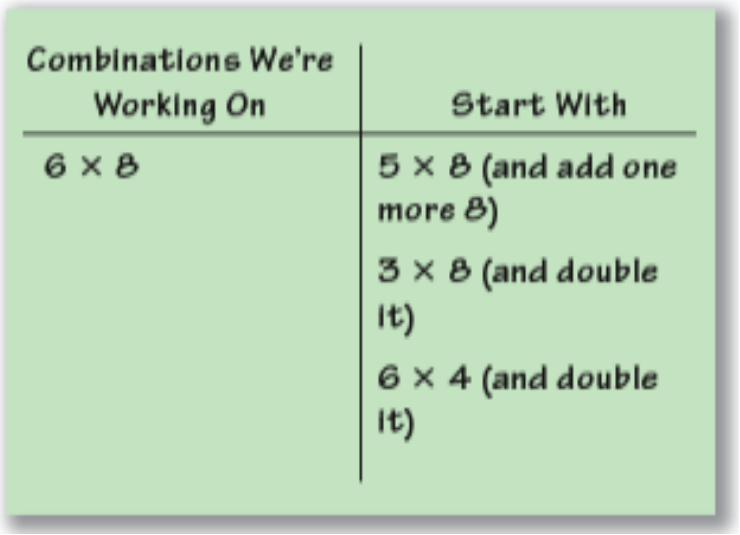
Grade 4 Math Content [1]

Number and Operations: Whole Numbers

Multiplication and Division

In Grade 4, three of the four curriculum units on number and operations with whole numbers focus on multiplication and division. This major component of students' work centers on reasoning about numbers and their factors and multiples, using models, representations, and story contexts to help them visualize and solve multiplication and division problems; and understanding the relationship between multiplication and division.





Combinations We're Working On	Start With
6 × 8	5 × 8 (and add one more 8) 3 × 8 (and double it) 6 × 4 (and double it)

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

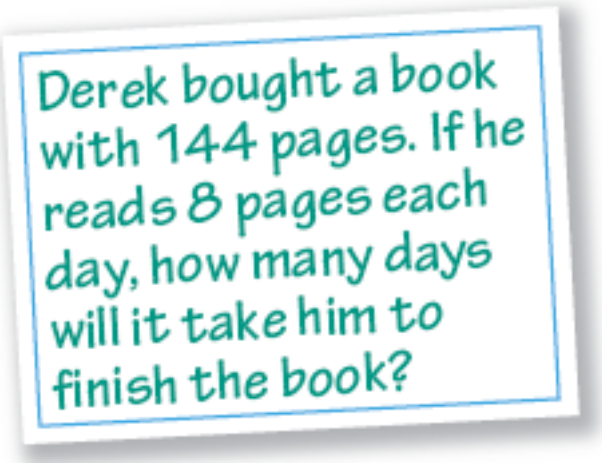
Students learn the multiplication combinations (facts) to 12 x 12 so that they can use these fluently to solve both multiplication and division problems. They develop strategies for solving multiplication and division problems based on looking at the problem as a whole, thinking about the relationships of the numbers in the problem, and choosing an approach they can carry out easily and accurately, often breaking the numbers apart or changing the numbers in some way. Visualizing how multiplication works is critical in applying the distributive property to solve problems and in keeping track of parts of the problem. Learning to multiply by multiples of 10 is also a key component of this work.

Examples of Multiplication Strategies

Breaking numbers apart by addition

48 x 42 =	**48 x 42 =**
40 x 40 = 1,600	48 x 40 = 1,920
40 x 2 = 80	48 x 2 = 96
8 x 40 = 320	1,920 + 96 = 2,016
8 x 2 = 16	
1,600 + 80 + 320 + 16 = 2,016	

Students interpret and solve division problems, both in story contexts and numerical contexts. They work with both grouping and sharing situations, and consider how to make sense of a remainder within the context of the problem. They use the inverse relationship between multiplication and division to solve division problems, including those related to the multiplication combinations to 12 x 12 (the division "facts"), and problems in which 3-digit numbers are divided by 1-digit and small 2-digit divisors.



The Algebra Connections pages in the three curriculum units that focus on multiplication and division show how students are applying the commutative and distributive properties of multiplication, as well as the inverse relationship between multiplication and division, as they solve problems. These pages also highlight particular generalizations about multiplication that students work on in Grade 4: If a number is a factor of a second number, are all the factors of the first number also factors of the second number? If one factor in a multiplication expression is halved and another factor is doubled, what is the effect on the product?

Emphases

Whole Number Operations

- Understanding and working with an array model of multiplication
- Reasoning about numbers and their factors
- Understanding and using the relationship between multiplication and division to solve division problems
- Understanding division as making groups of the divisor

Computational Fluency

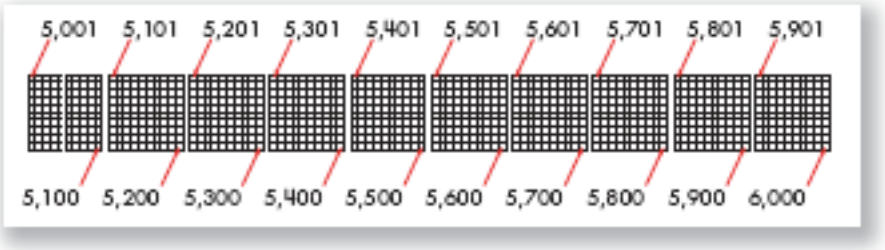
- Fluency with the multiplication combinations to 12 x 12
- Solving multiplication problems with 2-digit numbers

Benchmarks (compiled from Units 1, 3, and 8)

- Use known multiplication combinations to find the product of any multiplication combination to 12 x 12
- Use arrays, pictures or models of groups, and story contexts to represent multiplication situations
- Find the factors of 2-digit numbers
- Multiply 2-digit numbers by one-digit and small 2-digit numbers (e.g. 12, 15, 20), using strategies that involve breaking the numbers apart
- Solve division problems (2- and small 3-digit numbers divided by 1-digit numbers) including some that result in a remainder
- Use story problems, pictures, or concrete models to represent division situations
- Multiply by 10 and multiples of 10
- Demonstrate fluency with multiplication combinations to 12 x 12
- Multiply 2-digit numbers efficiently
- Solve division problems with 1- and small 2-digit divisors by using at least one strategy efficiently

Addition, Subtraction, and the Number System

In Grade 4, students extend their knowledge of the base-ten number system, working with numbers up to 10,000. Their work focuses on understanding the structure of 10,000 and how numbers are related within that structure, recognizing the place value of digits in large numbers, and using place value to determine the magnitude of numbers.

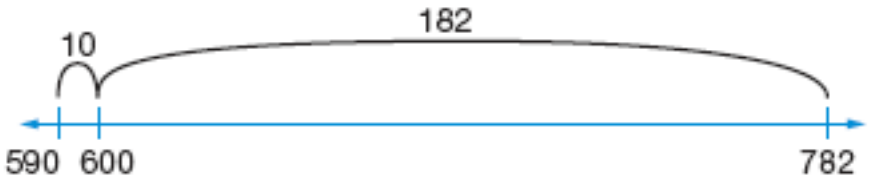


By discussing, refining and comparing their strategies for adding and subtracting 3- and 4-digit numbers, including studying the U.S. algorithm for addition, students continue expanding their understanding of addition and subtraction. Their strategies should involve good mental arithmetic, estimation, clear and concise notation, and a sound understanding of number relationships. By identifying and naming addition and subtraction strategies that they are using, students are adding to the repertoire of strategies they can use for flexible and fluent computation. Further, they consider how and why certain methods work. For example, some students change one or both numbers in an addition or subtraction expression to create an easier problem, then compensate as needed for that change. In this unit, students study *why* certain addition expressions are equivalent (e.g., 457 + 198 = 455 + 200) and *how* certain expressions in subtraction are related (e.g., 782 – 590 and 782 – 600).

782 – 590 = _____

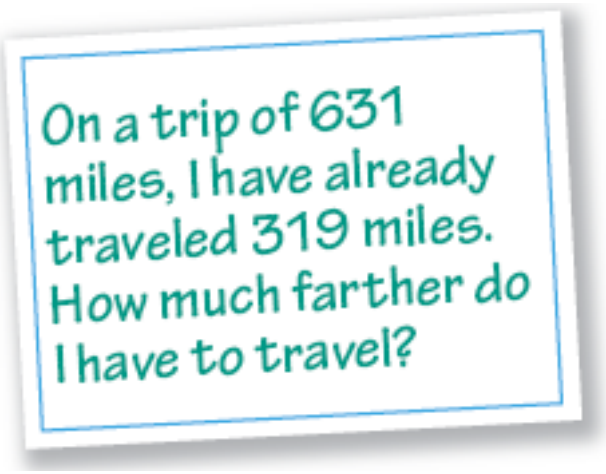
782 – 600 = 182

182 + 10 = 192



To help them make good decisions about strategies for subtraction and continue to develop their understanding of how subtraction operates, students use visual representations, such as number lines and 100 Charts, and story contexts that include

several types of subtraction situations—removal (or take away), comparison, and missing parts. Students focus particularly on missing part problems in the context of distance:



Some students visualize a problem like this one as adding up from the distance traveled to the total distance, while others visualize subtracting the distance traveled from the total distance. This provides another opportunity for students to consider the relationship between addition and subtraction.

319 + ____ = 631

631 – 319 = ____

The Algebra Connections page in the curriculum unit that focuses on addition and subtraction shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic ideas that underlie the generalizations students investigate and articulate when they create equivalent expressions in order to solve a problem (e.g., 124 – 89 = 125 – 90).

Emphases

The Base Ten Number System

- Extending knowledge of the base-ten number system to 10,000

Computational Fluency

- Adding and subtracting accurately and efficiently

Whole Number Operations

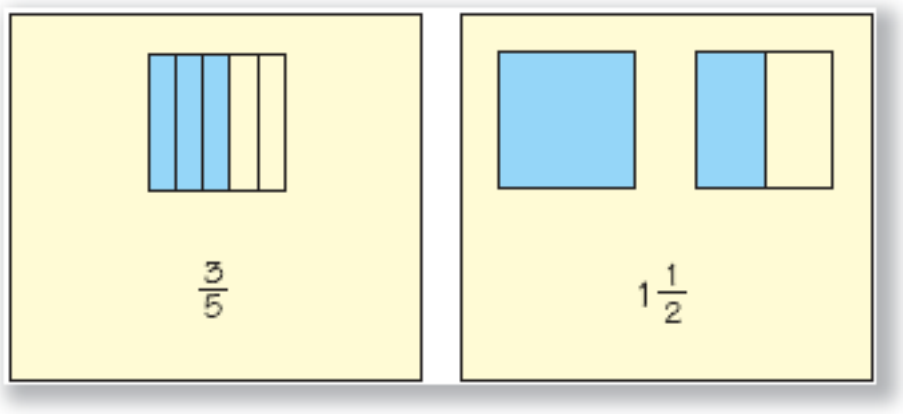
- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers
- Understanding different types of subtraction problems

Benchmarks

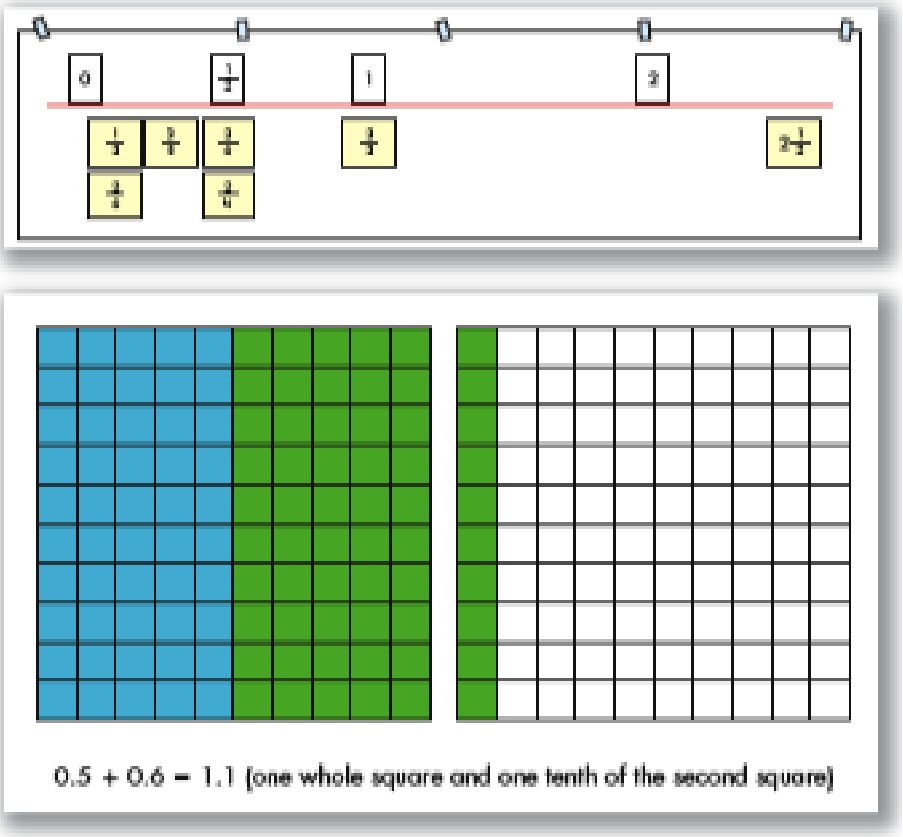
- Read, write, and sequence numbers to 10,000
- Add and subtract multiples of 10 (including multiples of 100 and 1,000) fluently
- Solve addition problems efficiently, choosing from a variety of strategies
- Solve subtraction problems with 3-digit numbers by using at least one strategy efficiently

Number and Operations: Rational Numbers

The major focus of the work on rational numbers in Grade 4 is on building students' understanding of the meaning, order, and equivalencies of fractions and decimals. Students continue to focus on the meaning of fractions as equal parts of a whole. They extend their images of equal parts to accommodate fractions that are greater than 1. Students work with fractions in the context of area (equal parts of a rectangle), as a group of things (e.g., a fractional part of the class), and on a number line. They work with fractions that represent halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths.



Students are introduced to decimal fractions in tenths and hundredths as an extension of the place value system they have studied for whole numbers. They relate decimals to equivalent decimals and fractions (for example, when they represent 0.25 as part of a rectangle, they can see how it is equal to ¼ and to 2 ½ tenths). Students draw on their mental images of fractions and decimals and on their knowledge of fraction and decimal equivalencies and relationships to reason about fraction comparisons, to order fractions on a number line, and to add fractions and decimals using representations.



Emphases

Rational Numbers

- Understanding the meaning of fractions and decimal fractions
- Comparing the values of fractions and decimal fractions

Computation with Rational Numbers

- Using representations to add rational numbers

Benchmarks

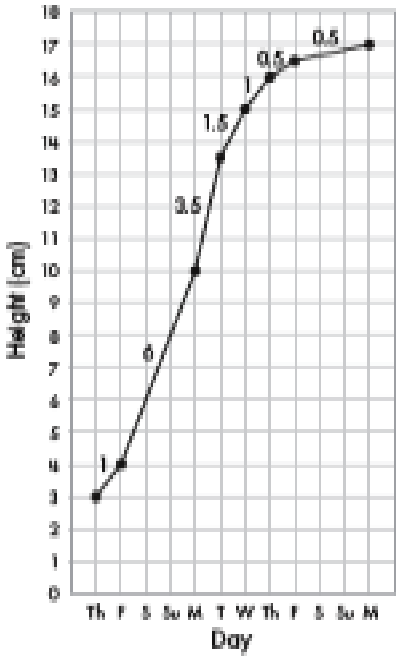
- Identify fractional parts of an area
- Identify fractional parts of a group (of objects, people, etc.)
- Read, write, and interpret fraction notation
- Order fractions with like and unlike denominators
- Read, write, and interpret decimal fractions in tenths and hundredths

Patterns, Functions, and Change

In Grade 4, students use graphs and tables to represent change. One focus of their work is how a line graph shows the *rate of change*, as they consider questions such as the following:
"How does this graph show the parts of the story that are about *speed* and the parts of the story that are about *changes in speed*?"

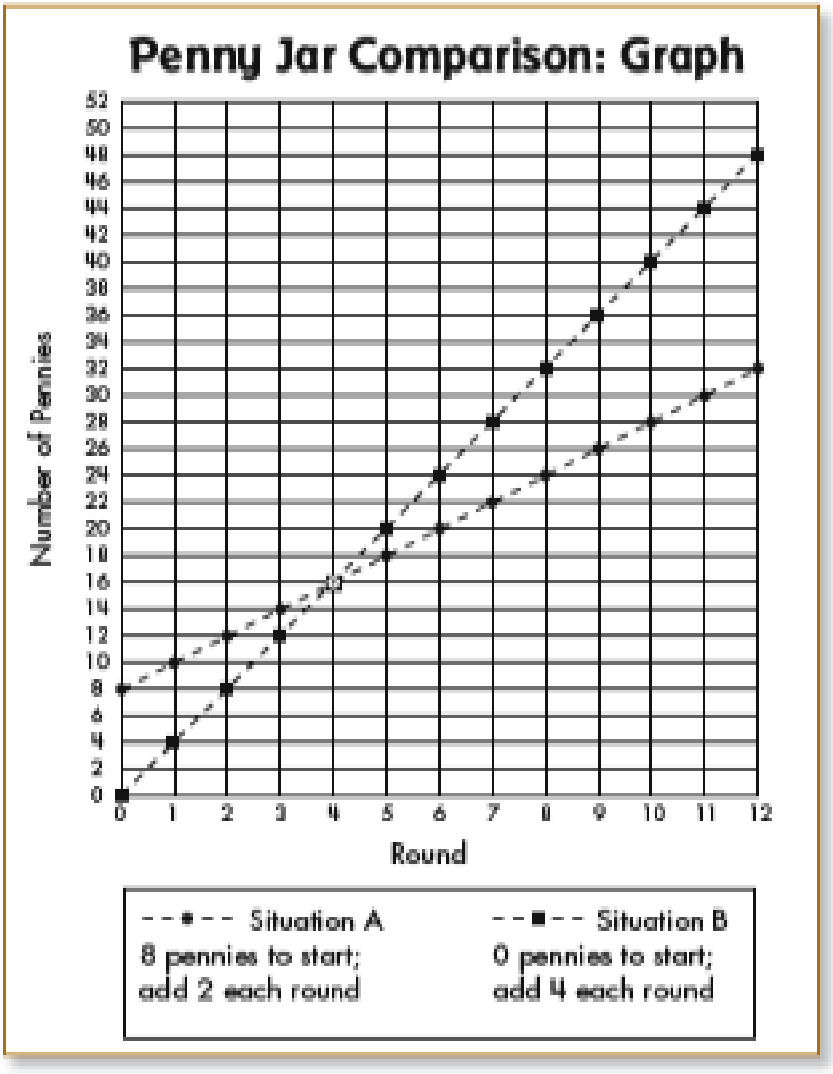
"What was the rate of growth for this plant? When was it growing more slowly or more quickly?"

Day	Height
Thursday	3 cm
Friday	4 cm
Monday	10 cm
Tuesday	13.5 cm
Wednesday	15 cm
Thursday	16 cm
Friday	16.5 cm
Monday	17.5 cm



Students create tables and graphs for situations with a constant rate of change and use them to compare related situations.

For example: Penny Jar A has 8 pennies in the jar to start and 2 pennies are added in each round; Penny Jar B has 0 pennies in the jar to start and 4 pennies are added in each round. Will the number of pennies in Penny Jar B, which starts with fewer pennies, ever "catch up" to the number of pennies in Penny Jar A?



By analyzing tables and graphs, students consider how the starting amount and the rate of change define the relationship between the two quantities (e.g., number of rounds, total number of pennies), and develop rules that govern that relationship. At first students articulate these rules in words (as they did in grade 3), but they also are introduced to the use of symbolic notation and equations to represent their rules. They use these rules to determine the value of one variable when the value of the other is known.

How many pennies are in Penny Jar A after 10 rounds?

Emphases

Using Tables and Graphs

- Using tables to represent change
- Using tables to represent change

Linear Relationships

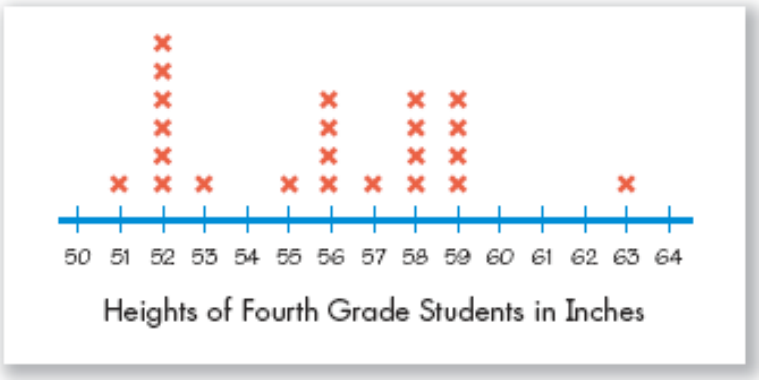
- Describing and representing a constant rate of change

Benchmarks

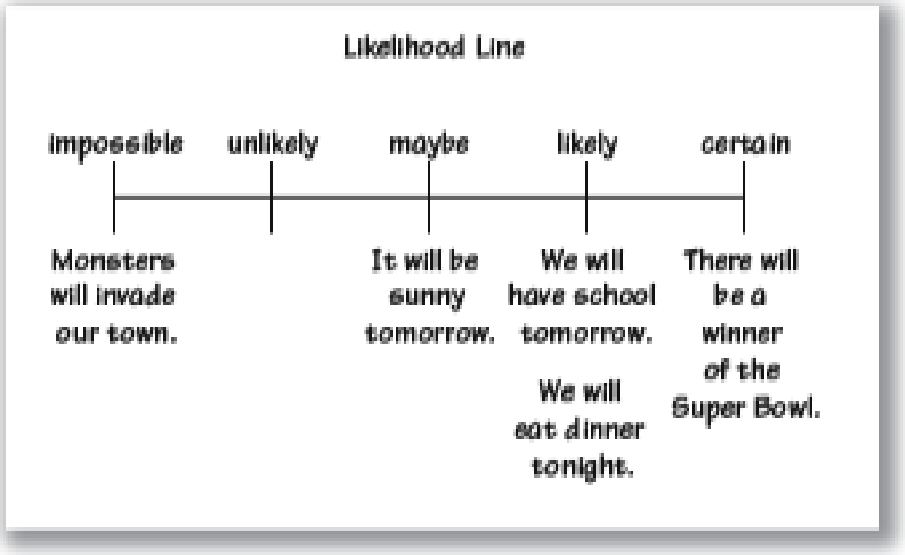
- Connect tables and graphs of change over time to each other and to the situations they represent.
- Make a graph on a coordinate grid from a table of values.
- Describe how a graph shows change: where the rate of change is increasing, decreasing, or remaining constant, and how differences in steepness represent differences in the rate of change.
- Take into account the starting amount and the amount of change in describing and comparing situations of constant change.
- In a situation of constant change, write rules (using words or arithmetic expressions) to determine the value of one quantity, given the value of the other.

Data Analysis and Probability

Students continue to develop their understanding of data analysis by collecting, representing, describing, and interpreting numerical data, in order to answer a question, investigate an issue, or provide information about something in the world that is of interest. Their work focuses on describing and summarizing data for comparing two groups. Using a *line plot* as a tool for showing the shape or distribution of a set of data—where the data are concentrated, how they are spread across the range—students represent data about two groups, and then consider how to characterize how the groups are similar or different. They develop conclusions and make arguments, based on the evidence they have collected.



Students also work on describing and predicting the likelihood of events in their world: what events are impossible, unlikely, likely, or certain? They consider situations in which there is a known number of possible outcomes—such as when rolling a number cube or pulling a red cube out of a bag holding a certain number of red and blue cubes. Students reason about how the theoretical chance (or *theoretical probability*) of, for example, rolling 1 on a number cube compares to what actually happens when a number cube is rolled repeatedly.



Emphases

Data Analysis

- Representing data
- Describing, summarizing, and comparing data
- Analyzing and interpreting data
- Designing and carrying out a data investigation

Probability

- Describing the probability of an event

Benchmarks

- Design an effective survey question to compare two groups
- Organize and represent data about two groups in order to compare the groups
- Describe the shape of the data from a numerical data set including where the data are concentrated and the highest, lowest and median values
- Use data to compare two groups
- Use evidence from a set of data to support an argument
- Describe the likelihood of an event in terms of a scale from impossible (probability of 0) to certain (probability of 1)

Geometry and Measurement

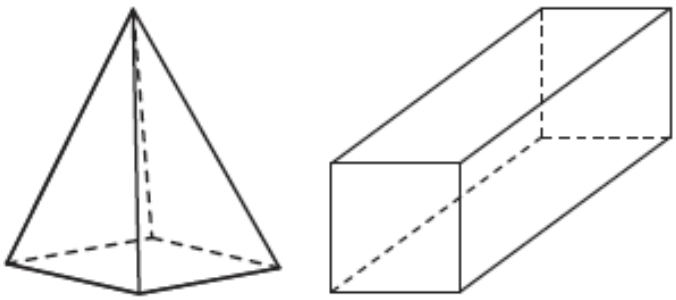
Students expand their understanding of the attributes of two-dimensional (2-D) and three-dimensional (3-D) shapes, and how these attributes determine their classification. Students consider the various attributes of 2-D shapes, such as number of sides, the length of sides, parallel sides, and the size of angles, expanding their knowledge of four-sided figures (quadrilaterals) to include parallelograms, rhombuses, and trapezoids.

Polygons

Not Polygons

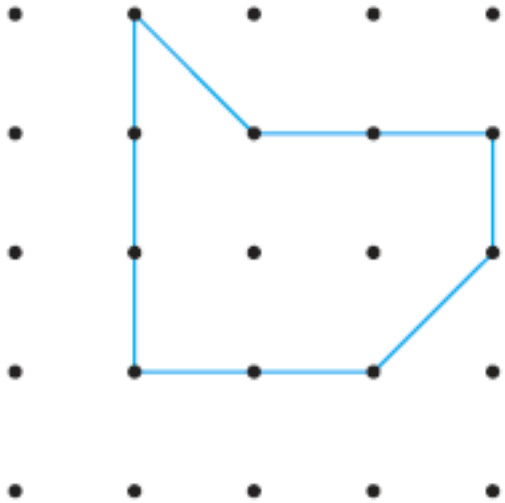
Students also describe attributes and properties of geometric solids (3-D shapes), such as the shape and number of faces, the number and relative lengths of edges, and the number of vertices. They describe classes of shapes, for example, how a pyramid has triangular faces meeting at a point.

They visualize how 3-D shapes can be represented in two dimensions, for example, by silhouettes projected by 3-D objects and structures.



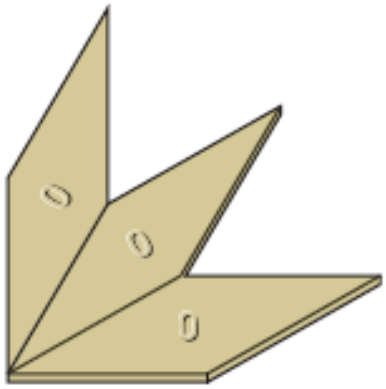
In Grade 4, students continue to build on measurement work from earlier grades, which includes linear measurement, area, angle measurement, and volume. They use both U.S. standard units (inches, feet and yards) and metric units (centimeters and meters) to measure lengths up to 100 feet, and they determine the perimeter of various shapes.

They measure the area of both regular and nonregular polygons in square units by using the understanding that area can be *decomposed*—that is, broken into smaller parts.



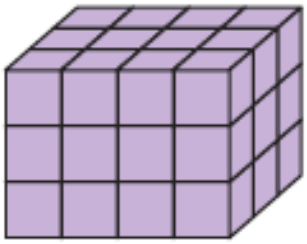
Measuring area on the Geoboard

Students work on determining the size of angles relative to a right angle, or 90 degrees. For instance, if three equal angles form a right angle, then each of the smaller angles must be 1/3 of 90 degrees or 30 degrees.



A right angle formed with Power Polygons

Finally, students work on understanding volume by structuring and determining the volume of one kind of geometric solid, a rectangular prism, in cubic units. They develop strategies for determining the number of cubes in 3-D arrays of cubes by mentally organizing the cubes--for example as a stack of three rectangular layers, each composed



of three rows of four cubes.

Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Describing properties of 3-D shapes
- Translating between 2-D and 3-D shapes

Linear Measurement

- Measuring with standard units

Area Measurement

- Understanding and finding area

Volume

- Structuring rectangular prisms and determining their volume

Benchmarks

- Use appropriate measurement tools to measure distance
- Identify quadrilaterals as any four-sided closed shape
- Know that a right angle measures 90 degrees, and use this as a landmark to find angles of 30, 45, and 60 degrees
- Find the area of polygons using a square unit of measure
- Identify 2-dimensional silhouettes of 3-dimensional solids (e.g. a cone can project a triangular silhouette)
- Draw 2-D representations showing different perspectives of a 3-D object
- Find the volume of cube buildings and rectangular prisms