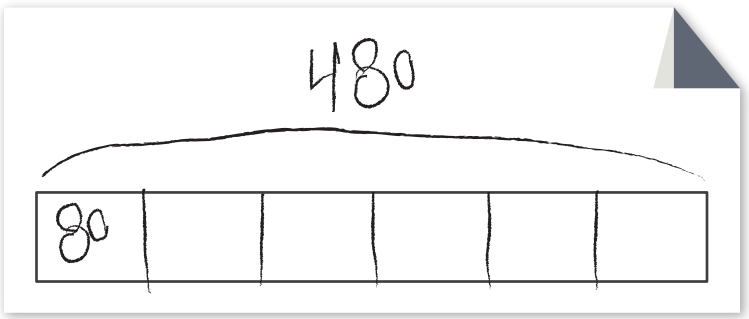
Why Can We Write $\frac{1}{6}$ of 480 as $\frac{1}{6} \times 480$?

Students have been finding fractional parts of a 480-mile bicycle race in Session 1.2. To start the discussion, the teacher displays the representation on a fraction bar that the class worked on together to find out how many miles Nora completed after the first day. (She has biked $\frac{1}{6}$ of the race.)



Teacher: Nora biked $\frac{1}{6}$ of the 480-mile race the first day. You found out that Nora biked 80 miles.

The teacher writes $\frac{1}{6}$ of 480 is 80.

Teacher: What equation could you write for that?

Rachel: $6 \times 80 = 480$.

Felix: $480 \div 6 = 80$.

Teacher: Those are both true, but what if I wanted to use $\frac{1}{6}$? The problem asks what $\frac{1}{6}$ of 480 miles is.

Deon: $\frac{1}{6} \times 480$.

Teacher: Why would that be the expression?

Lourdes: I know, but I'm not sure how to explain. If it's $480 \div 6$, you can do $\frac{1}{6} \times 480$. But I'm not sure how to explain.

Terrence: Since $\frac{1}{6}$ is below 1, you have to do the opposite of what it says.

Teacher: What do you mean?

Terrence: Well, like if you had $\frac{1}{2} \times 100$, you'd have to do $100 \div 2$.

The teacher writes $\frac{1}{2} \times 100 = 100 \div 2$.

Teacher: Why is this equation true?

Terrence: Because you're getting less of the number you started with—it's half of the number.

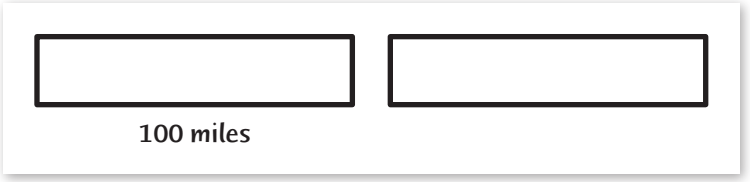
Teacher: Terrence, I'm glad you mentioned 100 because it's easy to work with.

The teacher draws a fraction bar and labels it 100 miles.

Teacher: Let's say this is a 100-mile bike race. If a boy biked the race two times, how far would that be? How would you show it?

Deon: You should draw another rectangle.

The teacher draws another fraction bar.



Teacher: What would the equation be?

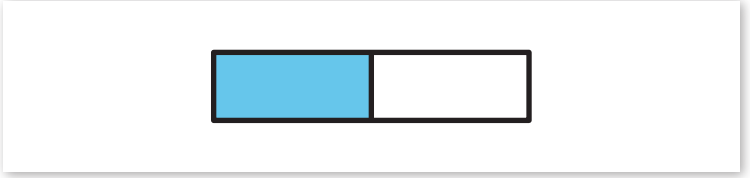
Deon: $2 \times 100 = 200$.

Teacher: Where is the 100 in this representation? Where is the 2? Where is the 200?

Hana points out where each number is in the representation.

Teacher: What if the boy only did $\frac{1}{2}$ of the race? How can we show that with fraction bars?

The teacher draws another fraction bar.



Georgia: You should shade in $\frac{1}{2}$ of the rectangle.

Olivia: You're getting part of 100. Like if it was 1×100, it would be one group, and 2×100 would be two groups, so $\frac{1}{2} \times 100$ is half a group.

Teacher: What would the equation be?

Zachary: $\frac{1}{2} \times 100 = 50$.

Talisha: It's the same as $\frac{1}{2}$ of 100. It's multiplying by a number smaller than 1.

Teacher: Let's look back at the equation we wrote for Nora's bike ride on Day 1. Does thinking about what we just did with 100 help you with explaining why we can write $\frac{1}{6} \times 480 = 80$?

Charles: Not really. This is kind of confusing.

Lourdes: I think I get it, but I can't explain it.

The teacher writes $\frac{1}{6}$ of 480 is 80 and next to that writes $\frac{1}{6} \times 480 = 80$.

Teacher: So why is this true? That we can write $\frac{1}{6}$ of 480 as $\frac{1}{6} \times 480$?

Janet: I'm not sure. Well, we divided 480 into 6 groups. So I think we can say each of these pieces is $\frac{1}{6}$ of the group of 480?

Tamira: I get it. 80 is $\frac{1}{6}$ of the whole thing.

Rachel: It's like $\frac{1}{2}$ *of* 100. This one is $\frac{1}{6}$ *of* 480. So you chop 480 into 6 parts and then you have the answer. It's kind of like multiplication *and* division.

Teacher: We are going to continue to work on problems like these and continue to think about how they are multiplication problems. Later in this unit we'll work on division with fractions as well.

As students are beginning to solve problems that involve multiplication with fractions, the teacher helps them make connections between the problems and the operation of multiplication. Students are used to thinking about multiplication as groups of equal groups. In order to understand how multiplication problems with fractions fit in with the operation of multiplication, students need to broaden their thinking about multiplication to include finding part of a group. The teacher helps students do this as she asks them to relate the problems with fractions to problems with whole numbers, and to make connections between multiplication equations, the problem context, and the representations they make to solve the problems. They continue to think and talk about multiplying with fractions in later sessions.