Investigations in Number, Data, and Space®

Grades K-5 Math Content: 2nd Edition

Kindergarten[1]

Number and Operations: Whole Numbers

Counting and the Number System

A main focus in Kindergarten is counting, which is the basis for understanding the number system and for almost all the number work in the primary grades. Students hear and use the counting sequence (the number names, in order) in a variety of contexts. They have many opportunities to connect the number names with the written numbers and with the quantities they represent. They have repeated experiences counting sets of objects, and matching and making sets of a given size. As students count sets of objects and make equal sets they begin to see the importance of counting each object once and only once, and of having a system for keeping track of what has been counted and what still remains to be counted. Students engage in repeated practice with counting and develop visual images for quantities to 10.

This student used pictures, numbers and words to show that he counted 13 nuts. He drew a circle for each nut and wrote both the number and the word – '13 nts.' When he recounted his circles to check, he realized he had one too many, so he scribbled out one circle.

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

As students are developing accurate counting strategies they are also building an understanding of how the numbers in the counting sequence are related: Each number is one more (or one less) than the number before (or after) it. Students develop an understanding of the concepts of greater than, fewer than, and equal to, and develop language for describing quantitative comparisons (e.g. bigger, more, smaller, fewer, less, same, equal) as they count and compare quantities.

Example: Write the names of the people in your home. Circle the name with the most letters.

Emphases

Counting and Quantity

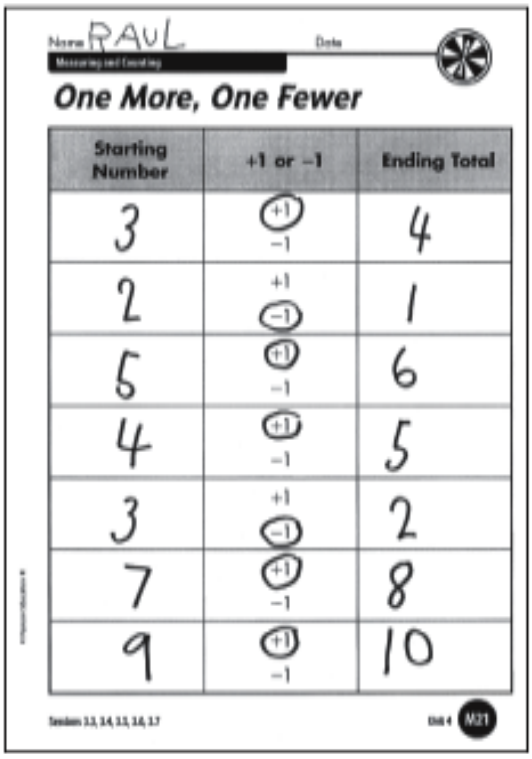
- Developing strategies for accurately counting a set of objects by ones
- Developing an understanding of the magnitude and position of numbers

Benchmarks

- Count a set of up to 10 objects
- Compare two quantities up to 10 to see which is greater
- Count a set of up to 15 objects
- Figure out what is one more or one fewer than a number
- Write the numbers up to 10
- Count a set of up to 20 objects

Addition and Subtraction and the Number System

Young students develop their understanding of the operations of addition and subtraction by having many opportunities to count, visualize, model, solve and discuss different types of problems. Many of the counting activities in Kindergarten build a bridge to the operations of addition and subtraction, as students add a small amount to a set or remove a small amount from a set and figure out, "How many now?" One of the ways students are introduced to addition and subtraction is via story problems about combining and separating. They retell the stories, act them out, and solve them, by modeling the action involved and using counting strategies. Students also play a variety of games that model the operations of addition and subtraction. They have repeated experiences joining two or more amounts, and removing an amount from a whole.



Name RAUL Date

Measuring and Counting

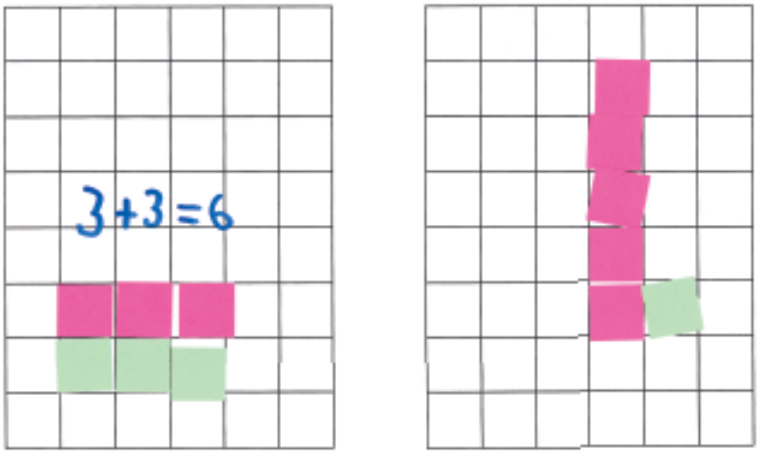
One More, One Fewer

Starting Number	+1 or –1	Ending Total
3	(+1) –1	4
2	+1 (–1)	1
5	(+1) –1	6
4	(+1) –1	5
3	+1 (–1)	2
7	(+1) –1	8
9	(+1) –1	10

Sessions 3.3, 3.4, 3.5, 3.6, 3.7

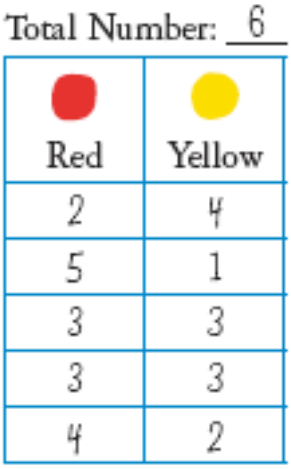
Unit 4 M21

Later in the year students work with combinations of quantities that they can count fluently. As they find ways to arrange and describe sets of 5-10 square tiles or record combinations of two-color counters, they begin to see that numbers can be composed in different ways. They work on activities that involve seeing and describing a given quantity (e.g. 6 tiles) as made up of groups (e.g. a group of 4 and a group of 2). They are also asked to decompose quantities (e.g. 6 can be split into 4 and 2) and to find one or more combinations of a quantity (e.g. 6 can be decomposed as 6 and 0, 3 and 3, or 5 and 1.) This work lays the foundation for making meaningful sense of 4 + 2 = 6 and 6 – 4 = 2 in subsequent years.



Students show their arrangements of six tiles and indicate how they know there are six tiles in all.

Example: Record the total number of chips. Toss the chips. Record the number that are red and the number that are yellow.



Students use mathematical tools and representations to model and solve problems to clarify and communicate their thinking. Kindergartners are just beginning to learn how to represent their mathematical work on paper and are encouraged to do so in ways that make sense to them. Many use combinations of pictures, words, and numbers.

The Algebra Connections pages of the units that focus on counting, addition, and subtraction show how students develop ideas about how numbers describe the size of a set–that the number of objects in a set is fixed no matter how it is arranged and counted, and different sets may have the same number of objects. Students' observations about the constancy of the total, no matter what the order of counting a set of objects, lays the foundation for what they will later call the commutative property of addition. These pages also show how students work on ideas of combining and decomposing quantities and on understanding how addition and subtraction operate. Thus, the generalization Kindergarten students are approaching might be stated as: When adding (with the numbers they know), the resulting amount is greater than you started with. When subtracting (with the numbers they know), the resulting amount is less than you started with.

Emphases

Whole Number Operations

- Making sense of and developing strategies to solve addition and subtraction problems with small numbers
- Using manipulatives, drawings, tools and notation to show strategies and solutions

Benchmarks

- Combine two small quantities
- Figure out what is one more or one fewer than a number

Data Analysis

Sorting and classifying are central to organizing and interpreting data. Students in Kindergarten have many opportunities to identify the attributes of groups of objects, determine how the objects are the same and different and sort them into groups according to their attributes. Students apply these skills to organizing data when they sort their favorite lunch foods into categories.

Students think about how these pieces of information are the same and different in order to determine how the data might be grouped and how those groups can be defined.

Important to any data collection activity is the need to establish the group of people or objects being considered. Students begin their work on data by determining the number of students in the class and finding a way to represent this number on paper. As students collect data about themselves, they think about the one to one correspondence between the number of people and the number of pieces of data. Developing strategies for keeping track of who has responded to a survey, recording data, as well as representing this information, are important parts of the Kindergarten work.

To begin to understand the processes involved in data analysis, Kindergarteners are involved in all phases of conducting a survey: They choose and pose a question, determine how to record responses, and count and make sense of the results.

Example: Do you like  ?

Yes	No
Carmen	Dennis
Mitchell	Timothy
Mary	Sarah
Tammy	Lisa
Raul	Kiyo
Jennifer	Latoya
	Lionel
	Manuel
	Yoshio
	Beth
	Russell

Students also use some of the data they collect to solve mathematical problems connected to their classroom. For example,

"25 students are in our class. 4 are absent. How many are here?"

Emphases

Data Analysis

- Representing Data
- Sorting and Classifying
- Carrying Out a Data Investigation

Benchmarks

- Represent a set of data
- Use data to solve a problem
- Sort a set of objects according to their attributes

Measurement

In Kindergarten, students are introduced to length and linear measurement through measuring by direct comparison. As they compare objects to determine the longest object, they discuss and make sense of important aspects of accurate measurement such as choosing which dimension to measure.

Students begin to think about the different dimensions of objects.

They also become comfortable with, and use language to describe length—long, short, wide, tall, high (and the comparative forms –longer, wider, etc). Later in the year students use multiple nonstandard units (e.g., craft sticks or cubes) to quantify length: "How many craft sticks long is this desk? the path from the window to the door?" "How many cubes long is my shoe? this pencil?" As they measure lengths around their classroom, students think about what happens if the units are (or are not) laid straight or if there are (or are not) gaps or overlaps between them.

Students begin to think about measuring accurately

Emphases

Linear Measurement

- Understanding length and using linear units

Benchmarks

- Decide which of two objects is longer
- Measure the length of an object by lining up multiple units

Patterns, Functions, and Change

Kindergarten students construct, describe, extend, and determine what comes next in repeating patterns. To identify and construct repeating patterns, students must be able to identify the attributes of the objects in the pattern. Therefore students first work on sorting objects by their attributes, before they begin constructing their own patterns. Students encounter patterns with two (AB, AAB, ABB) or three (ABC) elements. As students construct and describe many different patterns, they become more familiar with the structure of patterns, are able to identify what comes next in a pattern, and can begin to think about how two patterns are similar and different.

Example: What is the same about these cube trains? What is different?

After having many opportunities to construct their own patterns and extend patterns made by others, students begin to analyze the structure of a repeating pattern by identifying the *unit* of the pattern—the part of the pattern that repeats over and over.

Example: What is the repeating unit of this pattern?

Emphases

Data Analysis

- Sorting and Classifying

Repeating Patterns

- Constructing, describing, and extending repeating patterns
- Identifying the unit of a repeating pattern

Benchmarks

- Copy, construct, and extend simple patterns, such as AB and ABC
- Begin to identify the unit of a repeating pattern

Geometry

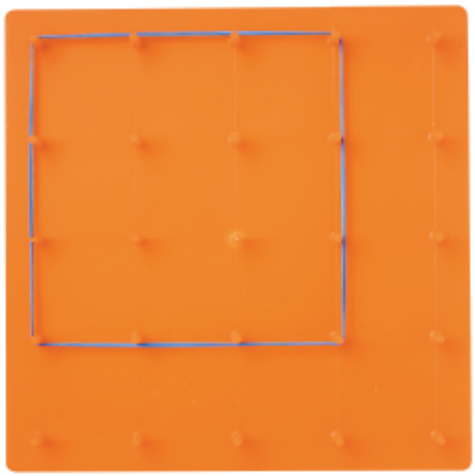
The geometry work in Kindergarten builds on students' firsthand knowledge of shapes to further develop their spatial sense and deepen their understanding of the two-and three-dimensional world in which they live. As students identify the different shapes that make up the world, they

are encouraged to use their own words to describe both 2-D and 3-D shapes. In this way, they form images of familiar shapes through associating them with familiar objects.

Students explore the geometric idea that shapes can be combined or subdivided to make other shapes. For example, they investigate how 3-D shapes can be combined to form a particular rectangular prism.

By putting shapes together and taking shapes apart, students deepen their understanding of the attributes of shapes and how shapes are related.

Students also construct 2-D and 3-D shapes with clay and on Geoboards. As they construct shapes they form mental images of the shapes and think about the attributes of particular shapes.



The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Composing and decomposing 2-D and 3-D shapes
- Describing, identifying, comparing, and sorting 2-D and 3-D shapes

Benchmarks

- Describe the overall size, shape, function, and/or features of familiar 2-D and 3-D shapes
- Construct 2-D and 3-D shapes
- Make 2-D and 3-D shapes by combining shapes

Grade 1[1]

Number and Operations: Whole Numbers

Counting and the Number System

Throughout first grade, students work on developing strategies for accurately counting a group of up to 50 objects. They have repeated practice with the counting sequence, both forwards and backward, and with counting and keeping track of sets of objects. They also connect the number names with the written numbers and the quantities that they represent.

5

Five

* * * * *

As students are developing accurate counting strategies they are also building an understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after) it. As students build this understanding, they compare and order quantities and develop a sense of the relative size of numbers and the quantities they represent.

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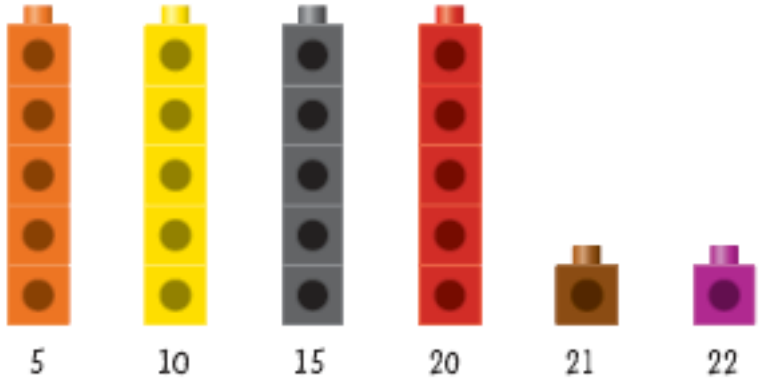
Students also make sense of counting by numbers other than 1. They connect the number sequence of counting by 2s, 5s, and 10s to the quantities they represent. As they work on activities that involve multiple groups of the same amount, they build an understanding that as they say each number in the counting sequence, they are adding 2, 5 or 10 more things. This leads to more efficient and accurate counting.



Emphases

Counting and Quantity

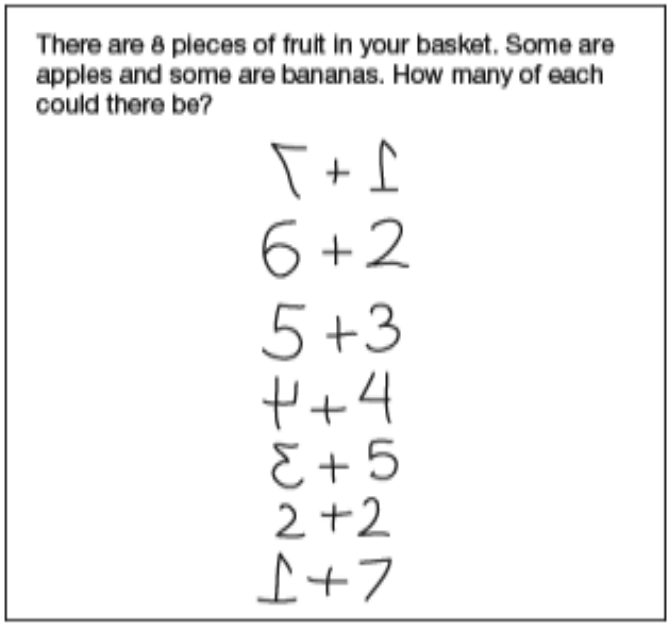
- Developing strategies for accurately counting a set of objects by ones
- Developing an understanding of the magnitude and position of numbers

Benchmarks

- Count a set of up to 20 objects
- Compare and order quantities up to 12
- Count a set of 40 to 50 objects
- Rote count, read, and write numbers up to 65
- Begin to use groups in meaningful ways
- Identify, read, write, and sequence numbers up to 105

Addition and Subtraction and the Number System

In first grade, students work with the important idea that quantities can be composed and decomposed in different ways, while the quantity remains the same. Students have repeated experiences breaking one number (a whole) into two parts, or combining two parts to form a whole. They consider the relationship between the parts, noticing, for example, that when the whole remains the same, as one part increases the other part decreases. Students work with composing and decomposing numbers to 20, and focus on the addition combinations of 10. Students are expected to develop fluency with the combinations of 10 by the end of the school year.



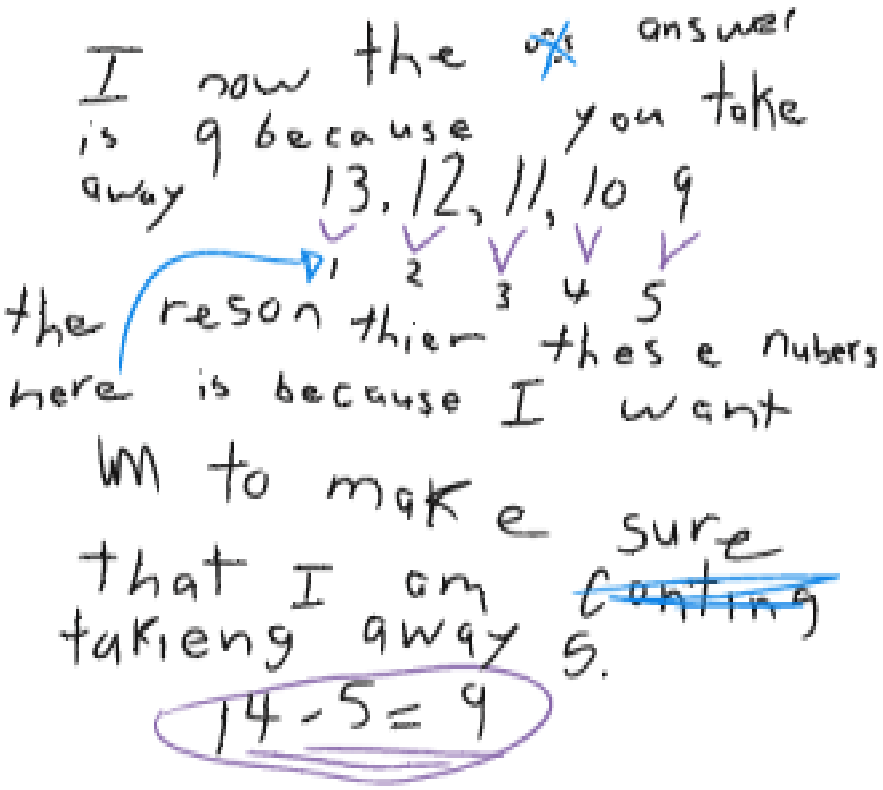
A student uses an ordered list to organize his responses to a How Many of Each? *Problem.*

The addition and subtraction work of first grade focuses on making sense of these operations, practicing adding and subtracting single-digit numbers, and solving addition and subtraction story problems. Many of the games and activities involve students in comparing and combining two amounts or removing one amount away from the other, which offers practice with single-digit addition and subtraction. The goal of the work with story problems is for students to learn to visualize the action of story problems and to solve the problems in ways that make sense to them.

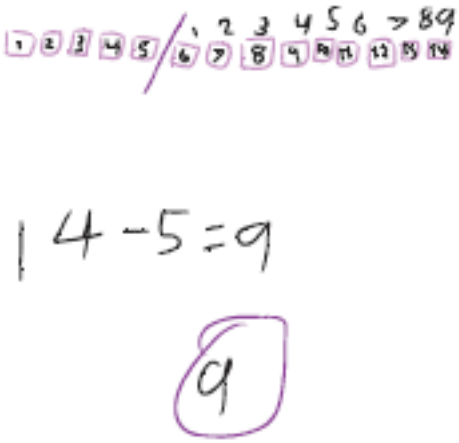
By the end of the year, it is expected that first graders will *count on* to combine two small quantities and that some students will *use a combinations they know* to solve related problems (e.g., 6 + 4 = 10 so 6 + 5 = 11). For subtraction, many students will still *show all, remove some, and count those that remain*. Others will *count back*, *count up*, or *use relationships they know* (e.g., 14 – 5 = 14 – 4 – 1)

1. There were 14 children playing in the park.
Then 5 children went home.
How many children were still in the park?

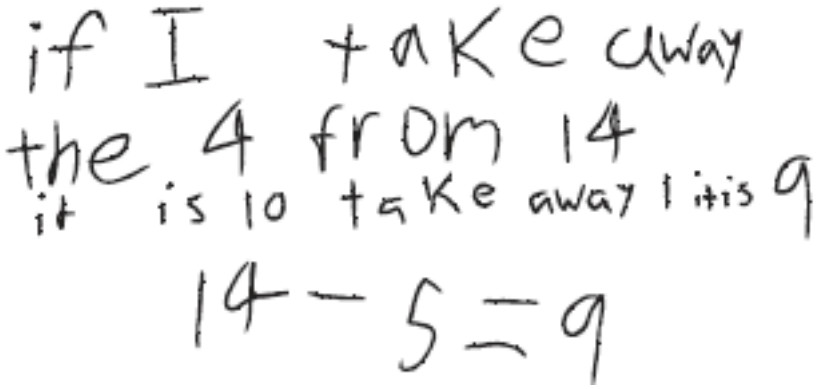
A sample subtraction problem from the Student Activity Book



Count Back



Show all, remove or cross out some, and then count how many are left



Use a combination you know

Students use mathematical tools, such as cubes and counters, and representations, such as the number line and 100 chart, to model and solve addition and subtraction problems and to clarify and communicate their thinking. They are encouraged to represent their work on paper in ways that make sense to them. Many use a combination of pictures, words, numbers and mathematical symbols.

The Algebra Connections pages of each of the four curriculum units that focus on addition and subtraction show how students are applying the commutative property of addition as they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 6 + 4 = 5 + 5 and 8 + 5 = 10 + 3) or when they use addition combinations they know to solve more difficult problems (e.g., since 5 + 5 = 10, 5 + 6 must equal 10 + 1, or 11).

Emphases

Number Composition

- Representing numbers using equivalent expressions
- Composing numbers up to 20 with two addends

Whole Number Operations

- Making sense of and developing strategies to solve addition and subtraction problems with small numbers
- Using manipulatives, drawings, tools and notation to show strategies and solutions

Computational Fluency

- Knowing addition combinations of 10
- Combine two small quantities

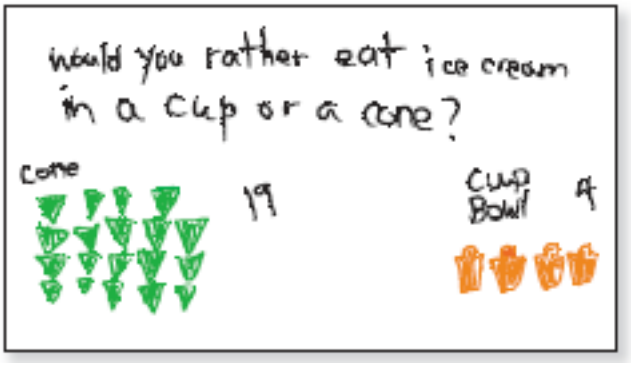
Benchmarks

- Find more than one combination of two addends for a number up to 10 (e.g., 7 is 4 and 3 and it's also 5 and 2)
- Find at least 5 two-addend combinations of 10
- Interpret (retell the action and sequence) and solve addition and subtraction story problems
- Find at least five combinations of two addends for a number up to 15
- Subtract one small quantity from another
- Represent numbers using equivalent expressions
- Combine two small quantities by at least counting on
- Demonstrate fluency with the two-addend combinations of 10

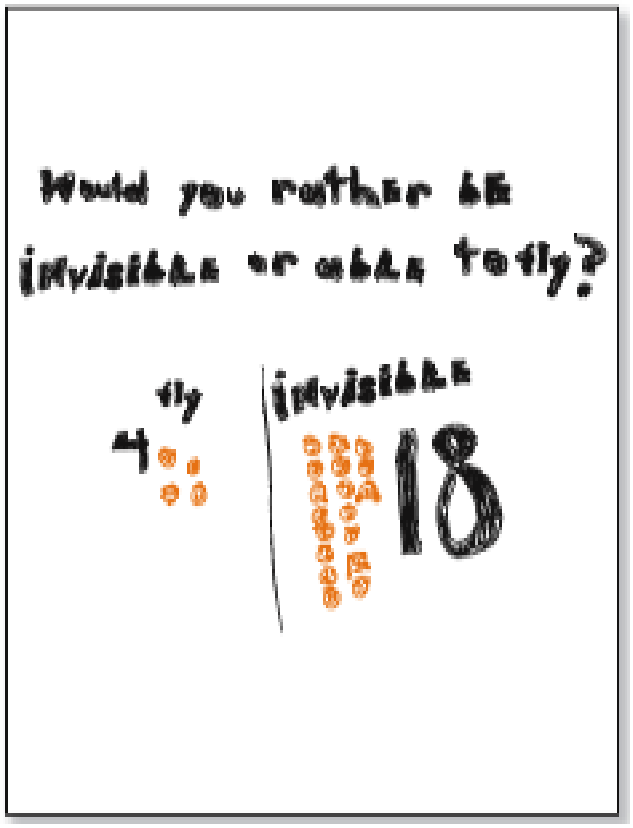
Data Analysis

In first grade, students sort groups of related objects, such as buttons, into groups, and describing what distinguishes one group from another. This early work in classification provides experience in considering only certain attributes of an object while ignoring others. Sorting a variety of sets lays the foundation for later work in classifying shapes and numbers and in working with categorical data.

First graders create their own representations of the data they collect, organizing their data and providing an image that helps them describe what the data show. Students are also introduced to several standard forms of representation, including picture graphs, tallies, charts, and bar graphs. By discussing and comparing representations, students consider what features of a representation help communicate a clear description of the data. As student describe data, the key question they consider is: What do these data tell us about our class [or the class next door, or our siblings]? In the context of this overall question, first graders' descriptions focus on two characteristics of the data: (1) "What is the number of pieces of data in each category or at each value?" and (2) "Which category has more data?"



Would you rather eat ice cream in a cup or in a cone?



Would you rather be invisible or be able to fly?

Students carry out their own data investigation. They develop a question, collect the data, represent the data, and describe and interpret the data, which may, in turn, bring up more questions. Once data are collected, the data are represented, examined, and analyzed to find out what information the data provide about the original questions.

Emphases

Data Analysis

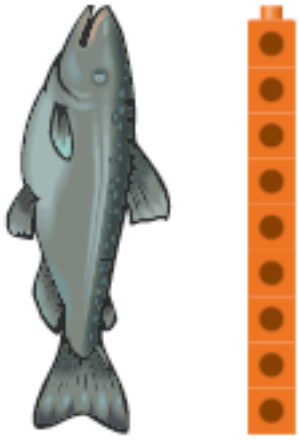
- Sorting and Classifying
- Representing Data
- Describing Data
- Designing and Carrying Out a Data Investigation

Benchmarks

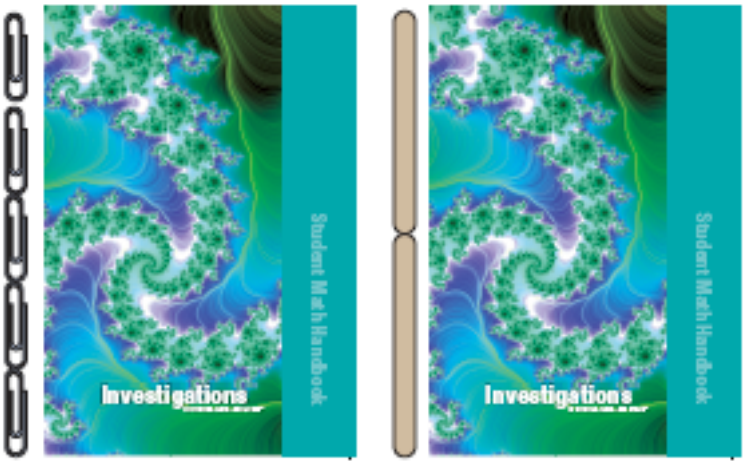
- Sort a group of objects according to a given attribute
- Represent a set of data with two categories
- Interpret a variety of representations of data with two categories
- Describe a set of data including how many are in each group, which group is greater, and how many people responded to the survey

Measurement

It is important for students to develop a sense of how measurement is used--and when it is helpful--in the real world. Unit 5, *Fish Lengths and Animal Jumps,* involves students in a real context in which measuring is used, that of measuring fish to determine if they are "keepers." They measure relatively small lengths (up to 18 inches) and larger distances (up to 5-6 feet), and see that measurement is applied to both objects and distances.



knowing where to start and stop measuring, understanding how measuring tools must be lined up so that there are no gaps or overlaps, knowing which dimension to measure, measuring the shortest line from point to point, and understanding that many measurements are not reported in whole numbers. Regardless of what is measured, students learn that when one measures an object twice--or when two different people measure it--the same results should be obtained, assuming the same measuring unit is used. Students also explore what happens when something is measured with small units versus larger units. Students begin to see that measuring an object in cubes will result in a different count than will measuring the same object in inch tiles or paper clips, but may not yet see the inverse relationship between size of unit and number of units needed to cover a distance.



Emphases

Linear Measurement:

- Understanding length
- Using linear units
- Measuring with standard units

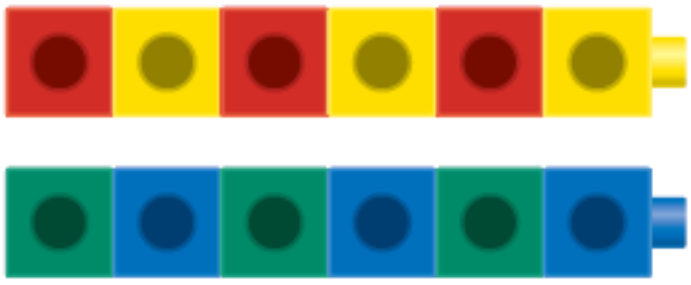
Benchmarks

- Demonstrate measuring techniques when measuring a distance with nonstandard or standard units. These techniques include starting at the beginning, ending at the end, leaving no gaps or overlaps, measuring in a straight line, and keeping track of the number of units
- Know at least one way of describing a measurement that falls between two whole numbers
- Understand that the same results should be obtained when the same object is measured twice, or when two different people measure the same object (using the same unit)
- Understand that measuring with different-sized units will result in different numbers

Patterns, Functions, and Change

Students begin their work on patterns in first grade by creating, describing, extending, and making predictions about repeating patterns. By building or acting out these patterns and thinking through how the pattern continues, students analyze the regularities of the pattern to determine what comes next or what will come several steps ahead in the pattern. Students analyze the structure of a repeating pattern by identifying the *unit* of the pattern—the part of the pattern that repeats over and over. By focusing on the unit of the repeating pattern, students shift their focus from seeing that "red follows yellow and yellow follows red" to how the repeating pattern is constructed of an iterated red-yellow unit. This focus allows students to analyze more complex patterns.

Students also compare patterns and begin to notice how patterns are the same. For example, a red, yellow, red, yellow pattern and a green, blue, green, blue pattern have the same structure.



Students then work with number sequences associated with repeating patterns. Associating the counting numbers with this pattern allows new kinds of questions about the pattern, such as the following: "What color will the 17th square be?" "Is the 20th square black?" Numbering the elements of a repeating pattern provides another way to describe that pattern.

Students also consider situations that have a constant increase. They investigate three different contexts— collecting pennies in a jar, making Staircase Towers from connecting cubes, and making repeating patterns with pattern blocks. In each situation, a sequence of numbers is generated by the situation.

Example: I have one penny in a jar, and each day I add three more pennies.

Comparison across contexts helps students focus on how the same start number and the same amount of constant increase can create the same number sequence in different situations.

Emphases

Repeating Patterns

- Constructing, describing, and extending repeating patterns
- Identifying the unit of a repeating pattern

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

Benchmarks

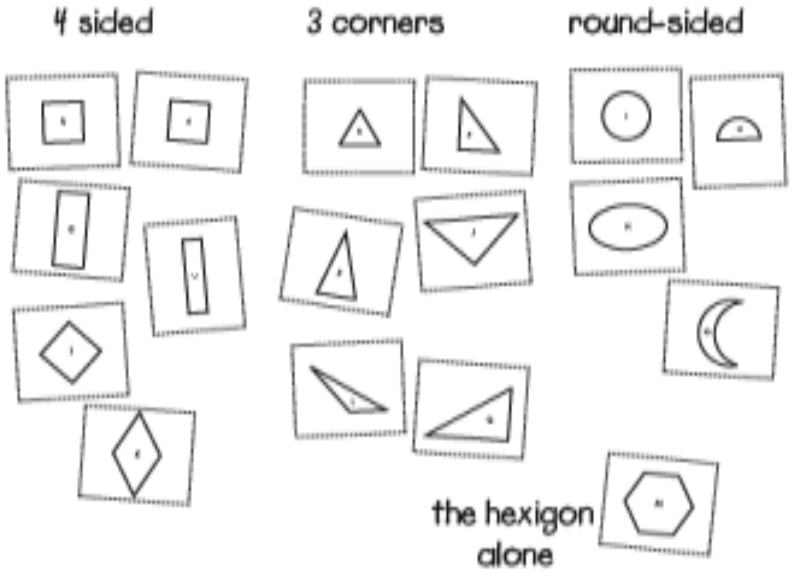
- Construct, describe, and extend a repeating pattern with the structure AB, ABC, AAB, or ABB
- Identify the unit of a repeating pattern for patterns with the structure AB or ABC
- Describe how various AB or ABC patterns are alike (e.g., how is a red-blue pattern like a yellow-green pattern?)
- Determine what comes several steps beyond the visible part of an AB, ABC, AAB, or ABB repeating pattern
- Construct, extend, and describe a pattern that has a constant increase for the sequences 1, 3, 5, …; 2, 4, 6, …; 1, 4, 7, …; 2, 5, 8, …; and 3, 6, 9, … through counting and building

Geometry

The emphasis of geometry work in 1st grade is on careful observation, description and comparison of two-dimensional (2-D) and three-dimensional (3-D) geometric shapes.

Students describe 2-D shapes, sort them and compare them, and they think about questions like the following: What makes a triangle a triangle? How are triangles different from squares?

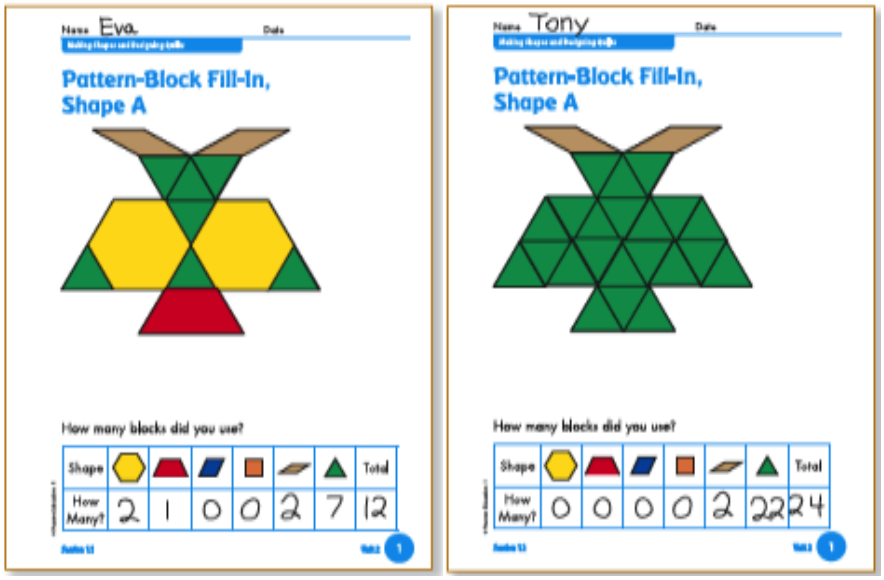
Developing visual images of shapes as well as drawing 2-D shapes are ways that students come to know the important features of shapes. When they sort 2-D shapes, they make groups of shapes that "go together," which requires them to look for similarities and differences among the attributes of different shapes.



One pair's sort of Shape Cards

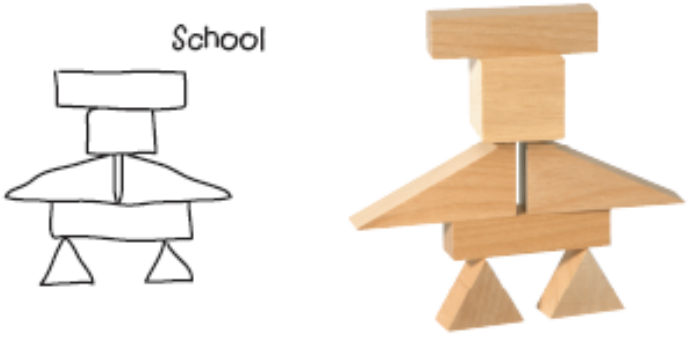
Students look for 3-D shapes in their own environment and they work with 3-D shapes (whose faces are familiar 2-D shapes) such as Geoblocks, manufactured boxes, and boxes made by students.

Students also learn about geometric relationships by composing and decomposing shapes. As they fill in the same shape outline with pattern blocks in different ways, they break apart or combine shapes in order to change how the shape is filled. When using the geoblocks, students notice, for example, that two cubes can be put together to make a rectangular prism and that two triangular prisms can be put together to make a cube.



Eva and Tony's student work of SAB 1, Pattern Block Fill-In, Shape A

Students investigate the relationship between 3-D shapes and 2-D representations of those shapes. By matching 3-D objects to outlines of their faces, to pictures, and to drawings of other students, they identify shapes by looking carefully at some parts of the shape and then visualizing what the whole shape looks like. Moving back and forth between 3-D objects and their 2-D representations helps students describe and compare the characteristics of common 3-D shapes.



A student draws a 2-D representation of his 3-D building.

The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Composing and decomposing 2-D shapes
- Describing, identifying, and comparing 2-D and 3-D shapes
- Exploring the relationships between 2-D and 3-D shapes

Benchmarks

- Fill a given region in different ways with a variety of shapes
- Use geometric language to describe and identify important features of familiar 2-D shapes
- Identify and describe triangles
- Describe and sort 2-D shapes
- Compose and decompose shapes
- Attend to features of 3-D shapes, such as overall size and shape, the number and shape of faces, and the number of corners
- Match a 2-D representation to a 3-D shape or structure

Grade 2[1]

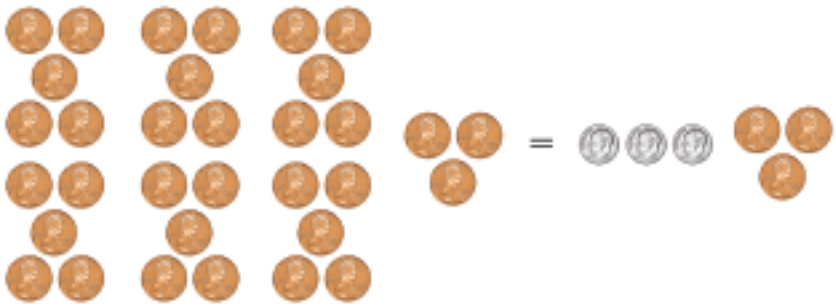
Number and Operations: Whole Numbers

Counting and the Number System

At the beginning of the school year, students have varied opportunities to count sets of objects by ones, write the number sequence, and explore and compare representations of the counting numbers on the number line and the 100 chart. As the school year progresses, most second graders shift from thinking and working primarily with ones to thinking and working with groups of ones. To help them make this shift, students have many opportunities to develop strategies for grouping and for counting by groups. The focus is first on contexts that encourage counting by groups of 2, 5, or 10 and then specifically on groups of 10 and the base ten structure of our number system.

Example: There are 6 people in our classroom. How many legs are there?

Example: If I have 33 pennies and I trade them for as many dimes as I can, how many dimes will I have? How many pennies?



3 groups of 10 pennies and 1 group of 3 pennies equals 3 dimes and 3 pennies.

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

In Grade 2, students work extensively with contexts and models that represent the place value structure of our base-ten number system. They use these contexts to build and visualize how two-digit numbers are composed. For example, 33 cents can be composed of 3 dimes and 3 pennies or 2 dimes and 13 pennies or 1 dimes and 23 pennies.

As an extention of their work with number composition, students investigate even and odd numbers through the context of partners (groups of two) and teams (two equal groups) and then develop definitions of even and odd numbers.

Example: If there are 12 students, can everyone have a partner?

Can there be 2 equal teams?

Is 12 even or odd? Why do you think so?

A student might respond: "Even because everyone would have a partner, and there would be two teams with no leftovers."

Emphases

Counting and Quantity:

- Developing strategies for accurately counting a set of objects by ones and groups
- Developing an understanding of the magnitude and sequence of numbers up to 100
- Counting by equal groups

The Base Ten Number System

- Understanding the equivalence of one group and the discrete units that comprise it

Benchmarks

- Count a set of objects up to 60 in at least one way
- Define even and odd numbers in terms of groups of two or two equal groups
- Recognize and identify coins and their values
- Interpret and solve problems about the number of tens and ones in a quantity
- Know coin equivalencies for nickel, dime, and quarter
- Count by 2s, 5s, and 10s, up to a number

Addition and Subtraction and the Number System

Throughout second grade, students work on making sense of the action of different types of addition and subtraction problems and on developing efficient strategies for solving them and for recording their work. They solve addition and subtraction problems in ways that make sense to them and practice using particular strategies.

Students study two particular strategies for addition –adding tens and ones and adding one number in parts. By the end of the school year, students are expected to have at least one strategy that they can use to accurately and efficiently solve an addition problem.

In Grade 2, students' work with place value becomes the basis for the development of strategies for adding and subtracting 2-digit numbers. The two strategies for addition, adding by place and adding one number in parts, and the strategy for subtraction of subtracting one number in parts, depend on an understanding of how to break numbers into tens and ones.

Adding tens and ones

14 + 32 = 46
10 + 30 = 40
4 + 2 = 6
40 + 6 = 46

Adding on one number in parts

14 + 32 = 46
32 + 10 = 42
42 + 4 = 46

Students consider and practice two strategies for subtraction– subtracting in parts and adding up. By the end of the year they are expected to have one strategy that they can use to accurately solve a subtraction problem.

Subtracting in parts

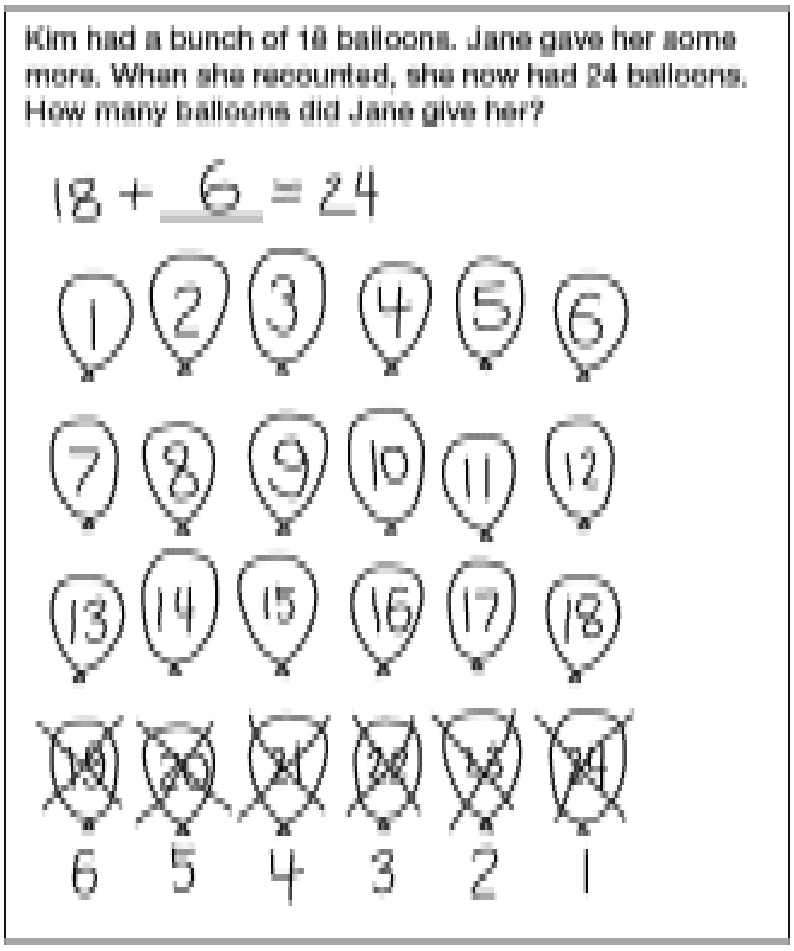
46 – 32 = 14
46 – 2 = 44
44 – 10 = 34
34 – 10 = 24
24 – 10 = 14

Adding up

46 – 32 = 14
32 + 10 = 42
42 + 4 = 46
10 + 4 = 14

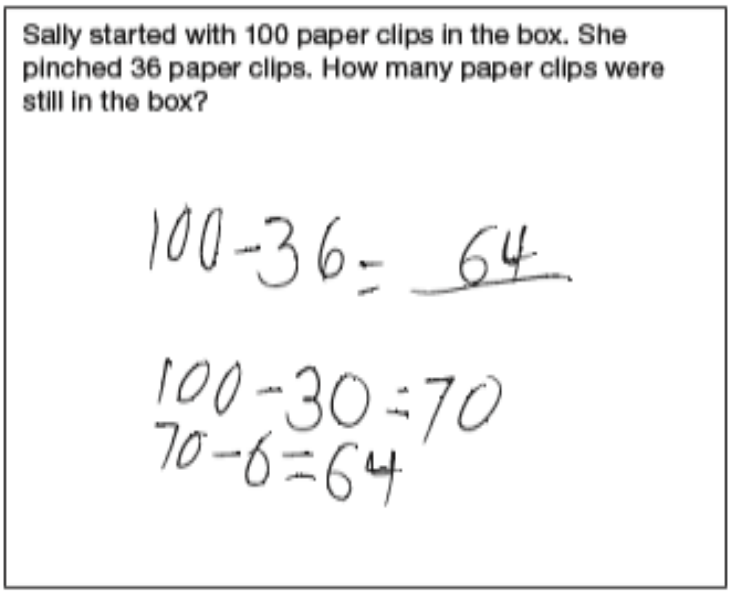
Knowing the single-digit addition combinations helps students as they work to develop efficient strategies for adding and subtracting. Students are expected to be fluent with addition combinations up to 10 + 10 by the end of the year. Students use the relationship between addition and subtraction to solve subtraction problems and to develop fluency with the subtraction expressions related to the addition combinations to 10 + 10.

Students use mathematical tools and representations to model and solve problems to clarify and communicate their thinking. They are encouraged to show their mathematics work on paper in ways that make sense to them; many use some combination of pictures, words, numbers and mathematical symbols and notation.



A student uses pictures, numbers, and notation to show his solution.

Students are expected to use standard notation to write equations to represent addition or subtraction problems. They are also expected to have methods for clearly showing their work, including: sticker notation, numbers, equations, the number line and 100 Chart, or some combinations of these.



A student writes an equation and uses a number line to show her work.

The Algebra Connections pages of each of the four curriculum units that focus on addition and subtraction show how students are applying the commutative and associative properties of addition as they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 5+ 9 = 4 + 10 or 5 + 9 = 5 + 10 - 1). In addition, these pages highlight the work that students do in proving generalizations about adding odd and even numbers.

Emphases

Whole Number Operations

- Using manipulatives, drawings, tools, and notation to show strategies and solutions
- Making sense of and developing strategies to solve addition and subtraction problems with totals to 100
- Understanding the properties of addition and subtraction
- Adding even and odd numbers

Computational Fluency

- Knowing addition combinations to 10+10

Benchmarks (compiled from Units 1, 3, and 8)

- Determine the difference between two numbers (up to 45)
- Interpret addition and subtraction story problems (read a story problem and determine what needs to be figured out)
- Have at least one strategy for solving addition and subtraction (as removal) story problems
- Demonstrate fluency with addition combinations to 10 + 10
- Understand what it means to double a quantity
- Use known combinations to add several numbers in any order
- Interpret and solve subtraction (removal) and unknown change story problems with totals up to 45
- Count on or break numbers apart to add two or more numbers up to a total of 45
- Write an equation that represents an addition or subtraction situation
- Determine the difference between a number and any multiple of 10 up to 100
- Add multiples of 5, up to 100
- Subtract two-digit numbers
- Reason about partners, teams, and leftovers to make and justify generalizations about what happens when even and odd numbers are added
- Add two 2-digit numbers accurately and efficiently

Data Analysis

Students' work on data begins with sorting activities in which they sort objects by their attributes, describing what distinguishes one group from another. This early work in classification provides experience in considering only certain attributes of an object while ignoring others. Students then apply these ideas to categorical data. They *classify* data with many different values, for example the responses to the question, "What is your favorite weekend activity?" by grouping the data into categories (outdoor and indoor activities; or things you do by yourself, things you do with one friend, and things you do with a group). By grouping the data in different ways, students can use the same data to answer different questions.

Example: What is your favorite activity to do on the weekend?

Indoor	Outdoor
reading	soccer
playing board games	walking my dog
	playing basketball

Alone	With More Than 1 Person
reading	soccer
walking my dog	playing basketball
	playing board games

Students use a variety of representations: Venn diagrams, towers of cubes, line plots, and their own representations. By comparing a variety of representations of the same data, they learn how different representations can make different aspects of the data set more visible. Students are introduced to line plots and other frequency distributions in which each piece of data is represented by one symbol (e.g., an X, a square, or a stick-on note). In using this kind of representation, students have to think through the meaning of two ways numbers are used in describing the data: Some numbers indicate the value of a piece of data (I have *8 pockets*); other numbers indicate *how often* a particular data value occurs (*7 children* have 8 pockets). Students describe data by considering the number of pieces of data that occur at each value, the mode and the highest and lowest values.

Through experiencing an entire data investigation from start to finish, students encounter many of the same issues encountered by statisticians as they decide how to collect, keep track of, organize, represent, describe, and interpret their data. They develop their own survey questions about "favorite things", and collect and organize the survey data. They also collect data from different grades about the number of teeth lost and represent and compare these data to their own class data.

Emphases

Data Analysis

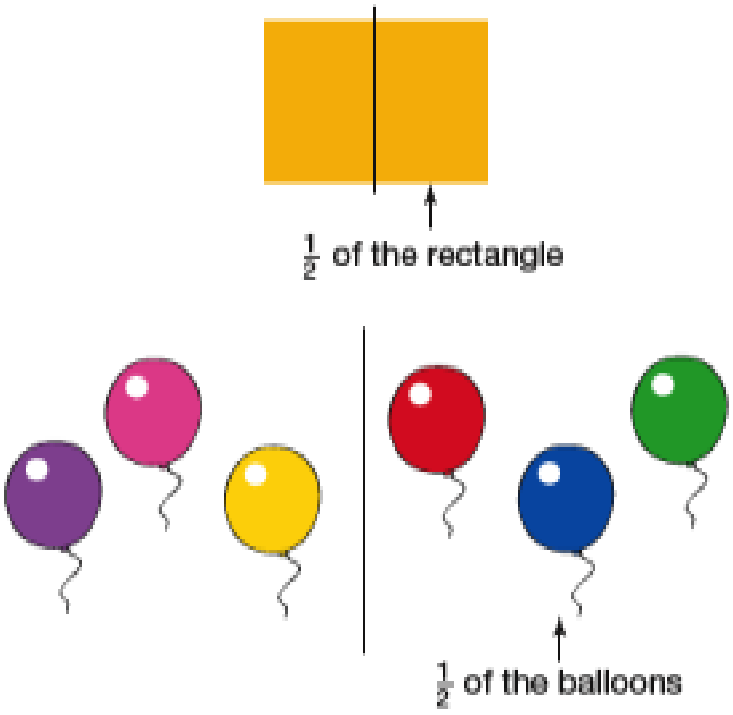
- Sorting and Classifying Data
- Representing Data
- Describing Data
- Designing and Carrying Out a Data Investigation

Benchmarks

- Use a Venn diagram to sort data by two attributes
- Identify categories for a set of categorical data and organize the data into the chosen categories
- Order and represent a set of numerical data
- Describe a numerical data set, including the highest and lowest values and the mode
- Read and interpret a variety of representations of numerical and categorical data
- Compare two sets of numerical data

Fractions

Second graders develop an understanding of what fractions are and how they can be used to name quantities. They learn that fractions are quantities that are equal parts of a whole whether the whole is a single object or a set of objects. Students work with 1/2, 1/3, 1/4, 2/3, 2/4, and 3/4 of single objects such as blocks, rectangles, squares and flags. They work with 1/2, 1/3, and ¼ of sets of objects such as balloons, sandwiches and other objects shared among a group of people.



Students learn how fractions are expressed in words—one half, two thirds—and represented using numbers—for example, 1/2, 2/3. They learn that the denominator represents the number of equal parts in the whole and that the numerator represents the number of the equal parts being considered, though they are not expected to use the words *denominator* and *numerator* when describing fractions. Students also learn the notation for mixed numbers through dividing sets. For example, if two girls share three sandwiches, each girl gets 1 1/2 sandwiches.

Emphases

What Fractions Are

- Understanding fractions as equal parts of a whole
- Using terms and notation

Benchmarks

- Identify 1/2, 1/3, and 1/4 of a region
- Find 1/2 of a set of objects
- Recognize that a fraction divides the whole into *equal* parts

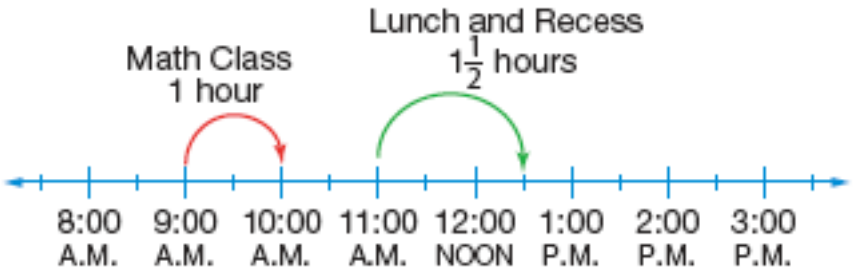
Measurement

In Grade 2, students continue to develop their understanding of length and how it is measured. They first compare lengths of objects by indirect and direct comparison and then use linear units to measure objects and compare measurements. Students learn about iterating a unit and about the relationship between sizes of units and the results of measuring: the smaller the unit, the greater the count for the same length.

By discussing their methods for measuring, students learn that agreeing on a common unit is critical for communicating measurement information to others and comparing results. This leads to work with standard measures: inches, feet, and centimeters.

As students move from using non-standard units (e.g., cubes) to measure objects to using standard tools of measurement such as rulers and yardsticks, the emphasis is on making sure that their use of a measuring tool is connected to making sense of length as an attribute of objects. Students begin their work with standard measurement tools by constructing their own inch rulers, which helps foster not only an understanding of the conventional units, but also the process of measuring with a tool and the principles that underlie the design and use of the tool. Students become accustomed to both systems of measurement: metric and U.S. Standard.

Students practice naming, notating, and telling time on digital and analog clocks. They also work with the idea that time can be represented as a horizontal sequence. Students work with timelines, associating events with a particular time. Students determine intervals of time with an emphasis on starting and ending times on the hour or half hour.



Emphases

Linear Measurement

- Understanding length
- Using linear units
- Measuring with standard units

Time

- Representing time and calculating duration

Benchmarks

- Identify sources of measurement error
- Recognize that the same count of different-sized units yields different lengths
- Recognize that, when measuring the same length, larger units yield smaller counts
- Measure objects using inches and centimeters
- Use a ruler to measure lengths longer than one foot
- Solve problems involving the beginning time of an event, ending time of an event, and duration of the event; given two of these, find the third for events beginning and ending on the hour or half-hour
- Use a timeline to record and determine duration to the hour or half-hour

Patterns, Functions, and Change

Second-grade students explore situations with constant ratios in two contexts: building cube buildings with the same number of "rooms" on each "floor" and covering a certain number of one pattern block shape with another pattern block shape. In both of these contexts, students build and record how one variable changes in relation to the other.

For example: it takes two of the red trapezoids to cover one of the yellow hexagons, 4 trapezoids to cover 2 hexagons and so forth. They use what they are learning to determine quantities later in the series.

Example: How many trapezoids will you need to cover 10 hexagons?

2 trapezoids cover 1 hexagon, 4 trapezoids cover 2 hexagons, and so on.

Tables are introduced and used as a central representation. Organizing data in a table can help students uncover a rule that governs how one quantity changes in relation to another. For example, for every increase of 1 for one quantity, the other quantity increases by 2, (e.g., for each additional trapezoid, from the pattern block set, 2 trapezoids are needed to cover it). Students compare tables that show different relationships, both within the same context and between the two contexts, and notice how different situations can have the same underlying relationship between quantities.

Number of Hexagons	Number of Trapezoids
1	2
2	4
3	6
4	8
5	10

Students also work with number sequences associated with repeating patterns that reveal important characteristics of the pattern and provide an avenue into studying the number sequences themselves.

Example: If the pattern keeps going in the same way, what color will the 12th cube be?

As students explore two-element and three-element repeating patterns, they encounter the odd number sequence, the even number sequence, and three different "counting by 3" sequences. An important part of second-grade students' work on pattern is considering how and why different situations generate the same number sequence.

Emphases

Linear Relationships

- Describing and representing ratios

Using Tables and Graphs

- Using tables to represent change

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

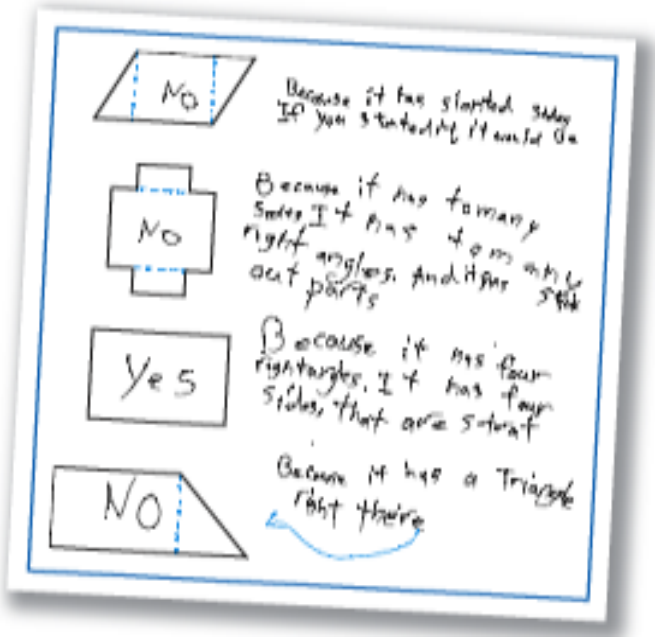
Benchmarks

- Explain what the numbers in a table represent in a constant ratio situation (involving ratios of 1:2, 1:3, 1:4, 1:5, and 1:6)
- Complete and extend a table to match a situation involving a constant ratio
- Extend a repeating pattern and determine what element of the pattern will be in a particular position (e.g., the 16^{th} position) if the pattern keeps going

Geometry

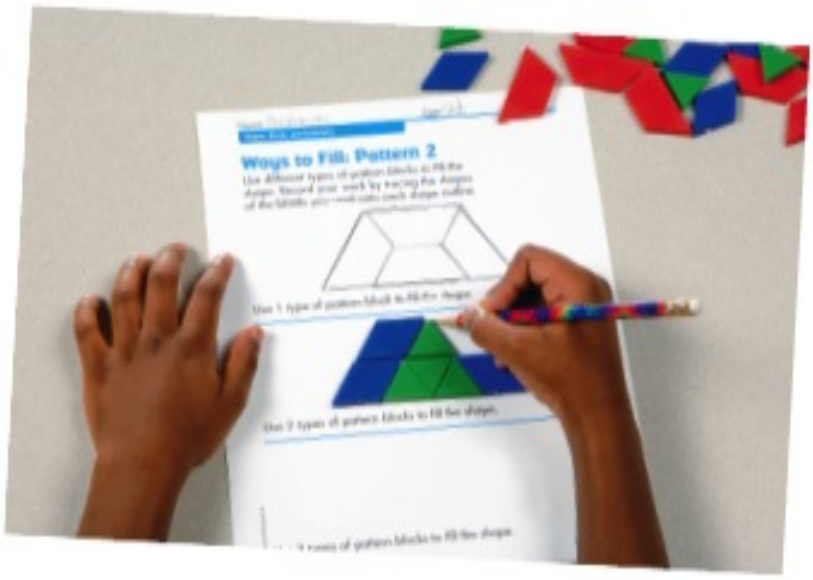
Students describe, sort and compare two-dimensional (2-D) and three-dimensional (3-D) shapes and think about questions like the following: What makes a rectangle a rectangle? How are rectangles different from squares?

Example: Is it a rectangle? Why or why not?



A student explains why shapes are (or are not) rectangles.

Second-grade students study rectangles and rectangular prisms, and consider which properties are important in describing these shapes. They combine and decompose both 2-D and 3-D shapes and explore the relationships between shapes, particularly as they work with pattern blocks and Geoblocks. As they develop knowledge about how shapes are related, they are learning about the important features of shapes.



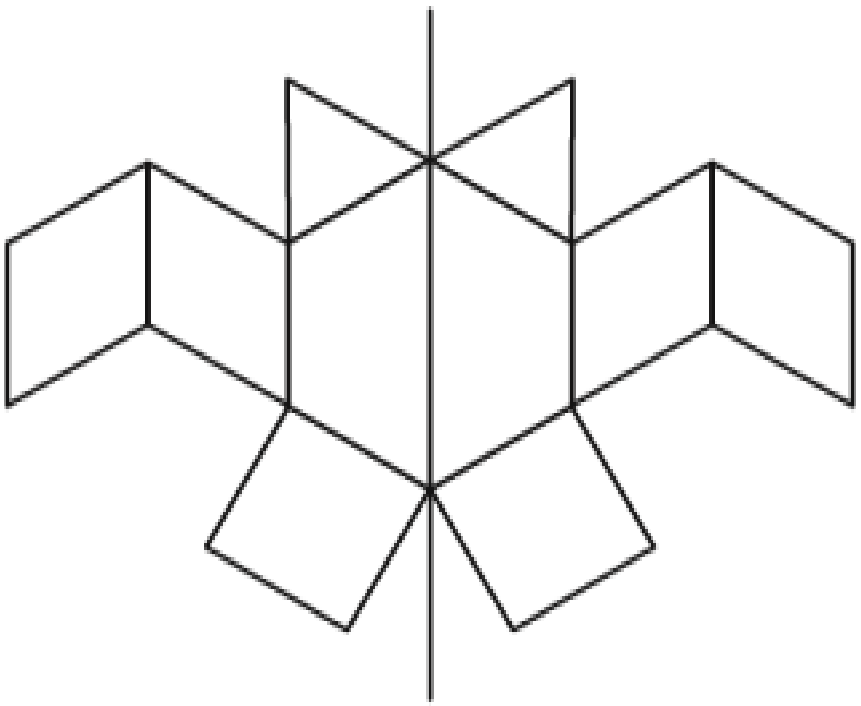
As they use pattern blocks to fill an outline, students find different ways to make the same 2-D shape.

Students find different ways to make a block of a different size.

Students begin their work with arrays, a visual representation that they will continue to use in Grades 3 through 5. As students create rectangles with square tiles, they learn about the structure of an array. In their study of arrays, work in geometry is closely linked with ideas about number. Students develop a variety of numerical strategies, based on the number of tiles in a row and the number of rows, to calculate the area of the rectangle.

A student might respond, "My rectangle has 2 rows with 3 tiles in each row."

Students develop an understanding of mirror symmetry as they identify objects that have mirror symmetry, create patterns and designs, fold and cut paper, and build 3-D structures with mirror symmetry. As they create and investigate symmetrical shapes, they develop language and ideas about what symmetry is and how it behaves.



The *Shapes* software is introduced as a tool for extending and deepening this work. This tool is designed for K-2 students to explore how different shapes can be combined to form other shapes, experiment with different sorts of geometric transformations (rotations, translation, reflection), make patterns, and investigate symmetry.

Emphases

Features of Shapes

- Combining and decomposing 2-D and 3-D shapes
- Describing, identifying, comparing, and sorting 2-D and 3-D shapes
- Exploring mirror symmetry

Area Measurement

- Visualizing the structure of arrays

Benchmarks

- Identify the number of sides of a polygon
- Identify the number of rows and the number of squares in each row in an array
- Identify rectangles as four-sided shapes with four right angles
- Identify the number of faces on a rectangular prism and show which faces are congruent
- Make a symmetrical picture based on an image provided

Grade 3[1]

Number and Operations: Whole Numbers

Addition and Subtraction and the Number System

In Grade 3, students build an understanding of the base-ten number system to 1,000 by studying the structure of 1,000 and using a base-ten context to represent the place value of two-digit and three-digit numbers. Students identify the hundreds digit as representing how many 100s are in the number, the tens digit as representing how many 10s, and the ones digit as representing how many 1s. They also break numbers into 100s, 10s, and 1s in different ways:

137 = 1 hundred, 3 tens, and 7 ones

137 = 1 hundred, 2 tens, and 17 ones

137 = 13 tens and 7 ones

137 = 12 tens and 17 ones

137 = 11 tens and 27 ones

. . .

In their work with number and operations in Grade 3, students focus particularly on addition and subtraction. Students solve addition and subtraction problems with two-digit and three-digit numbers, developing computation strategies that are built on adding and subtracting multiples of 10 and finding combinations that add to 100. Addition strategies include breaking the numbers apart and then either adding by place or adding on one number in parts. They also examine problems that lend themselves to changing the numbers in order to make them easier to add. Subtraction strategies include subtracting a number in parts, adding up, and subtracting back.

Addition Strategies

Adding by Place

349 + 175 =

300 + 100 = 400

40 + 70 = 110

9 + 5 = 14

400 + 110 + 14 = 524

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

Adding on number on in parts

349 + 175 =

349 + 100 = 449

449 + 50 = 499

499 + 25 = 524

Changing the numbers

349 + 175 =

350 + 175 = 525

525 – 1 = 524

Subtraction Strategies

Subtracting in parts

451 – 187 =

451 – 100 = 351

351 – 80 = 271

271 – 7 = 264

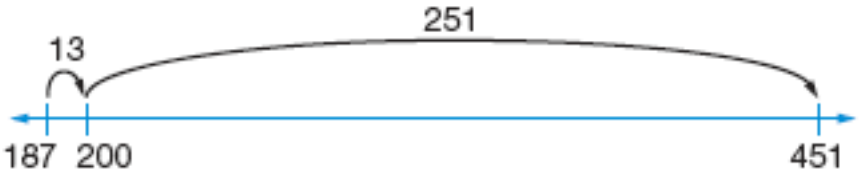
Adding up

451 – 187 =

187 + 13 = 200

200 + 251 = 451

13 + 251 = 264



Subtracting back

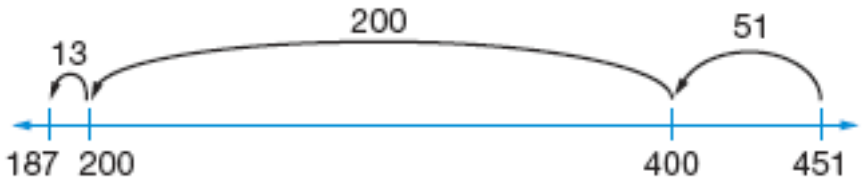
451 – 187 =

451 – 51 = 400

400 – 200 = 200

200 – 13 = 187

51 + 200 + 13 = 264



Students expand their understanding of subtraction to include other problem situations besides removal (or take away)—the type they are probably most familiar with from their work in Grade K-2. These include comparison problems and finding the missing part of a whole:

If I am collecting 1,000 baseball cards, and I have 250 so far, how many more do I need?

The ability to visualize what is happening in a subtraction situation is an important foundation for understanding the operation and how it works, and for making good decisions about strategies for computation. Students use visual representations (cubes, number lines, 300 charts, and 1,000 Charts) and story contexts. They work on developing fluency with the subtraction problems related to the addition combinations to 10 + 10 (the subtraction facts) so they are able to use these easily when solving subtraction problems with two-digit and three-digit numbers.

The Algebra Connections pages of each of the three curriculum units that focus on addition and subtraction show how students are applying the commutative and associative properties of addition as they develop strategies for solving addition problems. These pages also highlight students' application of the inverse relationship between addition and subtraction and how algebraic ideas underlie what students are doing when they create equivalent expressions in order to solve a problem (e.g., 48 + 72 = 50 + 70).

Emphases

The Base Ten Number System

- Understanding the equivalence of one group and the discrete units that comprise it
- Extending knowledge of the number system to 1,000

Whole Number Operations

- Understanding different types of subtraction problems

- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers

Computational Fluency

- Adding and subtracting accurately and efficiently

Benchmarks (compiled from Units 1, 3, and 8)

- Demonstrate fluency with the addition combinations up to 10 + 10
- Add multiples of 10 (up to 100) to and subtract them from 2-digit and small 3-digit numbers
- Solve addition problems with 2-digit numbers using strategies involving breaking numbers apart by place or adding one number in parts
- Break up 3-digit numbers less than 200 into 100s, 10s, and 1s in different ways (e.g. 153 equals 1 hundred, 5 tens, and 3 ones; 15 tens and 3 ones; 14 tens and 13 ones, etc.)
- Find combinations of 2-digit numbers that add to 100 or $1.00
- Read, write, and sequence numbers to 1,000
- Identify the value of each digit in a 3-digit number (100s, 10s, and 1s)
- Identify how many groups of 10 are in a 3-digit number (e.g. 153 has 15 groups of 10, plus 3 ones)
- Solve addition problems with 3-digit numbers (to 400) using strategies that involve breaking numbers apart, either by place value or by adding one number in parts
- Solve subtraction story problems in contexts that include removing a part from a whole, comparing two quantities, or finding a missing part
- Solve subtraction problems with 2-digit and 3-digit numbers (to 300) using strategies that involve either subtracting a number in parts, adding up, or subtracting back
- Add multiples of 10 and 100 (to 1,000) to and subtract them from any 3-digit number
- Solve 3-digit addition problems using at least one strategy efficiently
- Demonstrate fluency with problems related to the addition combinations to 10 + 10 (the subtraction facts)
- Solve subtraction problems with 3-digit numbers using strategies that involve either subtracting a number in parts, adding up, or subtracting back

Multiplication and Division

In Grade 3, students investigate the properties of multiplication and division, including the inverse relationship between these two operations, and develop strategies for solving multiplication and division problems. Their work focuses on developing the idea that multiplication involves some *number of equal-sized groups*, and that division also involves equal groups.

Students are introduced to arrays—rectangular arrangements of objects in rows and columns—to help them develop visual images that support their understanding of multiplication.

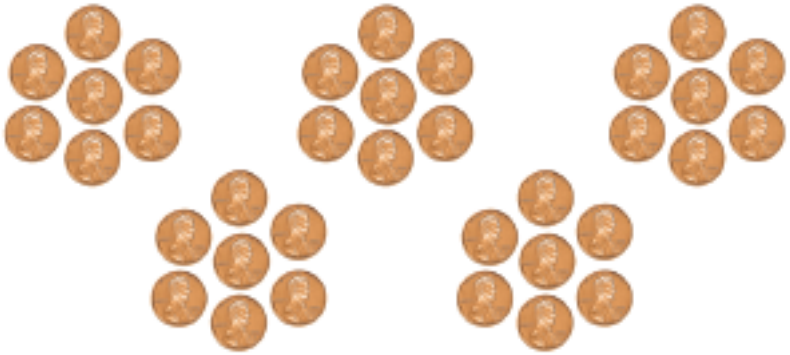
3 by 4 array

They use these rectangular arrays to represent the relationship between a product and its factors. Students determine, describe, and compare sets of multiples, noticing their characteristics and relationships, and use these to investigate important ideas about how multiplication works. They learn the multiplication combinations with products up to 50.

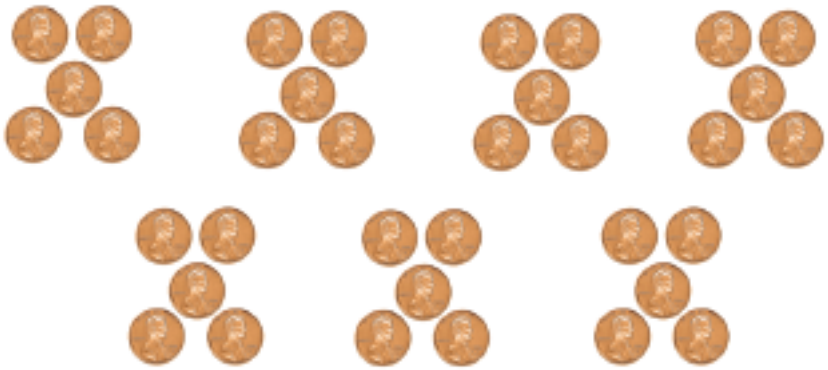
4 x 5 =	7 x 6 =
5 x 4 =	6 x 7 =
Start with	Start with
_____	_____

Students solve division situations that involve *sharing*, ("Divide 35 pennies among 5 people equally. How many pennies are in each share?") and those that involve *grouping* ("How many groups of 5 pennies can I make if I have 35 pennies?").

Sharing: Divide 35 pennies among 5 people equally. How many pennies are in each share?



Grouping: How many groups of 5 pennies can I make if I have 35 pennies?



Students use their knowledge of the relationship between division and multiplication by reasoning in ways like the following: "I know that five 5s is 25, and two more 5s make 35, so I have 7 groups of 5." Students are also introduced to two forms of division notation — 35 ÷ 5 and $5\overline{)35}$ — and learn how to interpret these numbers and symbols in terms of the meaning and actions of division.

The Algebra Connections page in the curriculum unit that focuses on multiplication and division shows how students are applying the commutative and distributive properties of multiplication as they solve problems. It also highlights students' application of the inverse relationship between multiplication and division.

Emphases

Whole Number Operations

- Understanding the meaning of multiplication
- Reasoning about numbers and their factors and multiples
- Understanding and working with an array model of multiplication
- Developing strategies for division based on understanding the inverse relationship between multiplication and division

Computational Fluency

- Learning the multiplication combinations with products to 50 fluently

Benchmarks

- Demonstrate an understanding of multiplication and division as involving groups of equal groups
- Solve multiplication combinations and related division problems using skip counting or known multiplication combinations
- Interpret and use multiplication and division notation
- Demonstrate fluency with the multiplication combinations with products up to 50 (by the end of Grade 3)

Number and Operations: Rational Numbers

Students use a variety of contexts to understand, represent, and combine fractions. These include rectangles representing "brownies," hexagonal pattern block "cookies," and groups of objects.

Students work with halves, fourths, eighths, thirds, and sixths as they learn how fractions represent equal parts of a whole. They learn the meanings of the numerator and denominator of a fraction, so that when comparing unit fractions (fractions with a numerator of 1), they understand that the larger the denominator the smaller the part of the whole: 1/6 is smaller than ½ of the same whole. Students also gain experience with common equivalencies, for example, that 3/6 and 2/4 are both equal to 1/2.

Using these equivalents in contexts, students find combinations of fractions that are equivalent to a whole or to another fraction. For example,

$$1/2 + 2/6 + 1/6 = 1$$
$$1/3 + 1/6 = 1/2$$

Students are introduced to decimal fractions (0.50 and 0.25), using the context of money, and gain familiarity with fraction and decimal equivalents involving halves and fourths.

Emphases

Rational Numbers

- Understanding the meaning of fractions (halves, fourths, eighths, thirds, sixths) and decimal fractions (0.50, 0.25) as equal parts of a whole (an object, an area, a set of objects)
- Using representations to combine fractions (halves, fourths, eighths, thirds, and sixths)

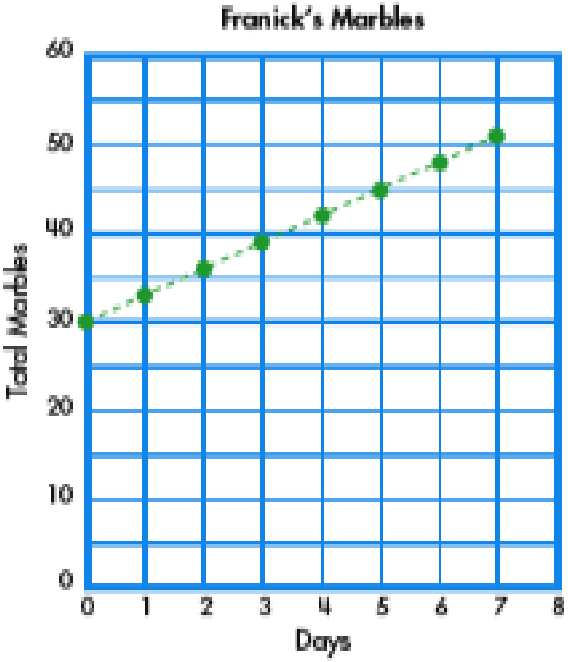
Benchmarks

- Divide a single whole or a quantity into equal parts and name those parts as fractions or mixed numbers
- Identify equivalent fractions (e.g. $\frac{3}{6} = \frac{1}{2}$ and $\frac{1}{3} = \frac{2}{6}$)
- Find combinations of fractions that are equal to 1 and to other fractions (e.g. $\frac{3}{6} + \frac{1}{2} = 1$; $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$; and $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$)

Patterns, Functions, and Change

Students study situations of change as they examine temperature change over time in different places around the world, analyze number sequences generated by repeating patterns, and consider a fantasy situation of constant change in which children receive a certain number of *Magic Marbles* each day. They make, read, and compare line graphs that show a relationship between two variables in situations of change over time. Students learn how to find the two values represented by a point on a coordinate graph by referring to the scales on the horizontal and vertical axes. Students focus on seeing a graph as a whole, thinking about the overall shape of a graph, and discussing what that overall shape shows about the change in the situation it represents. A class temperature graph is created over the course of the year and discussed regularly. Students learn to read and interpret temperatures using standard units.

Students also use tables as a representation that shows how one variable changes in relation to another variable. Emphasis is on how the numbers in the table relate to the situation they represent and to graphs of the same situation.



Day	Franick
Beginning	30
Day 1	33
Day 2	36
Day 3	39
Day 4	42
Day 5	45
Day 6	48
Day 7	51

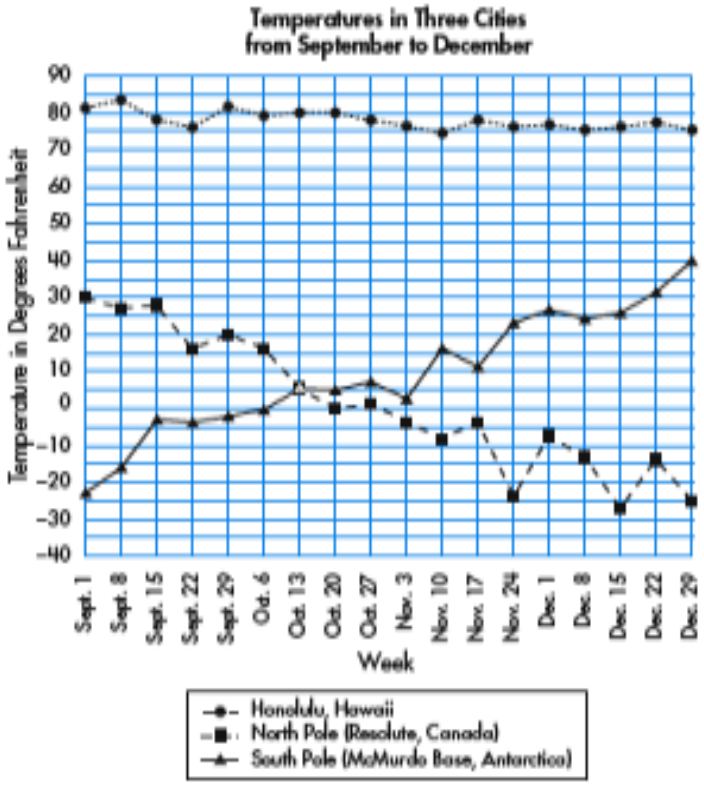


Table and graph from Grade 3 Unit 6 and *Line graph from Grade 3 Unit 6*

Students use both tables and graphs to examine and compare situations with a constant rate of change. They examine the relationship between columns of the table and consider why the points on graphs representing such situations fall in a straight line. By examining the tables and graphs, students consider any initial amount and the constant rate of change to develop general rules that express the relationship between two variables in these contexts.

Emphases

Using Tables and Graphs

- Using graphs to represent change
- Using tables to represent change

Linear Change

- Describing and representing a constant rate of change

Number Sequences

- Constructing, describing, and extending number sequences with constant increments generated by various contexts

Measuring Temperature

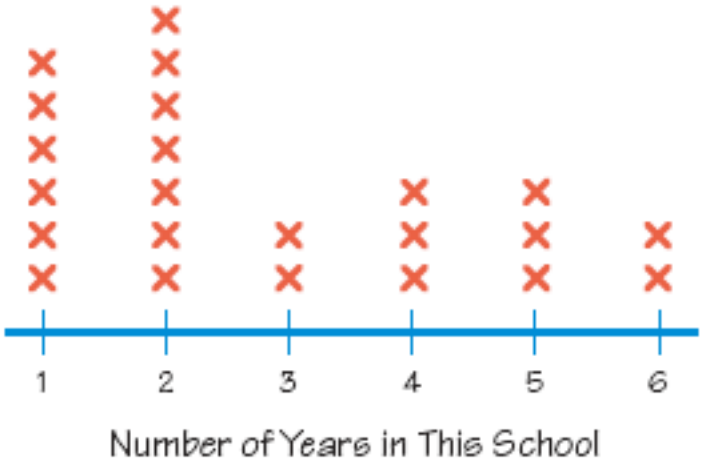
- Understanding temperature and measuring with standard units

Benchmarks

- Interpret graphs of change over time, including both the meaning of points on the graph and how the graph shows that values are increasing, decreasing, or staying the same
- Interpret temperature values (e.g., relate temperatures to seasons, to what outdoor clothing would be needed)
- Create a table of values for a situation with a constant rate of change and explain the values in the table in terms of the situation
- Compare related situations of constant change by interpreting the graphs, tables, and sequences that represent those situation

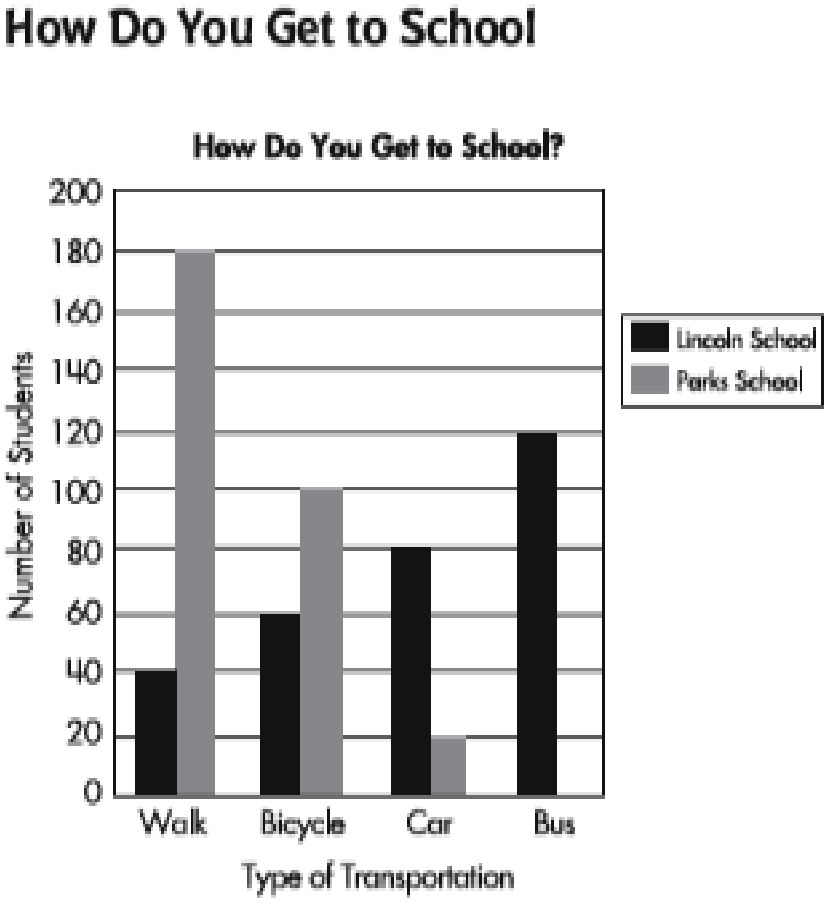
Data Analysis

Students collect, represent, describe, and interpret data. They work with both categorical and numerical data, and consider how to look at a data set as a whole and make statements about the whole group. In order to make sensible statements about a categorical data set that has many different values, students group the data into categories that help them see the data as a whole. Students order numerical data by value so that they can see the shape of the data—where the data are concentrated, where they are spread out, which intervals have many pieces of data, and which have very few. They describe what values would be typical or atypical, based on the data, and compare data sets in order to develop a sense of how data can be useful in describing and comparing some characteristic of a group.



Students work with their own data, creating representations, and then comparing and discussing these representations. Students use double bar graphs to compare groups, including some in which the scales have intervals greater than 1.

Students interpret line plots and create their own line plots to represent



numerical data. By conducting their own data investigations, students consider how the question they pose and the way they conduct their study affect the resulting data.

Emphases

Data Analysis

- Describing, summarizing, and comparing data
- Representing data
- Designing and carrying out a data investigation

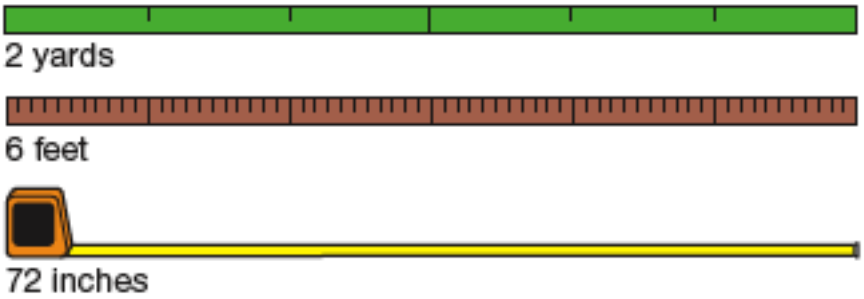
Benchmarks

- Organize, represent, and describe categorical data, choosing categories that help make sense of the data
- Interpret a bar graph
- Make a line plot for a set of numerical data
- Describe the shape of the data for a numerical data set, including where data are concentrated, where there are few data, what the lowest and highest values are, what the mode is, and where there is an outlier

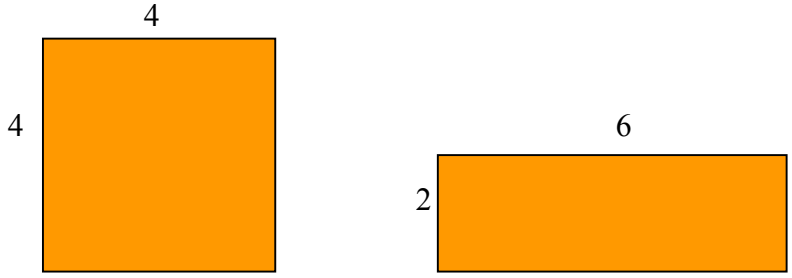
- Summarize a set of data, describing concentrations of data and what those concentrations mean in terms of the situation the data represent

Measurement

In Grade 3, students work on measurement in the data, 2D geometry and measurement, 3D geometry and measurement, and patterns and functions units. Measurement work in Grade 3 includes linear measurement, area, angle measurement, volume, and temperature. Students measure length and calculate perimeter with both U.S. standard units (inches, feet and yards) and metric units (centimeters and meters). Their work focuses on using measurement tools accurately, and understanding the relationship between measures when the same length is measured with different units.

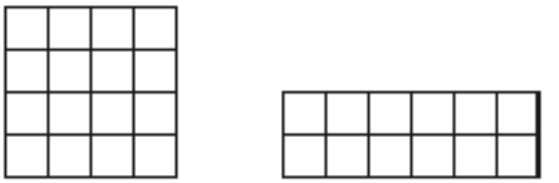


Students learn that the distance around the outside edges of a two-dimensional shape is called the *perimeter* and consider how different shapes can have the same perimeter.



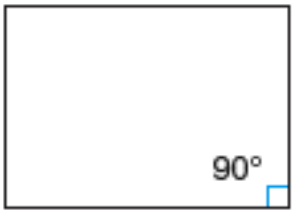
perimeter = 16 units *perimeter = 16 units*

They identify the amount of 2-D space a given shape covers as its *area,* and learn that area is measured in square units.



area = 16 square units *area = 12 square units*

They identify the internal angle of a rectangle or square as 90 *degrees*. They use right angles as a benchmark as they consider the sizes of angles of other polygons.



Students also learn how the term *degrees* is used differently when talking about measuring temperature. A class temperature graph is created over the course of the school year. Students learn to read and interpret temperature using standard units.

Students practice naming, notating, and telling time on digital and analog clocks. They begin at the start of the year with telling time at five-minute intervals and then move to telling time at any minute. Students also work on intervals of time. For example, they begin with a time and determine what time it will be after a given number of minutes have passed or they determine how many minutes have passed when given a starting and ending time.

Emphases

Linear Measurement

- Measuring length
- Measuring with standard units
- Understanding and finding perimeter

Area Measurement:

- Understanding and finding area

Features of Shape

- Describing and measuring angles

Volume:

- Structuring rectangular prisms and determining their volume

Measuring Temperature

- Understanding temperature and measuring with standard units

Benchmarks

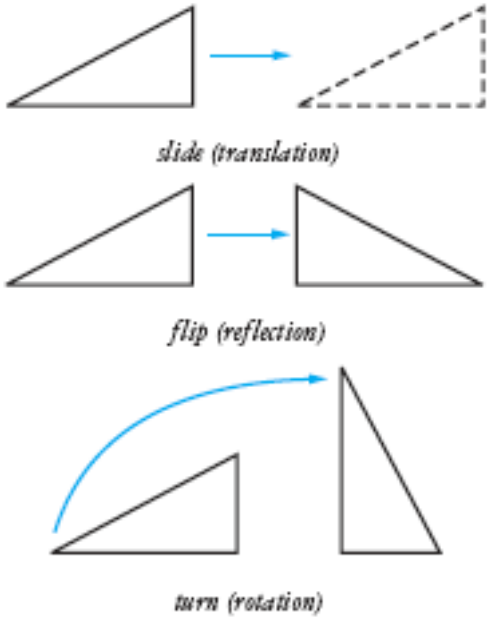
- Identify and measure the perimeter of a figure using U.S. standard and metric units
- Identify and find the area of given figures by counting whole and partial square units
- Identify right angles and recognize whether an angle is larger or smaller than a right angle
- Determine the number of cubes (volume) that will fit in the box made by a given pattern

Geometry and Measurement

Students study the attributes of two-dimensional (2-D) and three-dimensional (3-D) shapes, and how these attributes determine their classification. For example, a polygon is classified as a triangle or a quadrilateral based on the number of its sides.

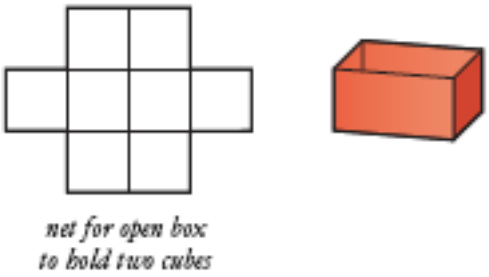
triangles *quadrilaterals*

Students also investigate the idea that one shape may have more than one name as they consider the properties of squares and rectangles. They describe shapes by whether or not they are *congruent* to other shapes, and use geometric motions—*slides (*translations*), flips* (reflections), and *turns* (rotations)—to determine if shapes are congruent.

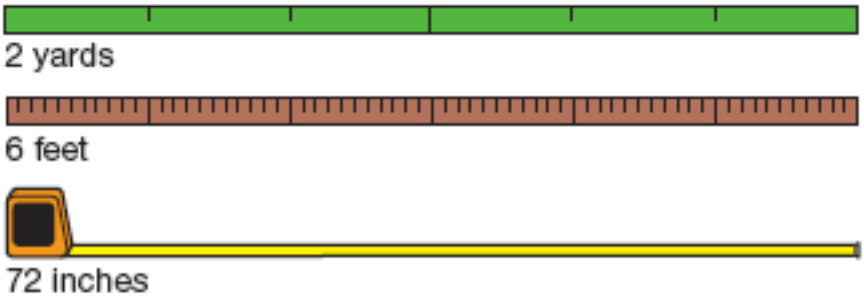


Students describe attributes of common geometric solids (3-D shapes), such as how many edges and faces a solid shape has, or how a pyramid has triangular faces coming to a point. They learn to distinguish between polyhedra (3-D shapes having only flat surfaces) and nonpolyhedra (3-D shapes that have curved surfaces) and, within the class of polyhedra, between prisms and pyramids.

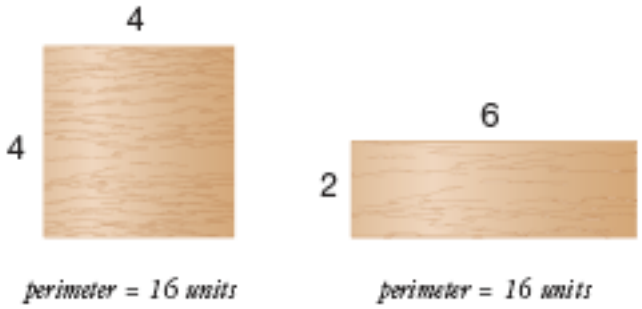
Students learn about how 3-D objects can be represented in 2-D space. For example, they design nets for open boxes that, if constructed in 3-D, would hold a certain number of cubes. They determine the volume of the rectangular prisms that fit into a variety of open boxes.



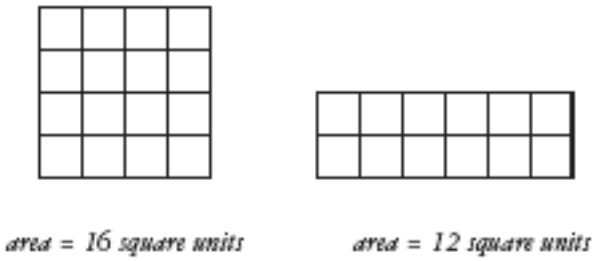
Students' measurement work in Grade 3 includes linear measurement, area, angle measurement, and volume. They measure length and perimeter with both U. S. standard units (inches, feet and yards) and metric units (centimeters and meters). Their work focuses on using measurement tools accurately, and understanding the relationship between measures when the same length is measured with different units.



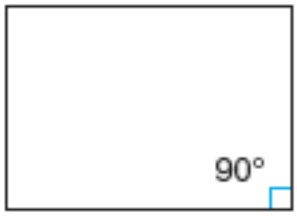
Students learn that the distance around the outside edges of a two-dimensional shape is called the *perimeter*, and consider how different shapes can have the same perimeter.



They identify the amount of 2-D space a given shape covers as its *area,* and learn that area is measured in square units.



They identify the internal angle of a rectangle or square as 90 *degrees*. They use right angles as a benchmark as they consider the sizes of angles of other polygons.



Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Describing properties of 3-D shapes
- Translating between 2-D and 3-D shapes

Linear Measurement

- Measuring length
- Measuring with standard units
- Understanding and finding perimeter

Area Measurement

- Understanding and finding area

Volume

- Structuring rectangular prisms and determining their volume

Benchmarks

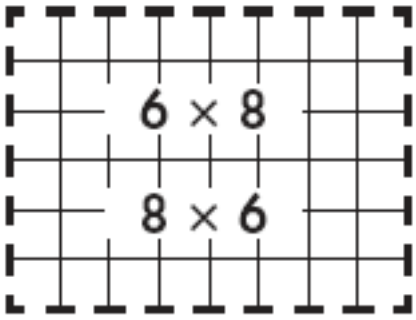
- Identify and accurately measure the perimeter of a shape using U.S. standard and metric units
- Identify and find the area of given figures by counting whole and partial square units
- Identify triangles as three-sided closed shapes with three vertices and three angles
- Identify right angles, and recognize whether an angle is larger or smaller than a right angle
- Identify and compare attributes of 3-D solids
- Determine the number of cubes (volume) that will fit in the box made by a given pattern
- Design patterns for boxes that will hold a given number of cubes

Grade 4[1]

Number and Operations: Whole Numbers

Multiplication and Division

In Grade 4, three of the four curriculum units on number and operations with whole numbers focus on multiplication and division. This major component of students' work centers on reasoning about numbers and their factors and multiples, using models, representations, and story contexts to help them visualize and solve multiplication and division problems; and understanding the relationship between multiplication and division.



Combinations We're Working On	Start With
6 × 8	5 × 8 (and add one more 8) 3 × 8 (and double it) 6 × 4 (and double it)

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

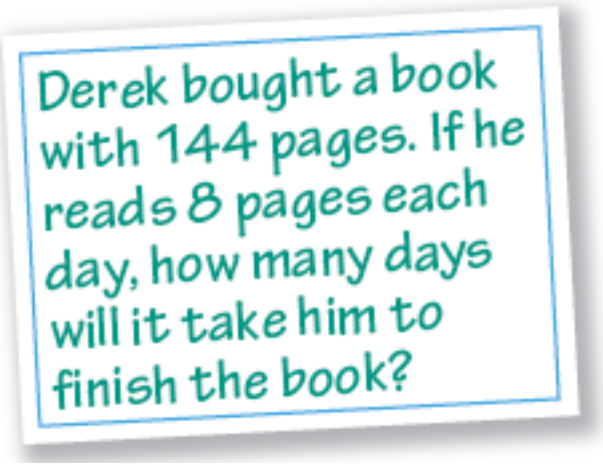
Students learn the multiplication combinations (facts) to 12 x 12 so that they can use these fluently to solve both multiplication and division problems. They develop strategies for solving multiplication and division problems based on looking at the problem as a whole, thinking about the relationships of the numbers in the problem, and choosing an approach they can carry out easily and accurately, often breaking the numbers apart or changing the numbers in some way. Visualizing how multiplication works is critical in applying the distributive property to solve problems and in keeping track of parts of the problem. Learning to multiply by multiples of 10 is also a key component of this work.

Examples of Multiplication Strategies

Breaking numbers apart by addition

48 x 42 =	**48 x 42 =**
40 x 40 = 1,600	48 x 40 = 1,920
40 x 2 = 80	48 x 2 = 96
8 x 40 = 320	1,920 + 96 = 2,016
8 x 2 = 16	
1,600 + 80 + 320 + 16 = 2,016	

Students interpret and solve division problems, both in story contexts and numerical contexts. They work with both grouping and sharing situations, and consider how to make sense of a remainder within the context of the problem. They use the inverse relationship between multiplication and division to solve division problems, including those related to the multiplication combinations to 12 x 12 (the division "facts"), and problems in which 3-digit numbers are divided by 1-digit and small 2-digit divisors.



The Algebra Connections pages in the three curriculum units that focus on multiplication and division show how students are applying the commutative and distributive properties of multiplication, as well as the inverse relationship between multiplication and division, as they solve problems. These pages also highlight particular generalizations about multiplication that students work on in Grade 4: If a number is a factor of a second number, are all the factors of the first number also factors of the second number? If one factor in a multiplication expression is halved and another factor is doubled, what is the effect on the product?

Emphases

Whole Number Operations

- Understanding and working with an array model of multiplication
- Reasoning about numbers and their factors
- Understanding and using the relationship between multiplication and division to solve division problems
- Understanding division as making groups of the divisor

Computational Fluency

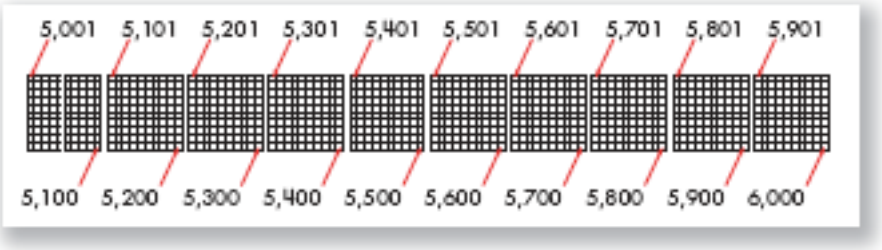
- Fluency with the multiplication combinations to 12 x 12
- Solving multiplication problems with 2-digit numbers

Benchmarks (compiled from Units 1, 3, and 8)

- Use known multiplication combinations to find the product of any multiplication combination to 12 x 12
- Use arrays, pictures or models of groups, and story contexts to represent multiplication situations
- Find the factors of 2-digit numbers
- Multiply 2-digit numbers by one-digit and small 2-digit numbers (e.g. 12, 15, 20), using strategies that involve breaking the numbers apart
- Solve division problems (2- and small 3-digit numbers divided by 1-digit numbers) including some that result in a remainder
- Use story problems, pictures, or concrete models to represent division situations
- Multiply by 10 and multiples of 10
- Demonstrate fluency with multiplication combinations to 12 x 12
- Multiply 2-digit numbers efficiently
- Solve division problems with 1- and small 2-digit divisors by using at least one strategy efficiently

Addition, Subtraction, and the Number System

In Grade 4, students extend their knowledge of the base-ten number system, working with numbers up to 10,000. Their work focuses on understanding the structure of 10,000 and how numbers are related within that structure, recognizing the place value of digits in large numbers, and using place value to determine the magnitude of numbers.

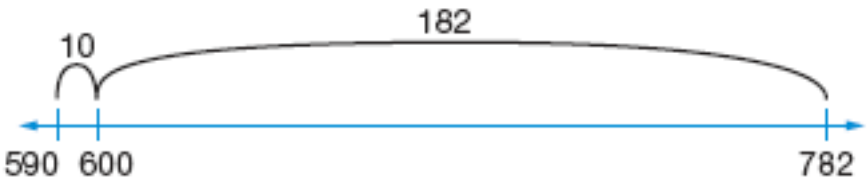


By discussing, refining and comparing their strategies for adding and subtracting 3- and 4-digit numbers, including studying the U.S. algorithm for addition, students continue expanding their understanding of addition and subtraction. Their strategies should involve good mental arithmetic, estimation, clear and concise notation, and a sound understanding of number relationships. By identifying and naming addition and subtraction strategies that they are using, students are adding to the repertoire of strategies they can use for flexible and fluent computation. Further, they consider how and why certain methods work. For example, some students change one or both numbers in an addition or subtraction expression to create an easier problem, then compensate as needed for that change. In this unit, students study *why* certain addition expressions are equivalent (e.g., 457 + 198 = 455 + 200) and *how* certain expressions in subtraction are related (e.g., 782 – 590 and 782 – 600).

782 – 590 = _____

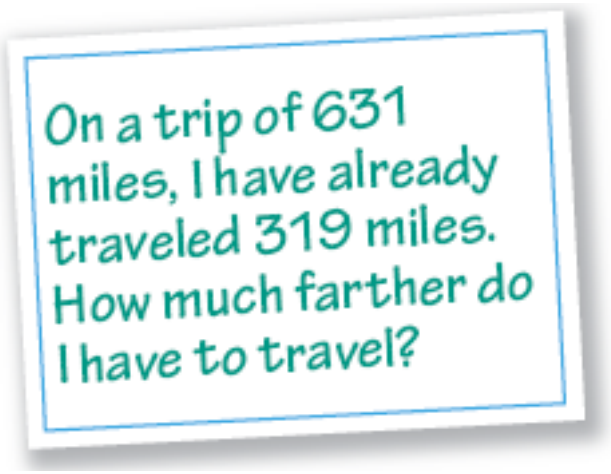
782 – 600 = 182

182 + 10 = 192



To help them make good decisions about strategies for subtraction and continue to develop their understanding of how subtraction operates, students use visual representations, such as number lines and 100 Charts, and story contexts that include several types of subtraction situations—

removal (or take away), comparison, and missing parts. Students focus particularly on missing part problems in the context of distance:



Some students visualize a problem like this one as adding up from the distance traveled to the total distance, while others visualize subtracting the distance traveled from the total distance. This provides another opportunity for students to consider the relationship between addition and subtraction.

319 + ____ = 631

631 – 319 = ____

The Algebra Connections page in the curriculum unit that focuses on addition and subtraction shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic ideas that underlie the generalizations students investigate and articulate when they create equivalent expressions in order to solve a problem (e.g., 124 – 89 = 125 – 90).

Emphases

The Base Ten Number System

- Extending knowledge of the base-ten number system to 10,000

Computational Fluency

- Adding and subtracting accurately and efficiently

Whole Number Operations

- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers
- Understanding different types of subtraction problems

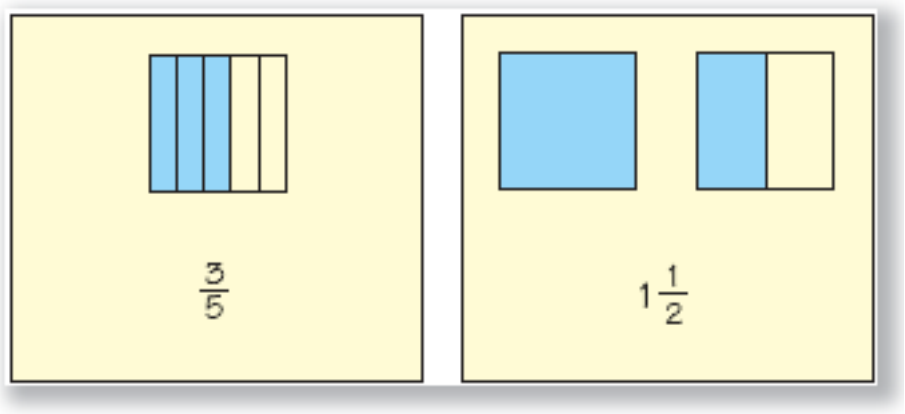
Benchmarks

- Read, write, and sequence numbers to 10,000
- Add and subtract multiples of 10 (including multiples of 100 and 1,000) fluently

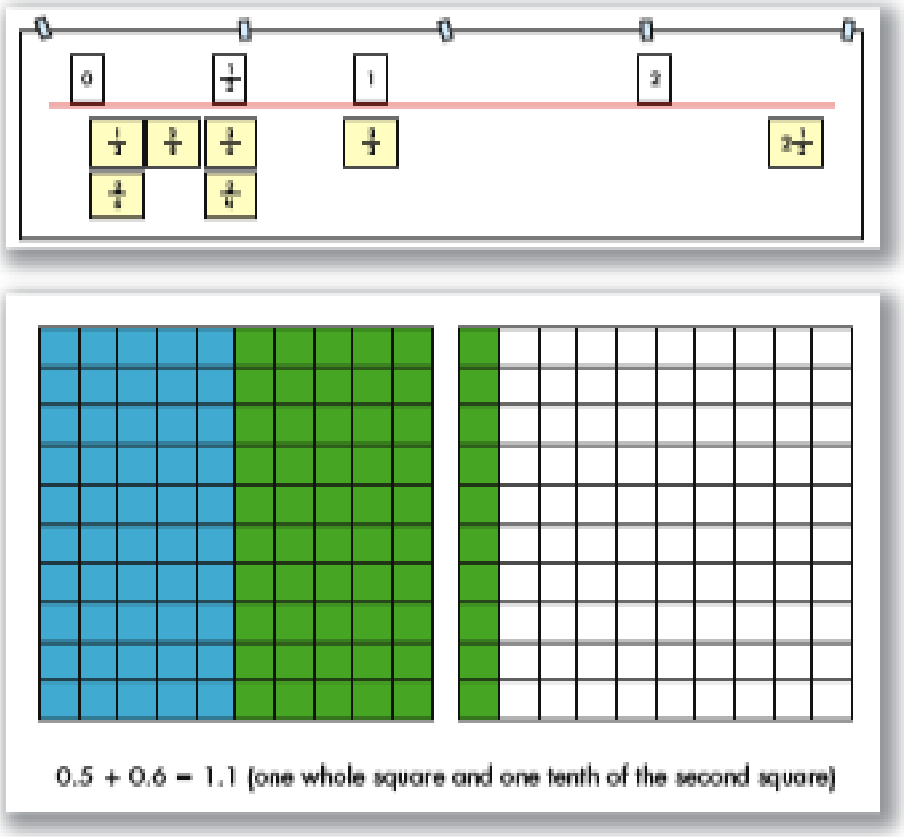
- Solve addition problems efficiently, choosing from a variety of strategies
- Solve subtraction problems with 3-digit numbers by using at least one strategy efficiently

Number and Operations: Rational Numbers

The major focus of the work on rational numbers in Grade 4 is on building students' understanding of the meaning, order, and equivalencies of fractions and decimals. Students continue to focus on the meaning of fractions as equal parts of a whole. They extend their images of equal parts to accommodate fractions that are greater than 1. Students work with fractions in the context of area (equal parts of a rectangle), as a group of things (e.g., a fractional part of the class), and on a number line. They work with fractions that represent halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths.



Students are introduced to decimal fractions in tenths and hundredths as an extension of the place value system they have studied for whole numbers. They relate decimals to equivalent decimals and fractions (for example, when they represent 0.25 as part of a rectangle, they can see how it is equal to ¼ and to 2 ½ tenths). Students draw on their mental images of fractions and decimals and on their knowledge of fraction and decimal equivalencies and relationships to reason about fraction comparisons, to order fractions on a number line, and to add fractions and decimals using representations.



Emphases

Rational Numbers

- Understanding the meaning of fractions and decimal fractions
- Comparing the values of fractions and decimal fractions

Computation with Rational Numbers

- Using representations to add rational numbers

Benchmarks

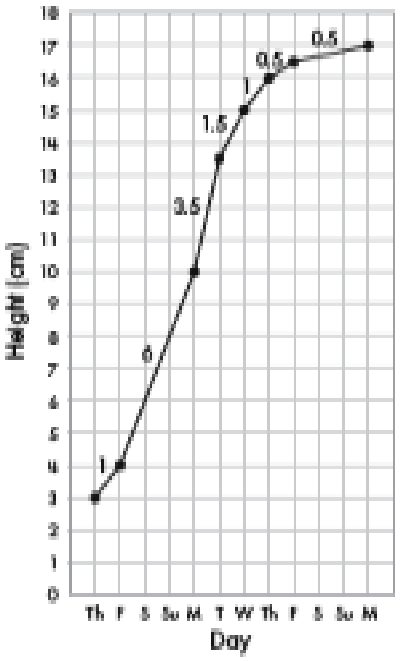
- Identify fractional parts of an area
- Identify fractional parts of a group (of objects, people, etc.)
- Read, write, and interpret fraction notation
- Order fractions with like and unlike denominators
- Read, write, and interpret decimal fractions in tenths and hundredths

Patterns, Functions, and Change

In Grade 4, students use graphs and tables to represent change. One focus of their work is how a line graph shows the *rate of change*, as they consider questions such as the following:
"How does this graph show the parts of the story that are about *speed* and the parts of the story that are about *changes in speed*?"

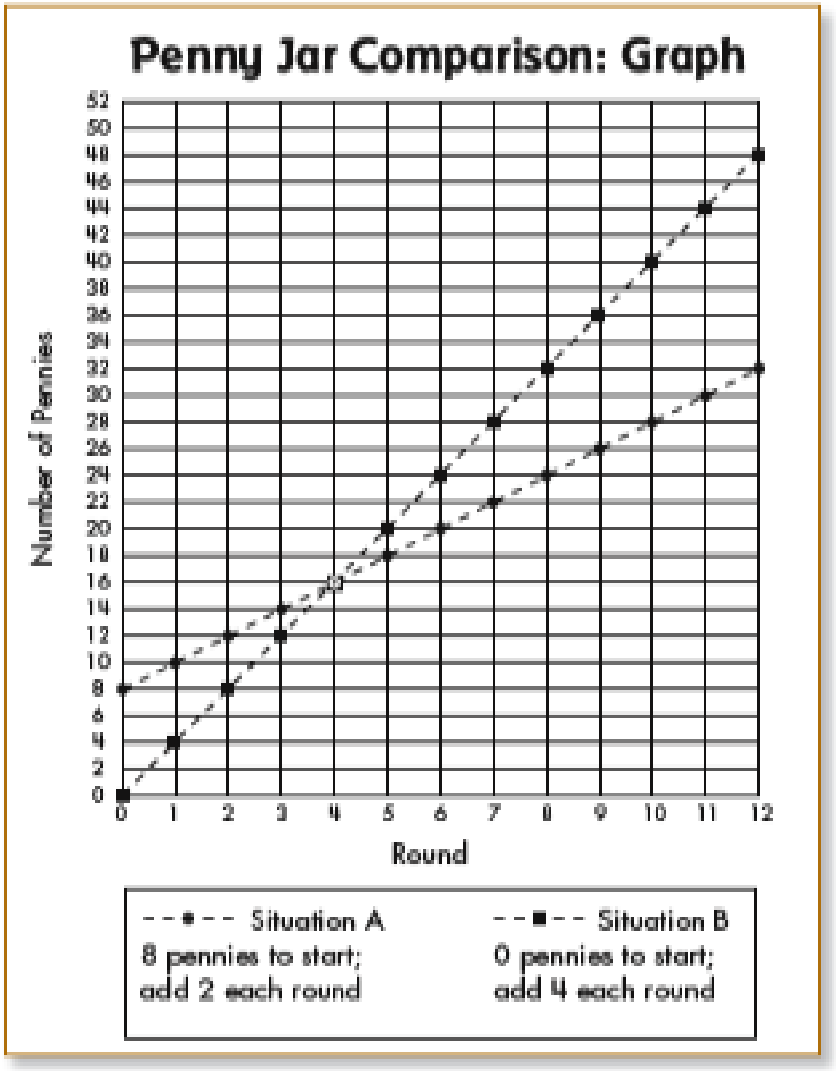
"What was the rate of growth for this plant? When was it growing more slowly or more quickly?"

Day	Height
Thursday	3 cm
Friday	4 cm
Monday	10 cm
Tuesday	13.5 cm
Wednesday	15 cm
Thursday	16 cm
Friday	16.5 cm
Monday	17.5 cm



Students create tables and graphs for situations with a constant rate of change and use them to compare related situations.

For example: Penny Jar A has 8 pennies in the jar to start and 2 pennies are added in each round; Penny Jar B has 0 pennies in the jar to start and 4 pennies are added in each round. Will the number of pennies in Penny Jar B, which starts with fewer pennies, ever "catch up" to the number of pennies in Penny Jar A?



By analyzing tables and graphs, students consider how the starting amount and the rate of change define the relationship between the two quantities (e.g., number of rounds, total number of pennies), and develop rules that govern that relationship. At first students articulate these rules in words (as they did in grade 3), but they also are introduced to the use of symbolic notation and equations to represent their rules. They use these rules to determine the value of one variable when the value of the other is known.

How many pennies are in Penny Jar A after 10 rounds?

Emphases

Using Tables and Graphs

- Using tables to represent change
- Using tables to represent change

Linear Relationships

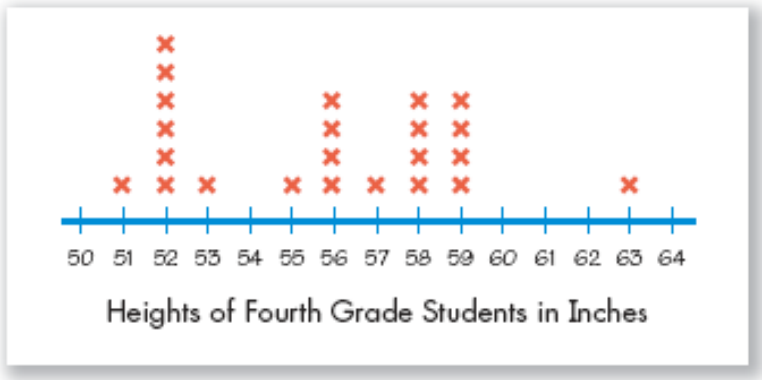
- Describing and representing a constant rate of change

Benchmarks

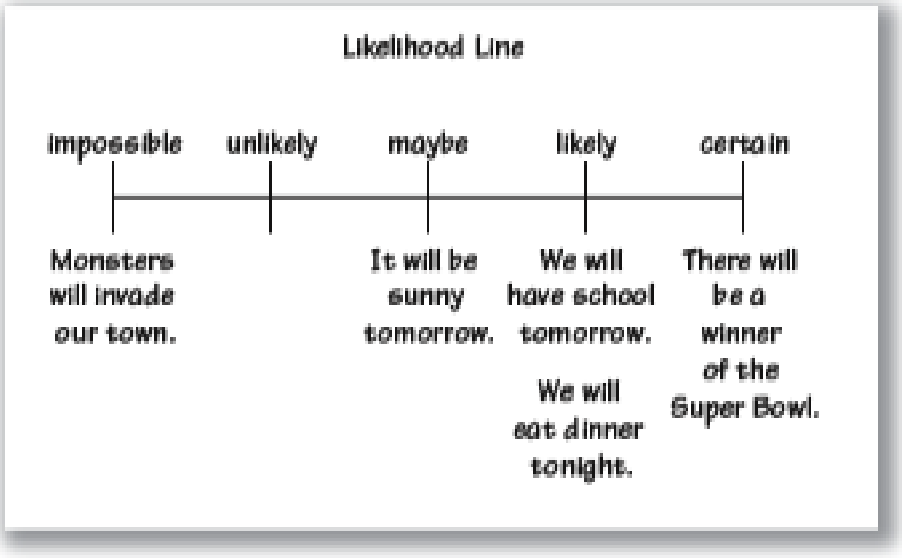
- Connect tables and graphs of change over time to each other and to the situations they represent.
- Make a graph on a coordinate grid from a table of values.
- Describe how a graph shows change: where the rate of change is increasing, decreasing, or remaining constant, and how differences in steepness represent differences in the rate of change.
- Take into account the starting amount and the amount of change in describing and comparing situations of constant change.
- In a situation of constant change, write rules (using words or arithmetic expressions) to determine the value of one quantity, given the value of the other.

Data Analysis and Probability

Students continue to develop their understanding of data analysis by collecting, representing, describing, and interpreting numerical data, in order to answer a question, investigate an issue, or provide information about something in the world that is of interest. Their work focuses on describing and summarizing data for comparing two groups. Using a *line plot* as a tool for showing the shape or distribution of a set of data—where the data are concentrated, how they are spread across the range—students represent data about two groups, and then consider how to characterize how the groups are similar or different. They develop conclusions and make arguments, based on the evidence they have collected.



Students also work on describing and predicting the likelihood of events in their world: what events are impossible, unlikely, likely, or certain? They consider situations in which there is a known number of possible outcomes—such as when rolling a number cube or pulling a red cube out of a bag holding a certain number of red and blue cubes. Students reason about how the theoretical chance (or *theoretical probability*) of, for example, rolling 1 on a number cube compares to what actually happens when a number cube is rolled repeatedly.



Emphases

Data Analysis

- Representing data
- Describing, summarizing, and comparing data
- Analyzing and interpreting data
- Designing and carrying out a data investigation

Probability

- Describing the probability of an event

Benchmarks

- Design an effective survey question to compare two groups
- Organize and represent data about two groups in order to compare the groups
- Describe the shape of the data from a numerical data set including where the data are concentrated and the highest, lowest and median values
- Use data to compare two groups
- Use evidence from a set of data to support an argument

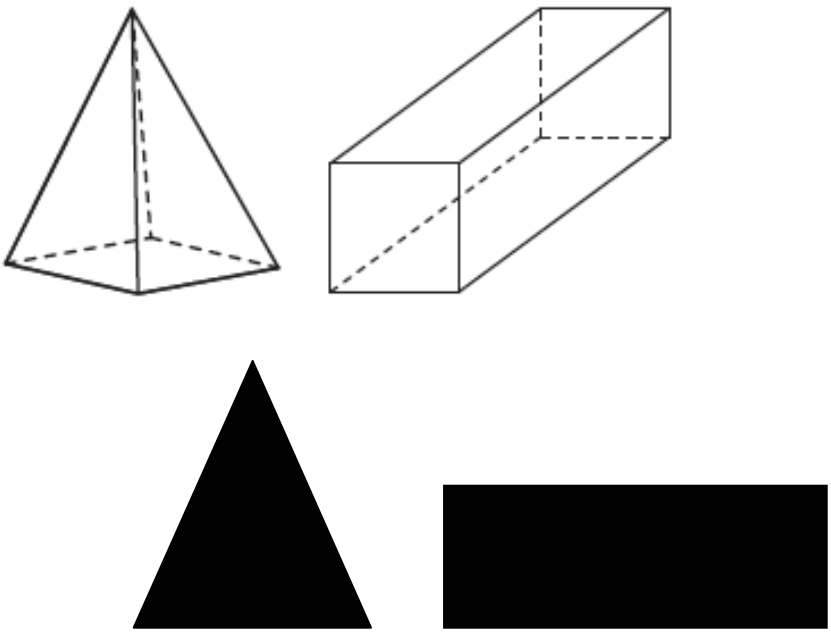
- Describe the likelihood of an event in terms of a scale from impossible (probability of 0) to certain (probability of 1)

Geometry and Measurement

Students expand their understanding of the attributes of two-dimensional (2-D) and three-dimensional (3-D) shapes, and how these attributes determine their classification. Students consider the various attributes of 2-D shapes, such as number of sides, the length of sides, parallel sides, and the size of angles, expanding their knowledge of four-sided figures (quadrilaterals) to include parallelograms, rhombuses, and trapezoids.

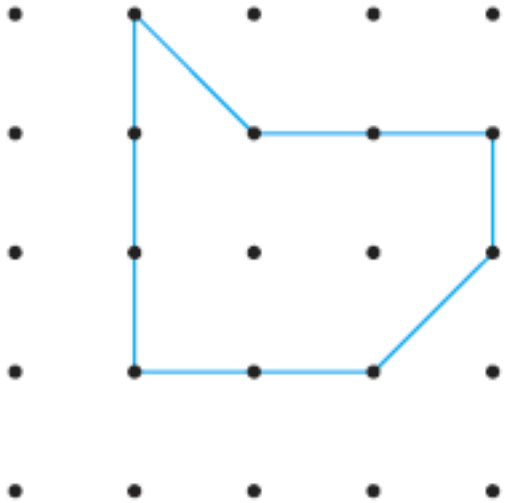
Students also describe attributes and properties of geometric solids (3-D shapes), such as the shape and number of faces, the number and relative lengths of edges, and the number of vertices. They describe classes of shapes, for example, how a pyramid has triangular faces meeting at a point.

They visualize how 3-D shapes can be represented in two dimensions, for example, by silhouettes projected by 3-D objects and structures.



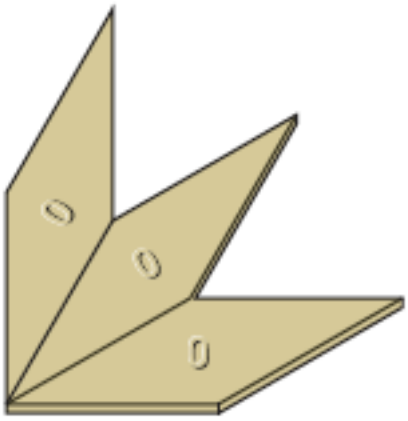
In Grade 4, students continue to build on measurement work from earlier grades, which includes linear measurement, area, angle measurement, and volume. They use both U.S. standard units (inches, feet and yards) and metric units (centimeters and meters) to measure lengths up to 100 feet, and they determine the perimeter of various shapes.

They measure the area of both regular and nonregular polygons in square units by using the understanding that area can be *decomposed*—that is, broken into smaller parts.



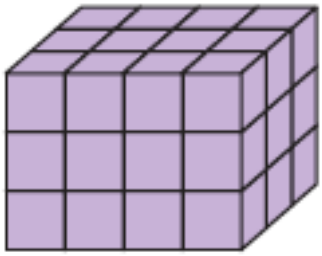
Measuring area on the Geoboard

Students work on determining the size of angles relative to a right angle, or 90 degrees. For instance, if three equal angles form a right angle, then each of the smaller angles must be 1/3 of 90 degrees or 30 degrees.



A right angle formed with Power Polygons

Finally, students work on understanding volume by structuring and determining the volume of one kind of geometric solid, a rectangular prism, in cubic units. They develop strategies for determining the number of cubes in 3-D arrays of cubes by mentally organizing the cubes--for example as a stack of three rectangular layers, each composed of three rows of four cubes.



Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Describing properties of 3-D shapes
- Translating between 2-D and 3-D shapes

Linear Measurement

- Measuring with standard units

Area Measurement

- Understanding and finding area

Volume

- Structuring rectangular prisms and determining their volume

Benchmarks

- Use appropriate measurement tools to measure distance
- Identify quadrilaterals as any four-sided closed shape
- Know that a right angle measures 90 degrees, and use this as a landmark to find angles of 30, 45, and 60 degrees
- Find the area of polygons using a square unit of measure
- Identify 2-dimensional silhouettes of 3-dimensional solids (e.g. a cone can project a triangular silhouette)
- Draw 2-D representations showing different perspectives of a 3-D object
- Find the volume of cube buildings and rectangular prisms

Grade 5[1]

Number and Operations: Whole Numbers

Multiplication and Division

In Grade 5, students consolidate their understanding of the computational strategies they use for multiplication. All students should be able to carry out strategies that involve breaking one or both factors apart, multiplying each part of one factor by each part of the other factor, then combining the partial products. They also practice notating their solutions clearly. They use representations and story contexts to connect these strategies, which are based on the distributive property of multiplication, to the meaning of multiplication. As part of their study of multiplication, students analyze and compare multiplication algorithms, including the U.S. algorithm for multiplication.

Examples of Multiplication Strategies

Breaking numbers apart by addition

148 x 42 =
40 x 100 = 4,000
40 x 40 = 1,600
40 x 8 = 320
2 x 100 = 200
2 x 40 = 80
2 x 8 = 16
4,000 + 1,600 + 320 + 200 + 80 + 16 = 6,216

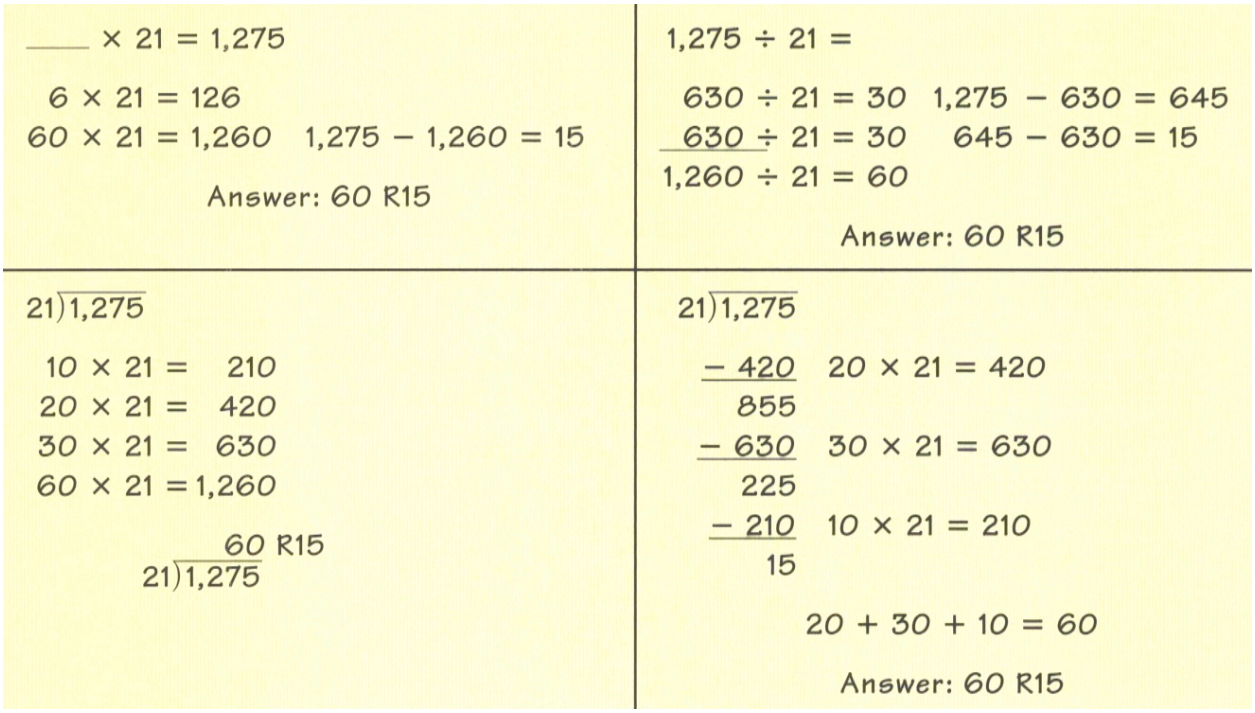
148 x 42 =
100 x 42 = 4,200
48 x 40 = 1,920
48 x 2 = 96
4,200 + 1,920 + 96 = 6,216

Changing one number to create an easier problem

148 x 42 =
150 x 42 = 6,300 (100 x 42 + 1/2 of 100 x 42)
2 x 42 = 84
6,300 – 84 = 6,216

[1] This document applies to the 2nd edition of *Investigations* (2008, 2012). See http://investigations.terc.edu/CCSS/ for changes when implementing *Investigations and the Common Core Standards*.

Students continue to learn ways to solve division problems fluently, focusing on the relationship between multiplication and division. They solve division problems by relating them to missing factor problems (e.g., 462 ÷ 21 = ____ and ____ x 21 = 462), by building up groups of the divisor, and by using multiples of 10 to solve problems more efficiently. As students refine their computation strategies for division, they find ways to use what they already know and understand well (familiar factor pairs, multiples of 10s, relationships between numbers, etc.) to break apart the harder problems into easier problems. They also work on notating their solutions clearly and concisely.

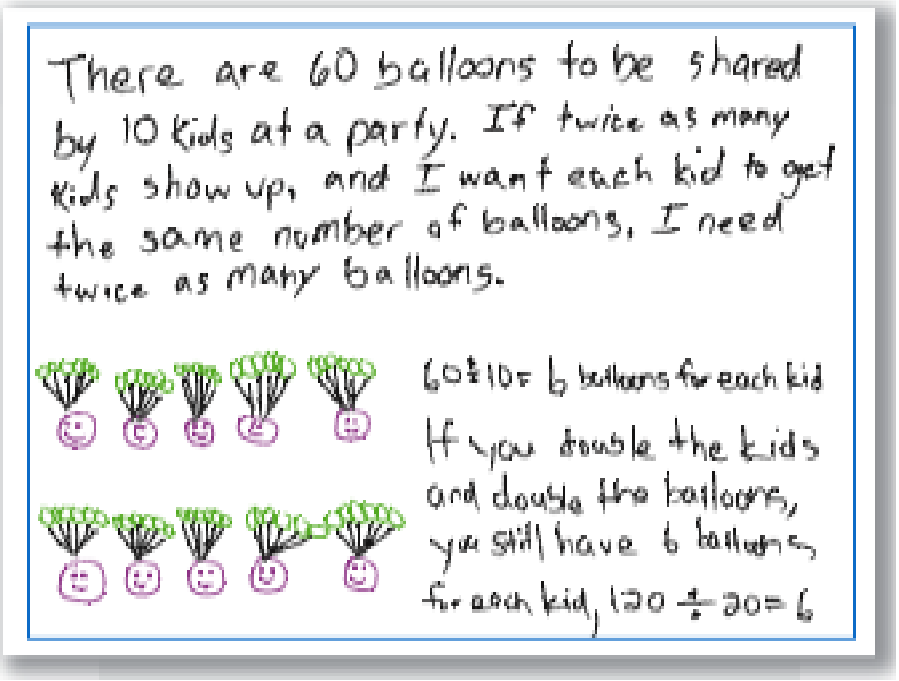


Examples of clear and concise notation

Students also study underlying properties of numbers and operations and make and justify general claims based on these properties. They study the relationship between a number and its factors, which supports mental computation strategies for multiplication and division with whole numbers. For example, students consider multiplication expressions related by place value (e.g., 3 x 6 = 18; 3 x 60 = 3 x 6 x 10 = 180), and equivalent multiplication expressions (e.g., 24 x 18 = 12 x 36 or 24 x 18 = 72 x 6). This work includes finding longer and longer multiplication expressions for a number and considering the prime factorization of a number.

Students also investigate equivalent expressions in multiplication and division. For example, they investigate why doubling one factor and halving the other factor (or tripling and thirding, etc.) in a multiplication expression of the form *a x b* maintains the same product. They also examine how and why the ratio between dividend and divisor

must be maintained to generate equivalent division expressions. In this work, students develop mathematical arguments based on representations of the operations.



Sample student work

The Algebra Connections pages in the two curriculum units that focus on multiplication and division show how students are applying the commutative and distributive properties of multiplication, as well as the inverse relationship between multiplication and division, as they solve problems. These pages also highlight particular generalizations about multiplication that students work on in Grade 5 as they create equivalent expressions for multiplication: If one factor in a multiplication expression is halved (or thirded) and another factor is doubled (or tripled), what is the effect on the product?

Emphases

Whole Number Operations

- Reasoning about numbers and their factors
- Understanding and using the relationship between multiplication and division to solve division problems
- Representing the meaning of multiplication and division
- Reasoning about equivalent expressions in multiplication and division

Computational Fluency

- Solving multiplication problems with 2-digit numbers
- Solving multiplication problems with 2- and 3-digit numbers
- Solving division problems with 2-digit divisors

Benchmarks

- Find the factors of a number
- Solve multiplication problems efficiently
- Solve division problems with 1-digit and 2-digit divisors
- Explain why doubling one factor in a multiplication expression (a x b) and dividing the other by 2 results in an equivalent expression
- Solve division problems efficiently

Addition, Subtraction, and the Number System

In Grade 5, students extend their knowledge of the base ten number system, working with numbers in the hundred thousands and beyond. In their place value work, students focus on adding and subtracting multiples of 100 and 1,000 to multi-digit numbers and explaining the results. This work helps them develop reasonable estimates for sums and differences when solving problems with large numbers. Students apply their understanding of addition to multi-step problems with large numbers. They develop increased fluency as they study a range of strategies and generalize the strategies they understand to solve problems with very large numbers.

90, 945 – 1,000 =

90,945 – 1,200 =

90,945 – 1,210 =

90,945 – 1,310 =

Students practice and refine their strategies for solving subtraction problems. They also classify and analyze the logic of different strategies; they learn more about the operation of subtraction by thinking about how these strategies work. Students consider which subtraction problems can be solved easily by changing one of the numbers and then adjusting the difference. As they discuss and analyze this approach, they visualize important properties of subtraction. By revisiting the steps and notation of the U.S. algorithm for subtraction and comparing it to other algorithms, students think through how regrouping enables subtracting by place, with results that are all in positive numbers.

Examples of Subtraction Strategies

Subtracting in parts

3,451 – 1,287 =

3,451 – 1,200 = 3,251

2,251 – 80 = 2,171

2,171 – 7 = 2,164

Adding up

3,451 – 1,287 =

1,287 + 13 = 1300

1,300 + 2,100 = 3,400

3,400 + 51 = 3,451

13 + 2,100 + 51 = 2,164

Subtracting back

3,451 – 1,287 =

3,451 – 51 = 3,400

3,400 – 2,100 = 1,300

1,300 – 13 = 1,287

51 + 2,100 + 13 = 2,164

Changing the numbers

3,451 – 1,287 =

3,451 – 1,300 = 2,151

2,151 + 13 = 2,164

3,451 – 1,287 =

(add 13 to both number to create an equivalent problem)

3,451 – 1,287 = 3,464 – 1300

= 2,164

The Algebra Connections page in the curriculum unit that focuses on addition and subtraction shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic ideas that underlie the generalizations students investigate and articulate when they create equivalent expressions in order to solve a problem (e.g., 892 – 567 = 895 – 570).

Emphases

The Base Ten Number System

- Extending knowledge of the base-ten number system to 100,000 and beyond

Computational Fluency

- Adding and subtracting accurately and efficiently

Whole Number Operations

- Examining and using strategies for subtracting whole numbers

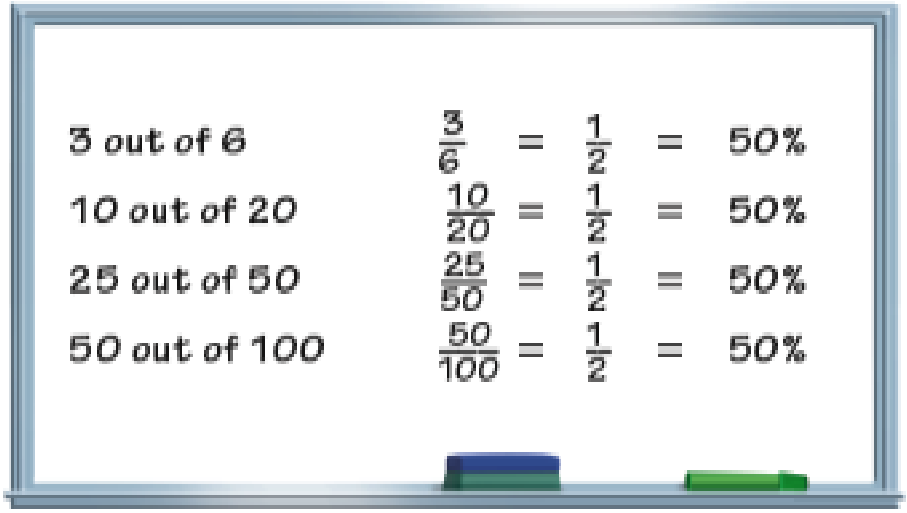
Benchmarks

- Read, write, and sequence numbers to 100,000
- Solve subtraction problems accurately and efficiently, choosing from a variety of strategies

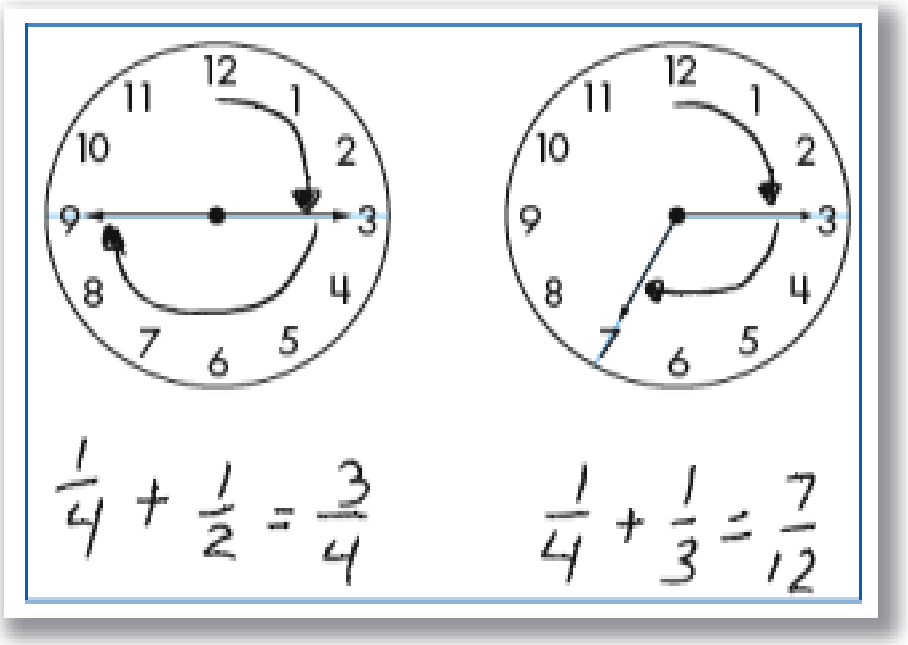
Number and Operations: Rational Numbers

The major focus of the work on rational numbers in grade 5 is on understanding relationships among fractions, decimals, and percents. Students make comparisons and identify equivalent fractions, decimals, and percents, and they develop strategies for adding and subtracting fractions and decimals.

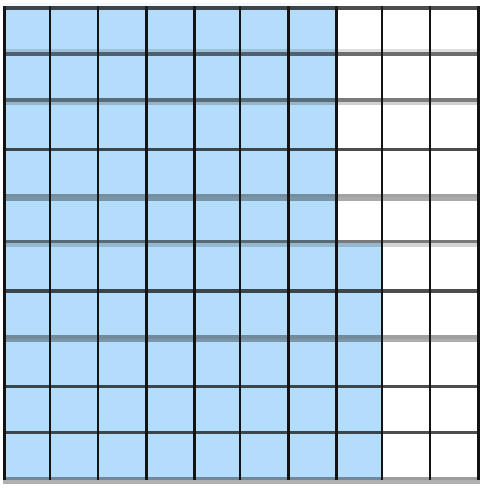
In a study of fractions and percents, students work with halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. They develop strategies for finding percent equivalents for these fractions so that they are able to move back and forth easily between fractions and percents and choose what is most helpful in solving a particular problem, such as finding percentages or fractions of a group.

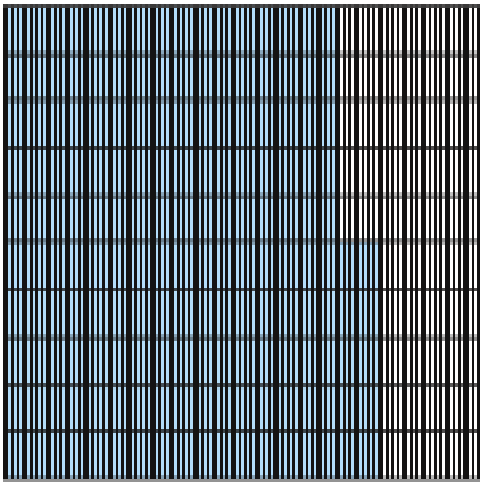


Students use their knowledge of fraction equivalents, fraction-percent equivalents, the relationship of fractions to landmarks such as ½, 1, and 2, and other relationships to decide which of two fractions is greater. They carry out addition and subtraction of fractional amounts in ways that make sense to them by using representations such as rectangles, rotation on a clock, and the number line to visualize and reason about fraction equivalents and relationships.

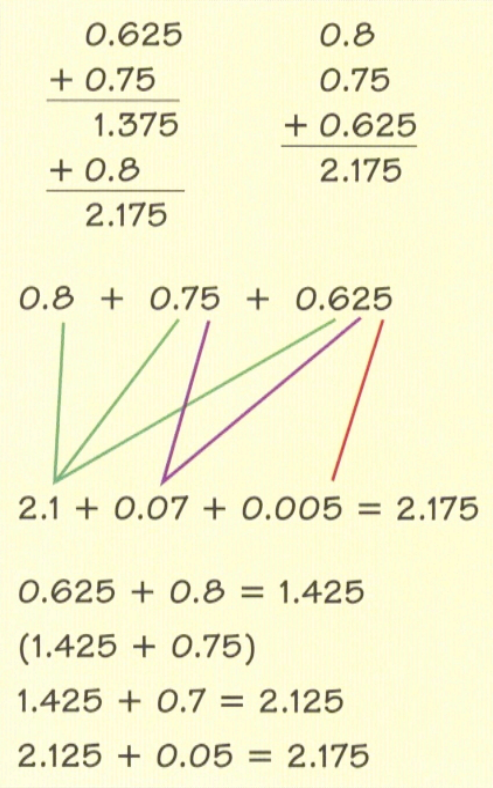


Students continue to develop their understanding of how decimal fractions represent quantities less than 1 and extend their work with decimals to thousandths. By representing tenths, hundredths, and thousandths on rectangular grids, students learn about the relationships among these numbers—for example, that one tenth is equivalent to ten hundredths and one hundredth is equivalent to ten thousandths—and how these numbers extend the place value structure of tens that they understand from their work with whole numbers.





Students extend their knowledge of fraction-decimal equivalents by studying how fractions represent division and carrying out that division to find an equivalent decimal. They compare, order, and add decimal fractions (tenths, hundredths, and thousandths) by carefully identifying the place value of the digits in each number and using representations to visualize the quantities represented by these numbers.



Emphases

Rational Numbers

- Understanding the meaning of fractions and percents
- Comparing fractions
- Understanding the meaning of decimal fractions
- Comparing decimal fractions

Computation with Rational Numbers

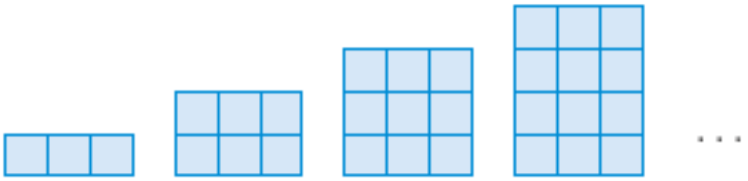
- Adding and subtracting fractions
- Adding decimals

Benchmarks

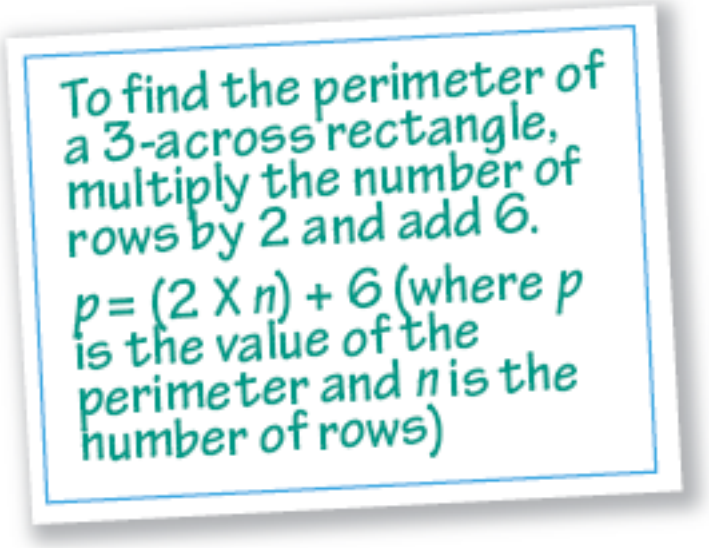
- Use fraction-percent equivalents to solve problems about the percentage of a quantity
- Order fractions with like and unlike denominators
- Add fractions through reasoning about fraction equivalents and relationships
- Read, write, and interpret decimal fractions to thousandths
- Order decimals to the thousandths
- Add decimal fractions through reasoning about place value, equivalents, and representations

Patterns, Functions, and Change

In Grade 5, students continue their work from Grades 3 and 4 by examining, representing, and describing situations in which the rate of change is constant. Students create tables and graphs to represent the relationship between two variables in a variety of contexts. They also articulate general rules for each situation. For example, consider the perimeters of the following set of rectangles made from rows of tiles with three tiles in each row:



If the value of one variable (the number of rows of three tiles) is known, the corresponding value of the other variable (the perimeter of the rectangle) can be calculated. Students express these rules in words and then in symbolic notation. For example:



For the first time in Grade 5, students create graphs for situations in which the rate of change is itself changing–for example, the change in the area of a square as a side increases by a constant increment–and consider why the shape of the graph is not a straight line as it is for situations with a constant rate of change.

Throughout their work, students move among tables, graphs, and equations and between those representations and the situation they represent. Their work with symbolic notation is closely related to the context in which they are working. By moving back and forth between the contexts, their own ways of describing general rules in words, and symbolic notation, students learn how this notation can carry mathematical meaning.

Emphases

Using Tables and Graphs

- Using graphs to represent change
- Using tables to represent change

Linear Change

- Describing and representing a constant rate of change

Nonlinear Change

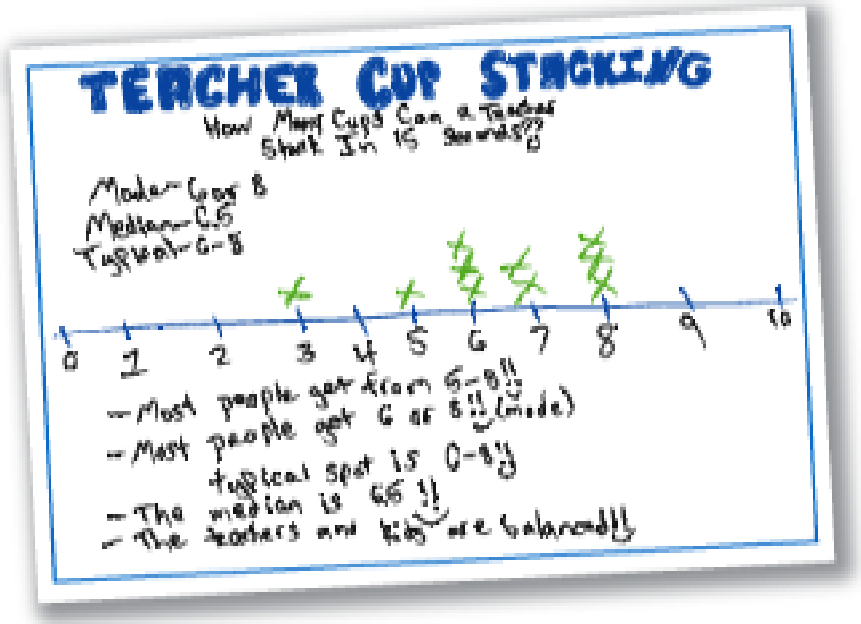
- Describing and representing situations in which the rate of change is not constant

Benchmarks

- Connect tables and graphs to represent the relationship between two variables
- Use tables and graphs to compare two situations with constant rates of change
- Use symbolic notation to represent the value of one variable in terms of another variable in situations with constant rates of change

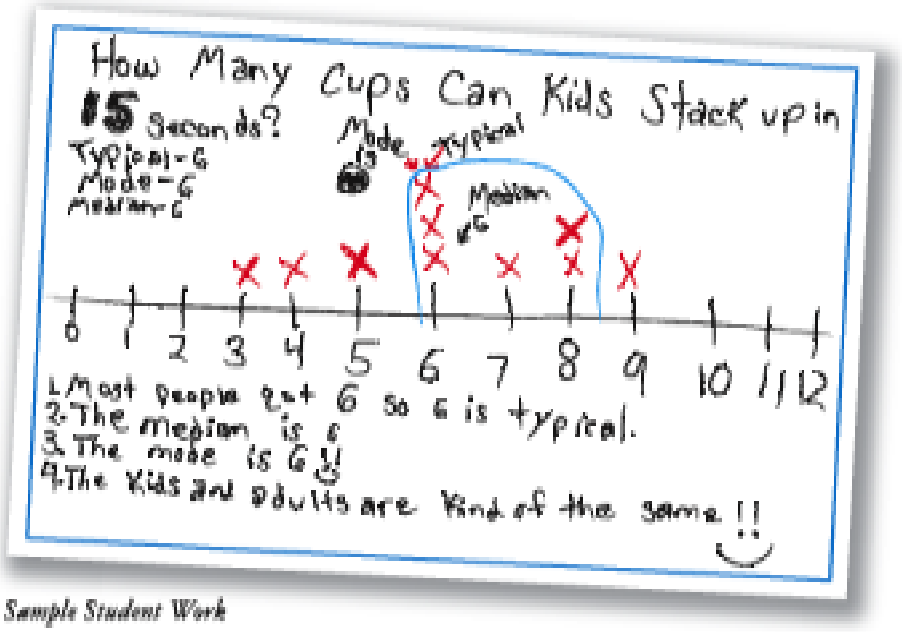
Data Analysis and Probability

Students continue to develop their understanding of data analysis in Grade 5 by collecting, representing, describing, and interpreting numerical data. Students' work in this unit focuses on comparing two sets of data collected from experiments. Students develop a question to compare two groups, objects, or conditions. (Sample questions: Which toy car goes farthest after rolling down the ramp? Which paper bridge holds more weight?). They consider how to ensure a consistent procedure for their experiment and discuss the importance of multiple trials. Using representations of data, including line plots and bar graphs, students describe the shape of the data—where the data are concentrated, how they are spread across the range. They summarize the data for each group or object or condition and use these summaries, including medians, to back up their conclusions and arguments. By carrying out a complete data investigation, from formulating a question through drawing conclusions from their data, students gain an understanding of data analysis as a tool for learning about the world.



Sample Student Work

In their work with probability, students describe and predict the likelihood of events and compare theoretical probabilities with actual outcomes of many trials. They use fractions to express the probabilities of the possible outcomes (e.g., landing on the green part of the spinner, landing on the white part of the spinner). Then they conduct experiments to see what actually occurs. The experiments lead to questions about theoretical and experimental probability, for example, if half the area of a spinner is colored green and half is colored white, why doesn't the spinner land on green exactly half the time?



Sample Student Work

Emphases

Data Analysis

- Representing data
- Describing, summarizing, and comparing data
- Analyzing and interpreting data
- Designing and carrying out a data investigation

Probability

- Describing the probability of an event
- Describe major features of a set of data represented in a line plot or bar graph, and quantify the description by using medians or fractional parts of the data

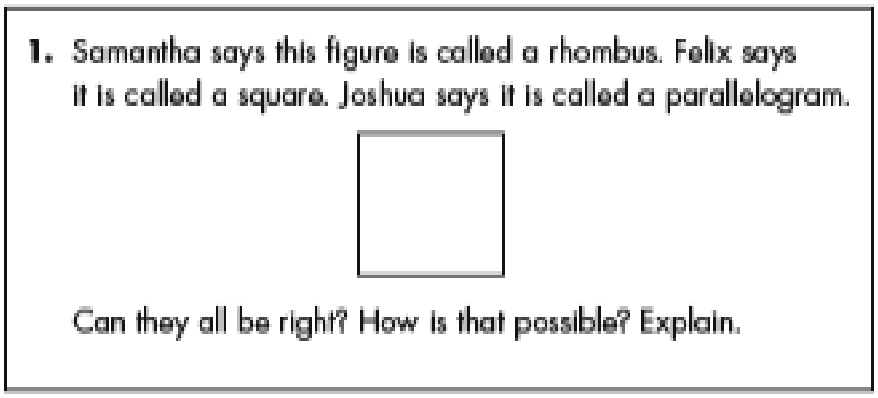
Benchmarks

- Draw conclusions about how 2 groups compare based on summarizing the data for each group
- Design and carry out an experiment in order to compare two groups
- Use a decimal, fraction, or percent to describe and compare the theoretical probabilities of events with a certain number of equally likely outcomes

Geometry and Measurement

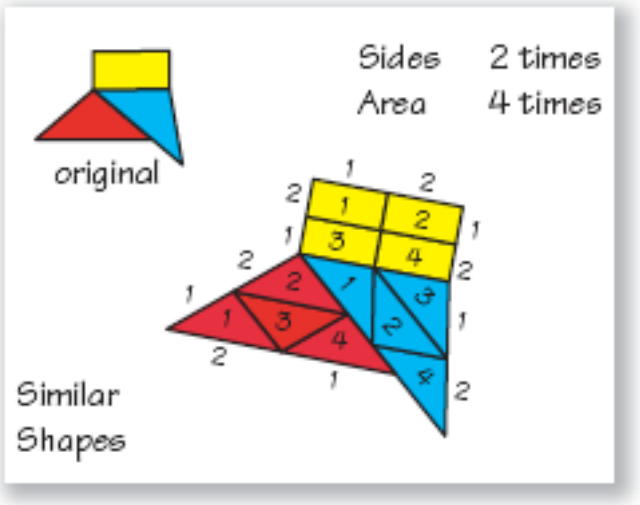
In their work with geometry and measurement in grade 5, students further develop their understanding of the attributes of two-dimensional (2-D) shapes, find the measure of angles of polygons, determine the volume of three-dimensional (3-D) shapes, and work with area and perimeter. Students examine the characteristics of polygons, including a variety of triangles, quadrilaterals, and regular polygons. They consider questions about the classification of geometric figures, for example:

> Are all squares rectangles?
>
> Are all rectangles parallelograms?
>
> If all squares are rhombuses, then are all rhombuses squares?



Question 1 from Resource Masters, M17 in Unit 5, Measuring Polygons

They investigate angle sizes in a set of polygons and measure angles of 30, 45, 60, 90, 120, and 150 degrees by comparing the angles of these shapes. Students also investigate perimeter and area. They consider how changes to the shape of a rectangle can affect one of the measures and not the other (e.g., two shapes that have the same area don't necessarily have the same perimeter), and examine the relationship between area and perimeter in similar figures.



Students continue to develop their visualization skills and their understanding of the relationship between 2-D pictures and the 3-D objects they represent. Students determine the volume of boxes (rectangular prisms) made from 2-D patterns and create patterns for boxes to hold a certain number of cubes. They develop strategies for determining the number of cubes in 3-D arrays by mentally organizing the cubes—for example as a stack of three rectangular layers, each three by four cubes. Students deepen their understanding of the relationship between volume and the linear dimensions of length, width, and height. Once students have developed viable strategies for finding the volume of rectangular prisms, they extend their understanding of volume to other solids such as pyramids, cylinders, and cones, measured in cubic units.

Emphases

Features of Shape

- Describing and classifying 2-D figures
- Describing and measuring angles
- Creating and describing similar shapes
- Translating between 2-D and 3-D shapes

Linear and Area Measurement

- Finding the perimeter and area of rectangles

Volume

- Structuring rectangular prisms and determining their volume
- Structuring prisms, pyramids, cylinders, and cones and determining their volume

Benchmarks

- Identify different quadrilaterals by attribute, and know that some quadrilaterals can be classified in more than one way
- Use known angle sizes to determine the sizes of other angles (30 degrees, 45 degrees, 60 degrees, 90 degrees, 120 degrees, and 150 degrees)
- Determine the perimeter and area of rectangles
- Identify mathematically similar polygons
- Find the volume of rectangular prisms
- Use standard units to measure volume
- Identify how the dimensions of a box change when the volume is changed
- Explain the relationship between the volumes of prisms and pyramids with the same base and height