

## TEACHER NOTE 3

## Comparing Fractions

Listen to students comparing fractions as they work on these problems during Investigation 2. Students develop a number of strategies that work for certain kinds of comparisons. You can help students become more explicit about the underlying regularities they are noticing as they use these strategies by asking questions such as these:

“How did you know which fraction was greater? Will that strategy always work? What types of fractions does your strategy work for? Can you give me another example of a comparison of two fractions when this strategy would work?”

Explicit discussions about comparing fractions are built into Sessions 2.4 and 2.5. But you can ask students about these ideas whenever they come up.

“Can you come up with a rule about your strategy—how it works and what kinds of fractions it works for?”

You may want to start a list of conjectures, worded by the students, based on what students are noticing. Here are some examples of strategies they develop and conjectures they might articulate, given the general ideas underlying each strategy.

### Comparing Fractions to 1

Example:  $\frac{3}{4}$  and  $\frac{4}{4}$

“I know that  $\frac{4}{4}$  is equal to 1 because it’s all of the fourths from 0 to 1.  $\frac{3}{4}$  has to be less.”

Conjecture: If one of the fractions has the same top and bottom numbers, then it’s always 1. A fraction with a smaller number on top and a larger number on the bottom has to be smaller than 1.

Example:  $\frac{3}{4}$  and  $\frac{3}{2}$

“To get to three halves you have to go past  $1\frac{1}{2}$ ,  $\frac{2}{2}$ ,  $\frac{3}{2}$ ,—and three fourths isn’t even one.”

Conjecture: If a fraction has a numerator that is less than the denominator (such as  $\frac{3}{4}$ ), it is less than one. If a fraction has a numerator that is more than the denominator (such as  $\frac{4}{3}$ ), it is greater than one, so it’s greater than the other fraction.

### Comparing Fractions to $\frac{1}{2}$

Example:  $\frac{2}{6}$  and  $\frac{2}{4}$

“ $\frac{2}{4}$  is equal to  $\frac{1}{2}$ . If the ant is going by sixths, it takes six steps to get to 1, so it takes 3 steps to get to  $\frac{1}{2}$ , so two sixths is smaller because it only has two of the  $\frac{1}{6}$  steps.”

Conjecture: If the numerator of a fraction is less than half the denominator, the fraction is less than  $\frac{1}{2}$ .

### Comparing Fractions with the Same Denominator

Example:  $\frac{5}{8}$  and  $\frac{2}{8}$

“All the moves on the number line are the same size. So if you have five, of course it’s more than two.”

Conjecture: When the denominators are the same, the fraction with the larger numerator is larger.

### Comparing Fractions with the Same Numerator

Example:  $\frac{3}{8}$  and  $\frac{3}{4}$

“The fourths are bigger than the eighths. So if you go three of the fourths, you go further than three of the eighths, so the three fourths has to be bigger.”

Conjecture: When the numerators are the same, the fraction with the smaller denominator is larger.