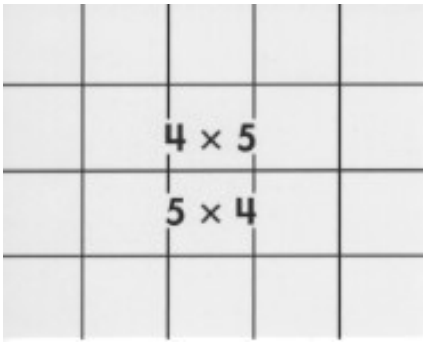
Investigations in Number, Data, and Space®

Array Cards in *Investigations*[1]

Array Cards are rectangular arrangements of squares. They help students develop a visual image of multiplication as equal groups (e.g. 6 rows of 8, or 8 rows of 6), and as area (e.g. a rectangle with dimensions 6 and 8). Used in Grades 3 and 4, Array Cards provide students with a way to practice and learn their multiplication combinations. They also help children develop an understanding of the operation of multiplication, its properties, and the relationship between multiplication and division.

What Are Array Cards?

An Array Card for 4 times 5 is a rectangular arrangement of 4 rows of 5 (or 5 rows of 4) squares. On one side of the card both dimensions of the rectangle are listed ("4 x 5" and "5 x 4"), and each individual square can be seen. On the other side, the total number of squares (20) and one dimension of the rectangle (5) are shown.



When students are looking at the side of a card showing both dimensions, they might count individual squares (i.e. 1, 2, 3…), count by one dimension (i.e. 5, 10, 15... or 4, 8, 12...), or use something they know (e.g. 2 rows of 5 is 10, double that is 20) to figure out the total number of squares. When they look at the side of the card showing the product and one dimension, some students think about multiplication (i.e. 5 x ? = 20) to find the missing factor, while others conceptualize the problem as either 20÷5=? or 20÷?=5.

How Are Array Cards Used?

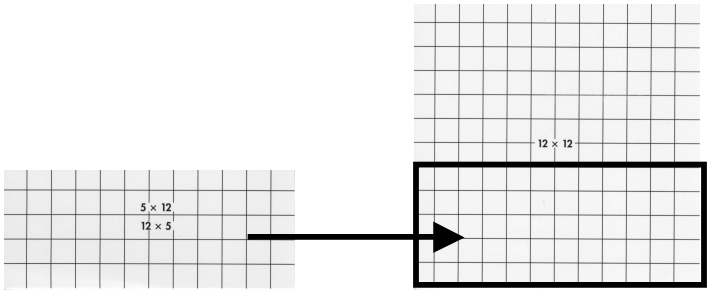
In *Investigations*, students use Array Cards in Grades 3 and 4 to learn the multiplication and division facts and to gain an understanding of the properties and relationships of multiplication and division. For example, they play games such as the following:

Factor Pairs. Players lay out all of the Array Cards, dimensions side up. They take turns pointing to a card and saying the total number of squares. If correct, the player takes the card. Play continues until all of the Array Cards have been picked up.

Count and Compare. Players deal out all of the Array Cards, dimensions side up. Players compare their top cards to see which array has more squares, without counting by 1's. The player with the larger array takes the cards.

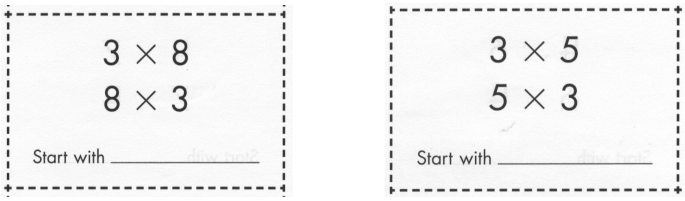
Missing Factors. Players lay out all of the Array Cards, dimensions side down. They take turns pointing to a card and identifying the missing factor (e.g. if the product is 16, and one dimension is 2, the missing factor is 8). If correct, they keep the card. If incorrect, they return the card to the table, dimension side up. In order to take the cards that are dimension side up, players must state the product. Play continues until all of the Array Cards have been picked up.

Small Array/Big Array. Two players each receive 10 Array Cards, dimensions side up. Six additional Array Cards are spread out between the two players, also dimensions side up. Players take turns placing one of their cards on top of a center card that matches it. "Matching" means that both arrays have one dimension, or one whole side, that is the same (e.g. a 5 x 12 "matches" a 12 x 12 because both have one dimension that is 12). The goal is to completely cover a larger array with 2 or 3 smaller arrays and thereby take all of those Array Cards. Array Cards that are played or removed are replaced with cards drawn from the pile. Play continues until there are no more cards or no more matches can be made.



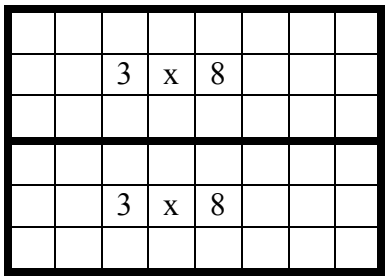
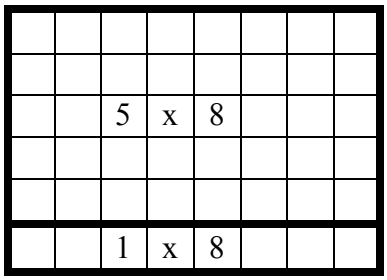
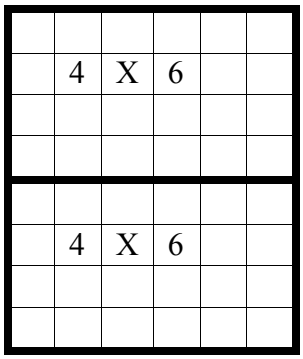
On his first turn, Matthew places a 5x12 Array Card on top of the 12x12 center card. They "match" because both have one dimension that is 12. In order to take the card, the remaining 7x12 needs to be covered.

Multiplication Combinations. Students also use Array Cards as they learn and practice the multiplication combinations. In addition to the work described in the games above, explicit practice with these combinations is built into the curriculum. Students use Multiplication Cards (see below) to figure out which combinations they know; to record ways to remember the facts they find difficult; and to practice the combinations they are still working on. Array Cards support students in finding and recording useful hints for their Multiplication Cards and in practicing the facts they don't know yet.



Why **Array Cards?**

Array Cards provide students with a visual image of multiplication. When presented with 6 x 8, students can connect the numbers in the problem to a visual image (e.g. 6 rows of 8, or 8 rows of 6). They can use this image to help construct a solution to an unknown or forgotten combination. For example, consider three of the hints one class generated for remembering that 6 x 8 and 8 x 6 equal 48, and how they can be "seen" in an array.

"I think (3 x 8) + (3 x 8). That's 24 + 24, and that's 48."	"I know 5x8=40. I need one more group of 8, so it's 48."	"8 x 6 is the same as (4 x 6) + (4 x 6). 24 + 24 = 48."
		
Example 1	Example 2	Example 3

One of the most powerful aspects of Array Cards is that they allow students to work on other, important, areas of mathematics while they are practicing their facts. These important math ideas include:

> **Commutativity.** Array Cards give students a visual, practical way to explore and make sense of the commutative property -- the idea that 6 x 8 = 8 x 6. Students often say things like, "All you have to do is turn the card -- if I turn this 6 by 8, now it's an 8 by 6. It still has 48 squares."
>
> **Distributivity.** As shown in Examples 1-3, Array Cards give students a visual, practical way to explore and make sense of the distributive property -- the idea that 6 x 8 = (3 x 8) + (3 x 8).
>
> **Mathematical Words and Ideas.** Array Cards enable students to explore, see, and understand concepts such as square numbers—"Hey, the card for 6 x 6 is a square! That's why 36 is called a square number!" Other math words and ideas that Array Cards can illuminate in a meaningful way include: prime numbers (numbers that have only 1 rectangle), composite numbers (numbers that have many rectangles), factor, factor pair, multiple, and dimensions.

Array Cards serve an important role in helping students develop fluency with the multiplication combinations and a robust understanding of the operation of multiplication and how it relates to division. They provide a concrete representation for multiplication and division, one they can manipulate with their hands. They also lay the groundwork for students' understanding and use of arrays in general, for problems beyond what can be shown with the Array Cards.

> "Students extend their understanding of multiplication through their work with arrays. For multiplication, the rectangular array is an important tool. It meets all the criteria for a powerful mathematical representation: it highlights important relationships, provides a tool for solving problems, and can be extended as students apply ideas about multiplication in new areas." (Grade 3 Unit 5, p. 157.)

[1] In the 2nd edition, students are expected to be fluent with the combinations with products to 50 by the end of 3rd grade, and the rest, through 12 x 12, by the end of 4th grade. *Investigations and the Common Core* adds 6 combinations to the 3rd grade benchmark to meet standard 3.OA.7: By the end of Grade 3, know from memory all products of two one-digit numbers.