SESSION 2.2

Locating and Comparing Fractions on Number Lines

MATH FOCUS POINTS

- Representing fractions on a number line
- Comparing fractions by reasoning about their size
- Understanding and explaining why fractions are equivalent

TODAY'S PLAN

MATERIALS

10 Class

TEN-MINUTE MATH: REVIEW AND PRACTICE

Today's Number

- Teacher Presentation

15 Class, Individuals

1 ACTIVITY

Introducing Comparing Fractions on a Number Line

- T54*

30 Class, Individuals, Pairs

2 ACTIVITY

Comparing Fractions on a Number Line

- *Student Activity Book*, pp. 369–370

15 Class

3 DISCUSSION

Equivalent Fractions

- *Student Activity Book*, pp. 369–370 (completed, from Activity 2)
- S64 (from Session 1.7; optional)
- Students' Fraction Sets (from Session 1.2; optional)

SESSION FOLLOW-UP: REVIEW AND PRACTICE

Daily Practice

- *Student Activity Book*, p. 371

* See *Materials to Prepare* in the Investigation 2 Planner.

Common Core State Standards		
	Ten-Minute Math: 3.NBT.A.2. 3.OA.C.7 **Session**: 3.NF.A.2a, 3.NF.A.2b, 3.NF.A.3c, 3.NF.A.3d	**Daily Practice**: 3.NF.A.2b, 3.NF.A.3d

TEN-MINUTE MATH: REVIEW AND PRACTICE

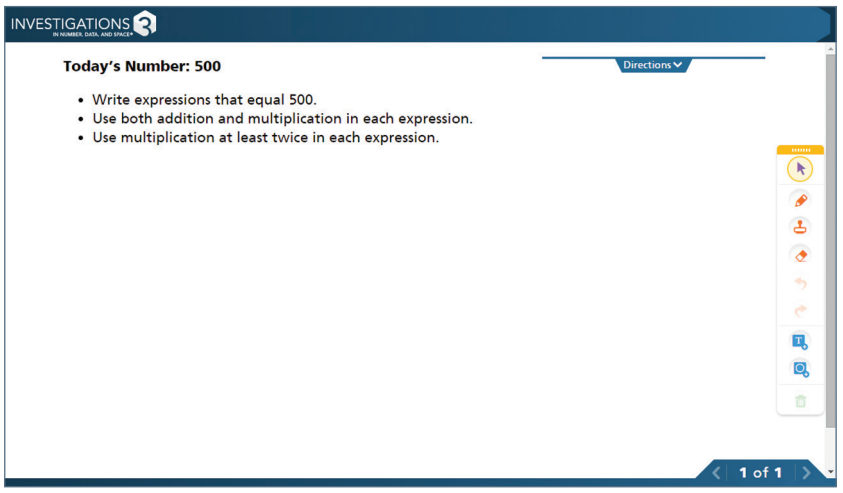
Today's Number

10

MATH FOCUS POINTS

- Generating expressions equivalent to a given number using particular constraints
- Practicing computation skills

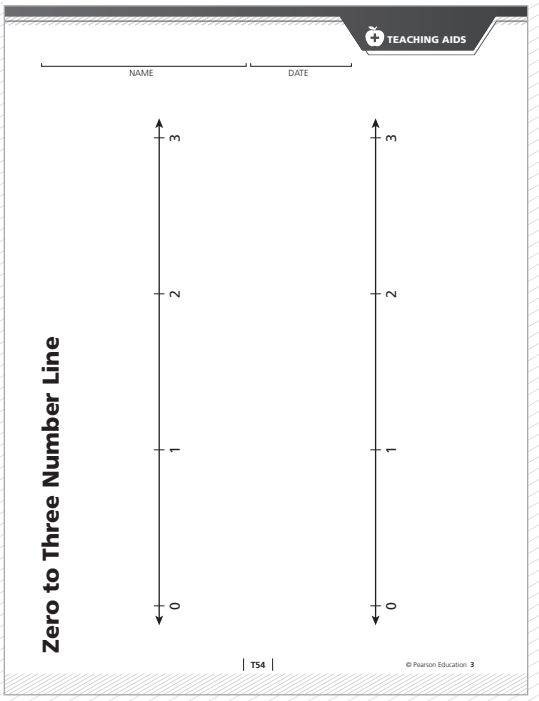
Display the Teacher Presentation or write the number 500.



Students create expressions using *addition and multiplication* that equal 500. They should use at least two multiplication expressions. Show an example and remind students how to use parentheses to show multiples of coin values in their expressions: $300 + (4 \times 25) + (10 \times 10) = 500$. Collect a few expressions to display and ask:

- **How do you know your expression equals 500?**
- **How did you decide what multiplication and addition expressions to use?**

RESOURCE MASTERS, T54



1 ACTIVITY

Introducing Comparing Fractions On a Number Line

15

Display Zero to Three Number Line (T54).

We're going to use number lines to figure out whether a fraction is greater, smaller, or the same as another fraction. Let's just review putting fractions on a number line. A few sessions back we used a part of a number line that showed 0, 1, and 2. Here is a number line that is marked off 0, 1, 2, and 3. If the distance from 0 to 1 is one whole unit, how many whole units are there from 0 to 2? 0 to 3? MN

Write $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$ under the appropriate numbers.

This is another way to write whole numbers in the form of fractions.

MATH NOTE

MN Number Lines Marked with Different Numbers In Session 1.5, students located fractions on a number line marked with 0, 1, and 2. In this session, the number line is marked with 0, 1, 2, and 3, so that students continue to extend their knowledge of how to locate and name points on the number line between whole numbers.

If I wanted to mark $\frac{2}{4}$ on the number line, where would I mark it? MWI1 MPN1

Ask a student to mark $\frac{2}{4}$ on the number line and explain how he or she knows where to put $\frac{2}{4}$.

STUDENTS MIGHT SAY

"I know that $\frac{2}{4}$ equals $\frac{1}{2}$ so I made a line halfway between 0 and 1 and that is $\frac{2}{4}$."

"I divided the distance between 0 and 1 into fourths and I counted two fourths."

Display the fraction $\frac{5}{4}$.

What do you think this fraction means? How would you say it? Think about what $\frac{1}{4}$ or $\frac{2}{4}$ means to help you think about what $\frac{5}{4}$ means.

If needed, look at $\frac{2}{4}$, and ask students to talk about what the 4 means and what the 2 means. Then return to the questions about $\frac{5}{4}$. Remind students that they can think about an ant traveling along the number line to explain their ideas.

Where would you mark $\frac{5}{4}$ on this number line?

Have a student mark $\frac{5}{4}$ on the number line.

How do you know that is where $\frac{5}{4}$ belongs? MPN2

As students share their ideas, illustrate them on the number line.

STUDENTS MIGHT SAY

"If an ant went $\frac{4}{4}$, it would land on 1. Then it still has to go another $\frac{1}{4}$, so I thought of dividing the space between 1 and 2 into fourths, and it went 1 of those fourths."

"I marked off the fourths between 0 and 1, and it wasn't enough, so I marked the space between 1 and 2 into fourths. You can just keep counting the fourths: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$. The four fourths take you up to the 1, so then you have to go past the 1 to get $\frac{5}{4}$."

Suppose there were two ants traveling along this line. One ant, let's call it Ant A, traveled $\frac{2}{4}$ of a block and then rested. The other ant, let's call it Ant B, traveled $\frac{5}{8}$ of the block. Which ant traveled farther? MWI2

MATH WORDS AND IDEAS

MWI1 Fractions on a Number Line

MWI2 Comparing Fractions with the Same Numerator or Denominator

MATH PRACTICE NOTES

MPN1 MP3 Construct viable arguments and critique the reasoning of others. Students base their arguments about equivalent fractions on representations. Help students articulate their arguments as fully as possible. Encourage them to explain not only *that* two fractions are equivalent but *why* they are equivalent. For example, if a student shows how $\frac{2}{6}$ matches $\frac{1}{3}$ with the Fraction Set, ask: "I can see from your demonstration that $\frac{2}{6}$ matches $\frac{1}{3}$. Does it make sense to you that $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent? Who can talk more about that?"

MPN2 MP4 Model with mathematics. While we have been talking about the number line as a mathematical tool for understanding fractions as numbers, the number line is also a mathematical model of our number system. In this unit, students are expanding their notion of the kinds of numbers that system includes—that there are numbers between the whole numbers and that those familiar whole numbers can also be expressed as fractions. Each time students place a fraction on the number line, they are modeling where that number fits in our system, and its relationship to whole numbers and to other fractions.

We already marked $\frac{2}{4}$ on the number line. Where should we mark $\frac{5}{8}$?

Ask a student to mark $\frac{5}{8}$ on the number line along with $\frac{2}{4}$ and $\frac{5}{4}$.

How do you know that is where $\frac{5}{8}$ belongs?

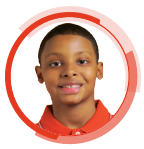
STUDENTS MIGHT SAY

"I marked off the eighths. Here is $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$ and $\frac{4}{8}$, so here is $\frac{5}{8}$."

"$\frac{1}{2}$ equals $\frac{4}{8}$, so $\frac{5}{8}$ is just one more eighth."

[Elena] says that Ant B traveled farther than Ant A. How do you know that $\frac{5}{8}$ is greater than $\frac{2}{4}$? TN

STUDENTS MIGHT SAY



"$\frac{2}{4}$ is equal to $\frac{1}{2}$, and $\frac{4}{8}$ is equal to $\frac{1}{2}$, so $\frac{5}{8}$ is just 1 more eighth."

"If you divide the number line into fourths and into eighths, $\frac{5}{8}$ is farther along the number line."

"Two eighths together make $\frac{1}{4}$. So if you have $\frac{2}{4}$, that is equal to $\frac{4}{8}$. So $\frac{5}{8}$ is 1 more eighth."

Ask students to show their thinking on the number line. If students just say $\frac{5}{8}$ is farther on the number line, ask them how they know it should be farther on the number line.

So $\frac{5}{8}$ is greater than $\frac{2}{4}$. How could we use notation to show that?

Remind students that the greater than, less than, and equal signs are symbols for showing the results of a comparison. Ask a student to use these symbols to compare the two fractions.

$$\frac{5}{8} > \frac{2}{4}$$

This statement says that $\frac{5}{8}$ is greater than $\frac{2}{4}$. Can someone use the less than symbol to compare these fractions? $\left(\frac{2}{4} < \frac{5}{8}\right)$

TEACHING NOTE

TN Strategies for Comparing Fractions

Students are likely to think about the relationship between related fractions. For example, to make sixths, you divide thirds in half. If $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, then $\frac{2}{6}$ is the same as $\frac{1}{3}$. Others may think about the distance on the number line—you have to take two jumps of $\frac{1}{6}$ to get to $\frac{1}{3}$. For other fraction comparisons, students may compare the fractions to $\frac{1}{2}$ (e.g., "$\frac{1}{3}$ is less than $\frac{1}{2}$, and $\frac{3}{6}$ is $\frac{1}{2}$.") or think about the relative size of the pieces (e.g., "Fourths are smaller than thirds because they're cut into more pieces.").

2 ACTIVITY

Comparing Fractions on a Number Line

30 min

On *Student Activity Book* pages 369–370, students compare fractions on number lines. MPN PD

ONGOING ASSESSMENT Observing Students at Work

Students reason about how fractions compare, using a number line. MWI

- **Do students correctly locate the fractions on the number line?** MP4
- **How do students explain how they know which fraction is greater?** Do they mark off fourths (or eighths, etc.) and eighths (or sixths, etc.) and find the specific fractions? Do they reason about the relative sizes of the pieces by using benchmarks such as $\frac{1}{2}$ or equivalent fractions?
- **Do students accurately notate which fraction is greater?**

DIFFERENTIATION Supporting the Range of Learners

INTERVENTION **Adapt the Task** If students are not sure how to locate each fraction on the same number line, they should focus first on locating the fractions on separate number lines, and then compare them. Encourage them to first mark off all of the fourths (eighths, sixths or thirds) and then find the specific fraction. They can also use the number lines they marked in Session 1.4 to help them.

EXTENSION **Adapt the Task** Challenge students to compare more difficult fractions (e.g., $\frac{5}{6}$ and $\frac{7}{8}$ or $\frac{2}{3}$ and $\frac{3}{4}$).

ENGLISH LANGUAGE LEARNERS **Provide Vocabulary Support** The *greater than* and *less than* terms and their relationships to the symbols [< and >] may be unfamiliar to some students. Use whole numbers to introduce the meanings of each symbol, and model how they are used. Continue modeling using fractions. Provide examples and ask students to determine which is *greater than* or *less than* using the appropriate symbols.

3 DISCUSSION

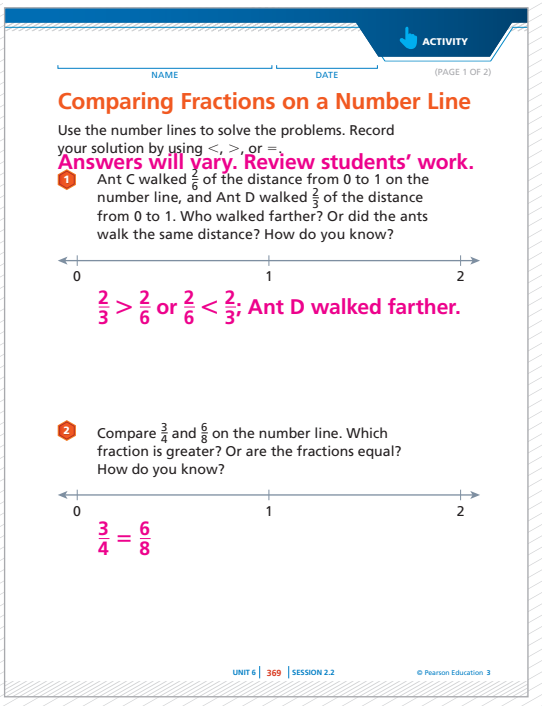
Equivalent Fractions

15 min

MATH FOCUS POINTS FOR DISCUSSION

- Comparing fractions by reasoning about their size
- Understanding and explaining why fractions are equivalent

STUDENT ACTIVITY BOOK, P. 369



ACTIVITY

NAME DATE (PAGE 1 OF 2)

Comparing Fractions on a Number Line

Use the number lines to solve the problems. Record your solution by using <, >, or =.

Answers will vary. Review students' work.

1. Ant C walked $\frac{2}{6}$ of the distance from 0 to 1 on the number line, and Ant D walked $\frac{2}{3}$ of the distance from 0 to 1. Who walked farther? Or did the ants walk the same distance? How do you know?

0 1 2

$\frac{2}{3} > \frac{2}{6}$ or $\frac{2}{6} < \frac{2}{3}$; Ant D walked farther.

2. Compare $\frac{3}{4}$ and $\frac{6}{8}$ on the number line. Which fraction is greater? Or are the fractions equal? How do you know?

0 1 2

$\frac{3}{4} = \frac{6}{8}$

UNIT 6 | 369 | SESSION 2.2 © Pearson Education 3

MATH PRACTICE NOTE

MPN MP4 **Model with mathematics.** While we have been talking about the number line as a mathematical tool for understanding fractions as numbers, the number line is also a mathematical model of our number system. In this unit, students are expanding their notion of the kinds of numbers that system includes—that there are numbers between the whole numbers and that those familiar whole numbers can also be expressed as fractions. Each time students place a fraction on the number line, they are modeling where that number fits in our system, and its relationship to whole numbers and to other fractions.

PROFESSIONAL DEVELOPMENT

PD **TEACHER NOTE 3:** Comparing Fractions

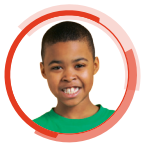
MATH WORDS AND IDEAS

MWI Comparing Fractions with the Same Numerator or Denominator

Question 2 on *Student Activity Book* page 369 asks you to compare $\frac{3}{4}$ and $\frac{6}{8}$. What did you find out about $\frac{3}{4}$ and $\frac{6}{8}$?

STUDENTS MIGHT SAY

"They are the same. There are 2 eighths in every fourth. So if there are three fourths, that's the same as 2, 4, 6 eighths."



"I divided between 0 and 1 into fourths and marked where $\frac{3}{4}$ is. Then I divided each fourth in half to make eighths. $\frac{6}{8}$ is at the same spot."

If students just say they are in the same place on the number line, ask them to explain how they know they are in the same place. They can use a number line, brownies, or Fraction Sets to show their thinking.

[Sam, Elena, and Alexi] found that $\frac{3}{4}$ and $\frac{6}{8}$ are equal. Can someone use brownies or Fraction Sets to show they are equal or not equal?

STUDENTS MIGHT SAY

"I took three $\frac{1}{4}$ pieces from my Fraction Sets and lined them up and then I lined up six $\frac{1}{8}$ pieces. They're the same."

"I cut a brownie into fourths and took 3 of the fourths which is $\frac{3}{4}$. If I cut each $\frac{1}{4}$ in half, it will be $\frac{1}{8}$ of a brownie. There are now six $\frac{1}{8}$ pieces which is $\frac{6}{8}$. So they are equal."

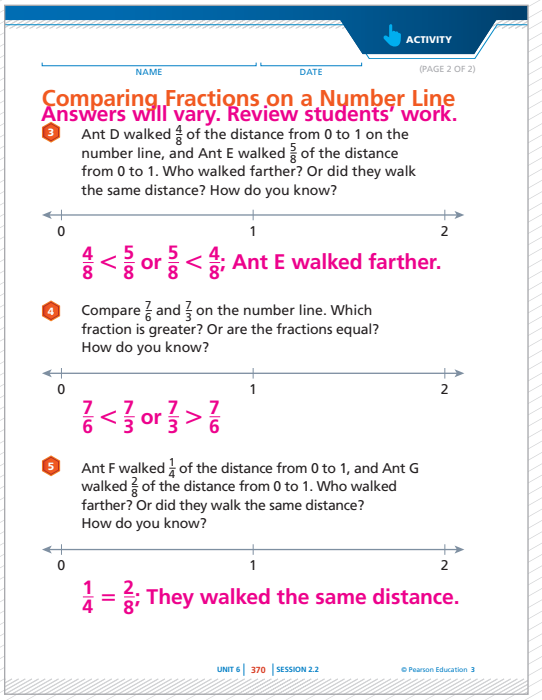
Does everyone agree $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions? Let's write them on our "Equivalent Fractions" chart.

SESSION FOLLOW-UP: REVIEW AND PRACTICE

Daily Practice

DAILY PRACTICE For reinforcement of this unit's content, have students complete *Student Activity Book* page 371.

STUDENT ACTIVITY BOOK, P. 370



ACTIVITY

NAME DATE (PAGE 2 OF 2)

Comparing Fractions on a Number Line

Answers will vary. Review students' work.

3. Ant D walked $\frac{4}{8}$ of the distance from 0 to 1 on the number line, and Ant E walked $\frac{5}{8}$ of the distance from 0 to 1. Who walked farther? Or did they walk the same distance? How do you know?

0 1 2

$\frac{4}{8} < \frac{5}{8}$ or $\frac{5}{8} < \frac{4}{8}$; Ant E walked farther.

4. Compare $\frac{7}{6}$ and $\frac{7}{3}$ on the number line. Which fraction is greater? Or are the fractions equal? How do you know?

0 1 2

$\frac{7}{6} < \frac{7}{3}$ or $\frac{7}{3} > \frac{7}{6}$

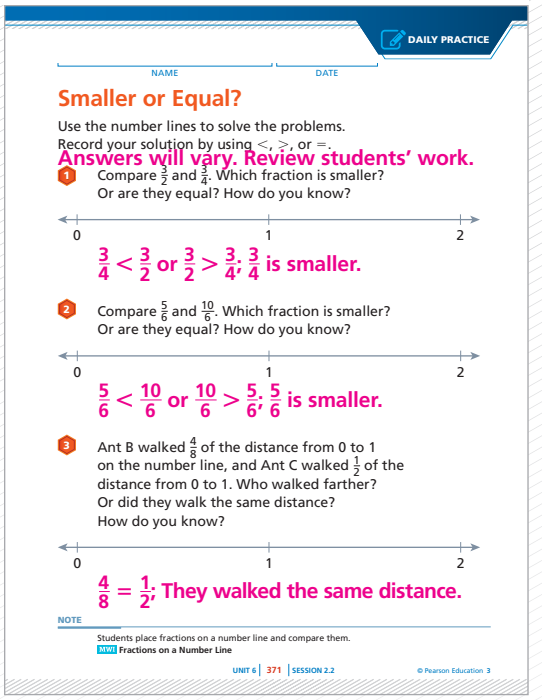
5. Ant F walked $\frac{1}{4}$ of the distance from 0 to 1, and Ant G walked $\frac{2}{8}$ of the distance from 0 to 1. Who walked farther? Or did they walk the same distance? How do you know?

0 1 2

$\frac{1}{4} = \frac{2}{8}$; They walked the same distance.

UNIT 6 | 370 | SESSION 2.2 © Pearson Education 3

STUDENT ACTIVITY BOOK, P. 371



DAILY PRACTICE

NAME DATE

Smaller or Equal?

Use the number lines to solve the problems.
Record your solution by using <, >, or =.

Answers will vary. Review students' work.

1. Compare $\frac{3}{2}$ and $\frac{3}{4}$. Which fraction is smaller? Or are they equal? How do you know?

0 1 2

$\frac{3}{4} < \frac{3}{2}$ or $\frac{3}{2} > \frac{3}{4}$; $\frac{3}{4}$ is smaller.

2. Compare $\frac{5}{6}$ and $\frac{10}{6}$. Which fraction is smaller? Or are they equal? How do you know?

0 1 2

$\frac{5}{6} < \frac{10}{6}$ or $\frac{10}{6} > \frac{5}{6}$; $\frac{5}{6}$ is smaller.

3. Ant B walked $\frac{4}{8}$ of the distance from 0 to 1 on the number line, and Ant C walked $\frac{1}{2}$ of the distance from 0 to 1. Who walked farther? Or did they walk the same distance? How do you know?

0 1 2

$\frac{4}{8} = \frac{1}{2}$; They walked the same distance.

NOTE

Students place fractions on a number line and compare them.
MMS Fractions on a Number Line

UNIT 6 | 371 | SESSION 2.2 © Pearson Education 3