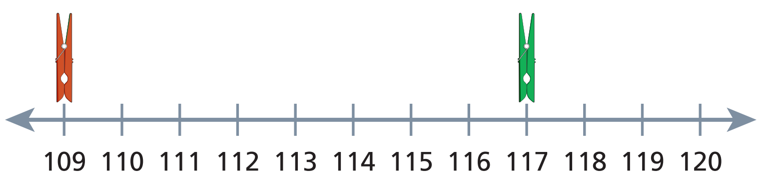
Number and Operations

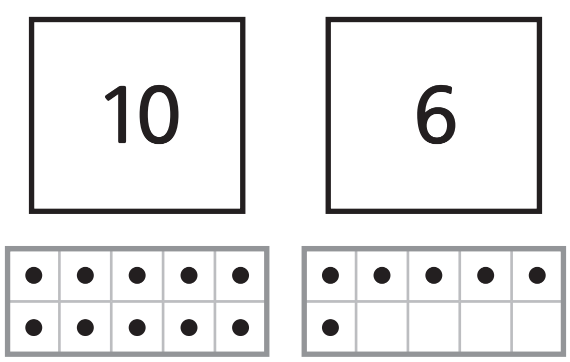
The Number System

Students extend their understanding of counting and the number sequence, building a strong foundation for their work with place value and the operations of addition and subtraction. They practice the rote counting sequence to 120 and read and write numbers within this range. Students connect number names with written numbers and the quantities that they represent as they count—and count out sets of—objects. They develop reliable and efficient strategies for accurately counting, first by 1s, and eventually by groups of 5 and 10. Being able to count flexibly forward and back from any number, by 1s and eventually by 10s, supports students' work with addition and subtraction.



[In the *Start With/Get To* routine, students practice counting forward and back.]

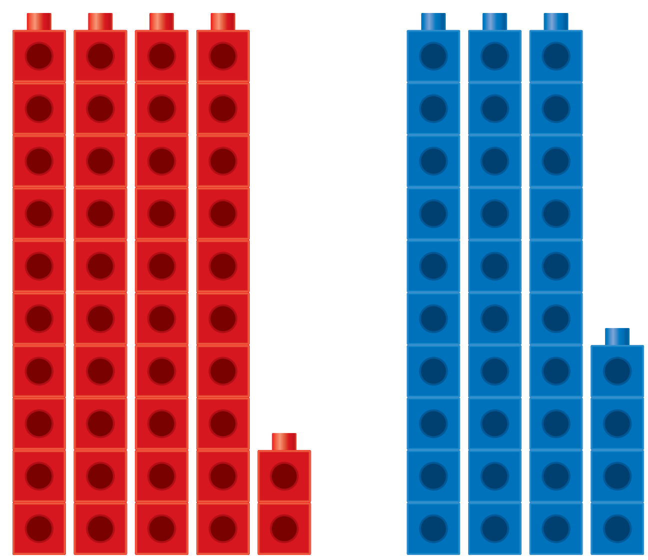
Students transition from understanding 10 as 10 ones to understanding 10 ones as 1 ten—a foundational idea in the base-10 number system. Students extend this idea to the teen numbers, coming to see every teen number as one group of ten and some number of ones. They also examine the relationship between the written number and how it is represented with Ten Frame cards, a tool that represents and reinforces the structure of tens and ones, noticing how the written numeral also represents the tens and ones structure of our place-value system.



[10 + 6 = 16]

Students use this understanding as they think about the number of fingers in a group of students, cubes in so many towers of 10, and dots on so many Ten Frame cards, as well as how many there would be if one (or more) group(s) of 10 were added (or subtracted). This work helps students come to understand the way the counting-by-10 numbers are written. For example, they might reason, "If I build the number 40 with towers of 10, I need 4 towers. The 4 in the number 40 tells me that."

With this knowledge of tens, students investigate the place value of 2-digit numbers. As they represent given numbers with cubes organized in towers of tens and ones, and as they determine the total when given groups of cubes, students come to understand that in a 2-digit number, the tens digit represents the number of tens and the ones digit represents the number of ones. They apply this knowledge as they revisit and use *greater than* and *less than* notation to compare 2-digit numbers, thinking about how the number of tens helps with the comparison.



[Some students use knowledge of the counting sequence to compare numbers. They might say, "42 is greater because it's higher on the number line. Others reason about tens and ones, thinking "42 has more groups of 10, so it's greater than 34."]

Composing and decomposing 2-digit numbers and representing 2-digit numbers as the sum of a multiple of ten and some number of ones (e.g., 22 = 20 + 2) lays the foundation for adding and subtracting larger numbers.

MAIN MATH IDEAS

- Understanding and extending the counting sequence
- Understanding place value

Benchmarks

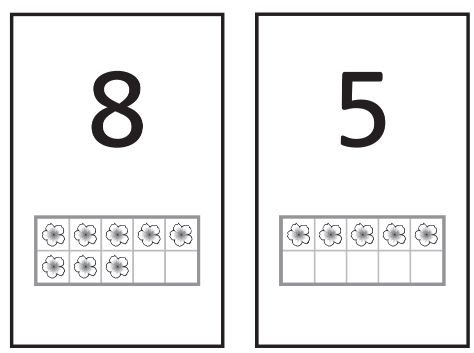
- Understand ten ones as one ten, and the teen numbers as one ten and some number of ones. **(Unit 3)**
- Rote count, read, and write numbers to 120. **(Unit 3)**
- Understand that the multiples of 10 through 90 refer to 1-9 tens and 0 ones. **(Unit 7)**
- Use a numeral to represent a number of objects organized into tens and ones and, given a numeral, represent it with tens and ones. **(Unit 7)**
- Use standard notation (<, >) to represent the comparison of two 2-digit numbers. **(Unit 7)**
- Add or subtract 10 to/from any 2-digit number. **(Unit 7)**

Addition and Subtraction

Students make an important connection between counting and quantity and the operations of addition and subtraction. They come to recognize that counting on (or back) 1 or 2 (or any amount) from a given number is the same as adding (or subtracting) 1 or 2 (or any amount) to (or from) that given number.

Such discussions begin students' work with addition and subtraction within 20. Activities, Classroom Routines, and games that ask students to combine two or more quantities, to remove one quantity from another, and to compose/decompose numbers in different ways provide repeated practice with these ideas throughout the year and support the development of fluency within 10. Discussions focus on strategies for efficiently adding and subtracting numbers, with a particular focus on using what students know about the combinations of 10 and the relationship between addition and subtraction.*

Students come to understand that 6 = 4 + 2, 2 + 2 + 2 = 6, and 2 + 4 = 3 + 3 are all valid equations because the equal sign signifies equivalence—whatever is on one side of the equation has to balance or have the same value as whatever is on the other side. They have many opportunities to see, discuss, and use a variety of equation formats as they solve and discuss story problems and record their work on games (e.g., *Dot Addition*) and in activities (e.g., *Today's Number*). Students are also explicitly asked to consider given equations, to decide whether they are true or false, and to explain why they think so. This understanding can help students think about and make sense of helpful strategies, such as *making a 10,* to solve an addition problem (e.g., 8 + 5 = 10 + ___).



["Move two from the 5, and put them with the 8 to make 10. 10 plus 3 is 13."]

* When solving subtraction problems, some students count or add up to find the difference. They are turning a subtraction problem into an unknown addend problem. While not a benchmark in a single place, teachers are encouraged to look for and track evidence of such strategies over the course of the year.

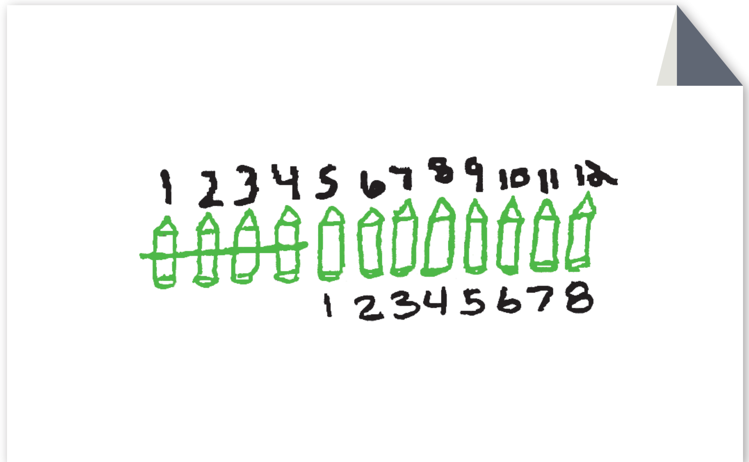
Number and Operations, *continued*

Understanding and solving a variety of story problem types, with unknowns in all positions, is a major focus. Throughout, the emphasis is on making sense of the problem and what it is asking. Students listen to and retell the story and consider whether the resulting amount will be more or less than the initial amount. They solve the problem, using available tools as needed (e.g., cubes, Ten Frames, and number lines), and show their work so that someone else could tell how they solved it. Most use a combination of numbers, symbols, pictures, and words. Finally, they share and compare solution strategies. Notation is introduced and modeled, with a focus on understanding how the quantities and actions described in a story problem can be represented with an equation. Note that some of this work is located in units focused on Measurement or Data, which contain contexts that especially lend themselves to adding and subtracting. Students also solve related problems that encourage them to notice the relationship between problems (often involving a property of addition or the relationship between addition and subtraction) and to use one problem to solve the other.*

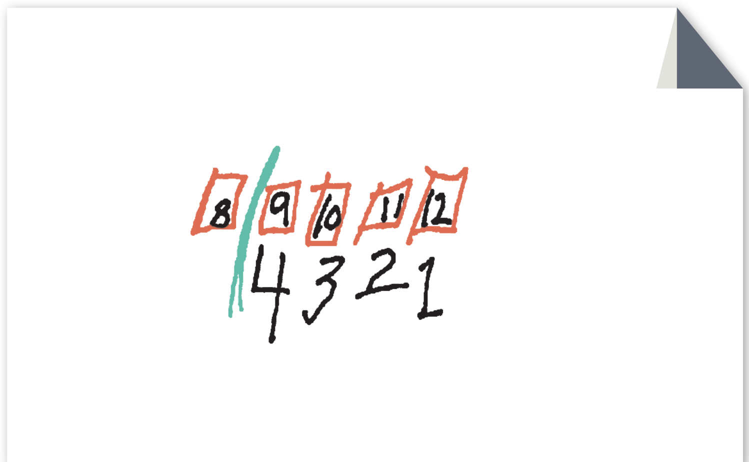
Problem Type	Description and Location
Add To	In these problems, two amounts are joined. The unknown can be the result (U1), the amount added (U5), or the starting amount (U6).
Take From	In these problems, one amount is removed from another. The unknown can be the amount remaining (U1), the amount removed (U5), or the initial amount (U6).
Put Together/ Take Apart	These problems involve two groups, but no action (e.g., joining or removing). The unknown can be the total (U1), the number in one group (U5), or the number in both groups (U3).
Comparison	These problems involve comparing two amounts. The unknown can be the difference (U4) or either of the amounts (bigger or smaller) (U6, U7).

[For more information, see **Teacher Note 8:** Types of Story Problems, Unit 1.]

Depending on the problem type and the time of year, students' strategies typically fall into three categories: counting all of the quantities in the problem by 1s; counting on (or back); or using something they know about a particular fact, or about the operation, to solve the problem.

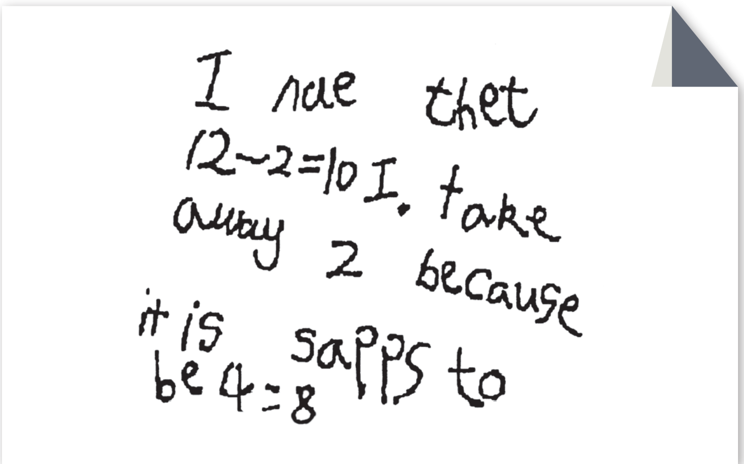


[To solve 12 − 4, this student showed all 12 of the pencils, crossed out 4, and then counted the ones that were left.]



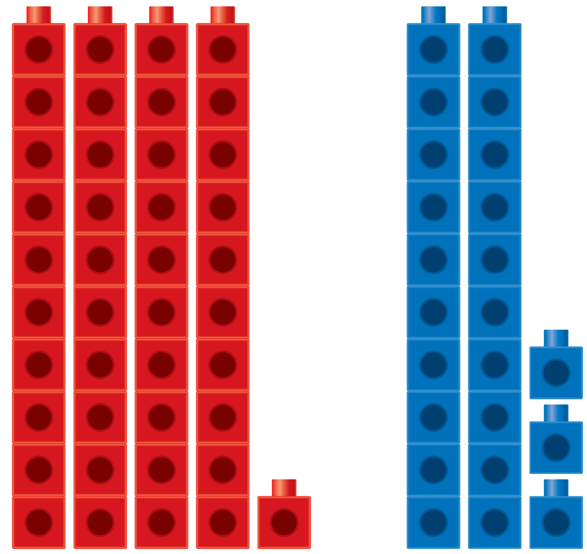
[This student *counted back* 4 from 12.]

* When solving problems about combining, separating, and/or comparing, some students use strategies that involve the properties of operations. While not a benchmark in a single place, teachers are encouraged to look for and track evidence of such strategies over the course of the year.

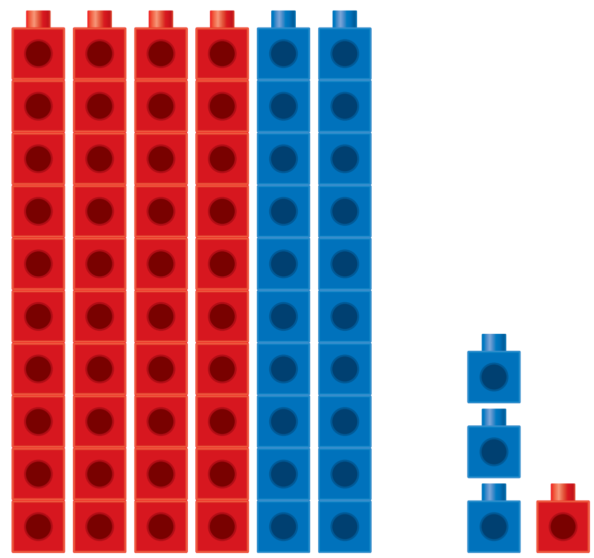


[This student used something he knew to subtract 4 from 12. First he subtracted 2, to get to 10, and then he subtracted 2 more.]

Students apply their understanding of place value and the operations as they add and subtract larger numbers. They use a place-value model (i.e., cubes in towers of 10) to add 2-digit numbers and to subtract multiples of 10 from multiples of 10. Students approach this work in a variety of ways, almost all of which rely on place value. To solve 41 + 23, some students add one number on in parts (e.g., 41 + 10 = 51, 51 + 10 = 61, and 61 + 3 = 64), while others add by place (e.g., 40 + 20 = 60, 1 + 3 = 4, and 60 + 4 = 64). They encounter situations where there are more than 9 ones and consider what happens to the answer in those situations. The steps of students' strategies are modeled with equations, and discussions focus on relating the numbers and symbols to the model and actions and thinking about what part of the number changes and why.



["There are 41 reds. Plus 10 is 51, plus 10 is 61, plus 3 more is 62, 63, 64."]



["40 + 20 = 60. Then there's 4 more, so 64."]

In units that focus on addition and subtraction, the **Algebra Connections in This Unit** Teacher Notes illustrate the kinds of generalizations students make about addition and subtraction, show how students explicitly encounter the commutative and associative properties of addition, show how students' strategies depend on properties of the operations, and demonstrate how they come to understand the relationship between addition and subtraction.

Number and Operations, *continued*

MAIN MATH IDEAS

- Understanding, representing, and solving problems involving addition and subtraction
- Understanding equivalence
- Using knowledge of place value to add and subtract

Benchmarks

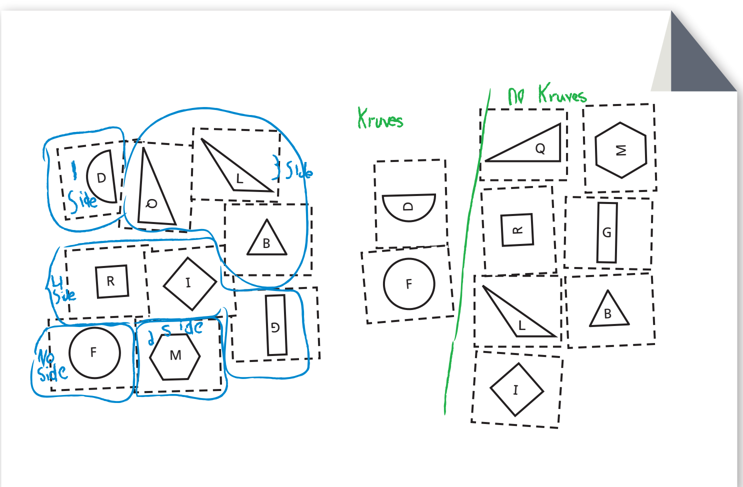
- Understand that you can count on/back to add/subtract 1 or 2. **(Unit 1)**
- Fluency with the +1, +2, −1, −2 facts. **(Unit 1)**
- Determine which of two pairs of numbers to 10 is greater. **(Unit 1)**
- Solve a take from story problem with result unknown. **(Unit 1)**
- Solve an add to story problem with result unknown. **(Unit 1)**
- Solve a put together story problem with total unknown. **(Unit 1)**
- Understand that you can count on/back to add/subtract. **(Unit 3)**
- Find at least 5 solutions to a put together/take apart problem with both addends unknown. **(Unit 3)**
- Solve story problems with 3 addends. **(Unit 3)**
- Represent numbers with equivalent expressions. **(Unit 3)**
- Solve comparison story problems with the difference unknown (how many more and how many fewer). **(Unit 4)**
- Fluency with addition and subtraction within 10. **(Unit 5)**
- Solve a put together/take apart problem with one addend unknown. **(Unit 5)**
- Understand the meaning of the equal sign. **(Unit 5)**
- Determine the unknown in an addition or subtraction equation relating 3 numbers (e.g., 5 + ___ = 8). **(Unit 5)**
- Solve add to and take from problems with unknown change. **(Unit 5)**
- Solve comparison story problems with a bigger or smaller unknown. **(Unit 6)**
- Subtract multiples of 10 from multiples of 10 using concrete models that represent tens and ones. **(Unit 7)**
- Add within 100 using concrete models that represent tens and ones. **(Unit 7)**

Geometry

The geometry work focuses on careful observation, description, and comparison of two-dimensional (2-D) and three-dimensional (3-D) geometric shapes.

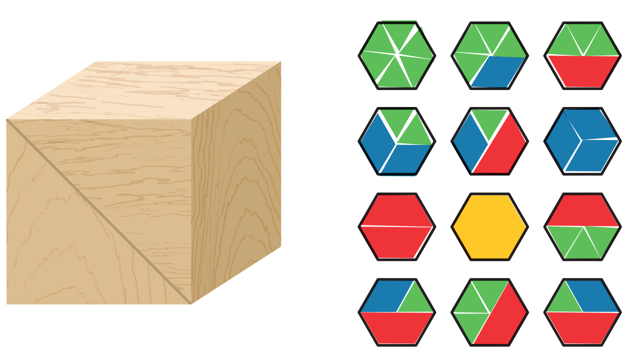
Students become more competent at determining which attributes of shapes are, or are not, important as they describe, build, and draw 2-D and 3-D shapes; as they sort and compare them; and as they think about questions such as, "What makes a triangle a triangle?" and "How is a cylinder different from a cone?"

In order to sort 2-D shapes into groups that "go together," students must look for similar and different attributes among a collection of shapes. While their categories may not always be conventional, explaining why certain shapes are grouped together helps students develop vocabulary for naming and describing the defining attributes of shapes.



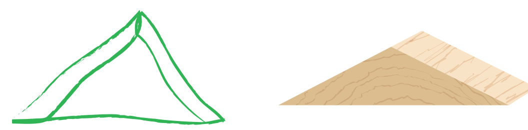
[Deshawn sorted by number of sides, and Paula used curves and no curves as her categories.]

Composing and decomposing 2-D and 3-D shapes helps students learn about geometric relationships and structure. While working with GeoBlocks, students notice that they can "put together" two triangular prisms to build a cube or, with pattern blocks, that they can use two trapezoids to make a hexagon.



[Students explore ways to make composite shapes with GeoBlocks and pattern blocks.]

Work with 2-D and 3-D shapes is connected as students look carefully at two of the defining attributes of 3-D shapes—the number and shapes of faces. As students describe the 2-D faces of 3-D prisms in the GeoBlock set, they notice that some faces are square while others are (non-square) rectangles or triangles. Similarly, as they match blocks to the outlines of faces or to drawings of blocks made by other students, they are considering specific attributes, as well as developing a visual image of the 3-D shape as a whole. In doing so, they deepen their understanding of both 3-D and 2-D shapes.



[A student makes a 2-D sketch of a 3-D GeoBlock.]

As students have a variety of experiences working with geometric shapes from various perspectives, they deepen their understanding of specific shapes and their defining attributes.

MAIN MATH IDEAS

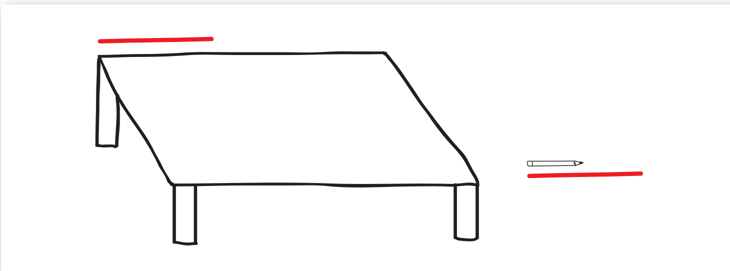
- Describing, identifying, and comparing attributes of 2-D shapes
- Composing and decomposing 2-D shapes
- Describing, identifying, and comparing attributes of 3-D shapes
- Composing and decomposing 3-D shapes
- Relating 2-D and 3-D shapes

Benchmarks

- Compose and decompose shapes in different ways. **(Unit 2)**
- Build and draw familiar 2-D shapes. **(Unit 2)**
- Use geometric language to describe and identify important attributes, and use those attributes to sort familiar 2-D shapes. **(Unit 2)**
- Use geometric language to describe and identify defining attributes of familiar 3-D shapes. **(Unit 8)**
- Compose 3-D shapes. **(Unit 8)**
- Match a 2-D representation of a 3-D shape to the outline of one of its faces. **(Unit 8)**

Measurement

Students develop an understanding of length as a stable and measurable dimension through experiences comparing objects. They determine which of two objects is longer (or shorter) and how much longer (or shorter); order sets of objects from shortest to longest; and indirectly compare objects using a third object.

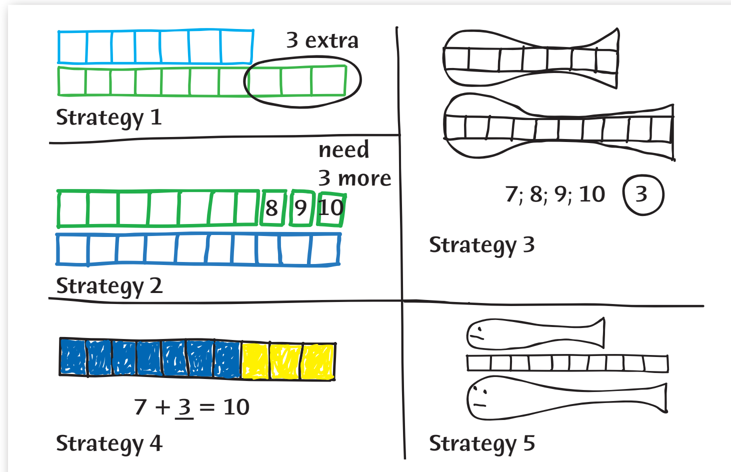


[If the desk is longer than the string, and the string is longer than the pencil, then the desk is longer than the pencil.]

Students practice foundational skills for accurate linear measurement such as knowing when to start and stop measuring, understanding how units must be lined up so as not to create gaps or overlaps, and measuring in a straight line from point to point. Regardless of what object is measured, students learn that when one measures an object twice—or when two different people measure it—the answer should be the same if the unit was the same. Students use string, cubes, paper clips, and inch tiles as units. Though focused on measuring lengths that are a whole number of units, the idea of halves or partial units naturally arises as students measure various objects.

Students practice measuring the length of classroom objects using different units.

This work provides the context for introducing comparison problems about length. Students compare the lengths of fish, which they measure with inch tiles, to determine which is longer (or shorter) than another and how much longer (or shorter). They visualize and model the problems with inch tiles or sketches, and they solve them using a variety of methods.



[The teacher records students' strategies for determining how much longer a 10-inch fish is than a 7-inch fish.]

Understanding time and making sense of the clock as a measurement tool is another form of measurement that students encounter across all Grade 1 units. Students first think about time as they look at the daily schedule. As they consider the day's activities, they hear and use vocabulary that relates to time and sequence (e.g., *first, next, last, before, after, during, early, later, at the beginning or end of*) and times of day (e.g., *morning, afternoon*). They associate times on the hour with familiar classroom activities, becoming familiar with digital and analog clock formats. With this foundation, they learn to name and notate times on the hour and half hour, in analog and digital form, and they learn that there are various ways to refer to half hour times (e.g., *two-thirty, 30 minutes past 2 o'clock,* and *half past two*).

MAIN MATH IDEAS

- Understanding length
- Using linear units
- Understanding time

Benchmarks

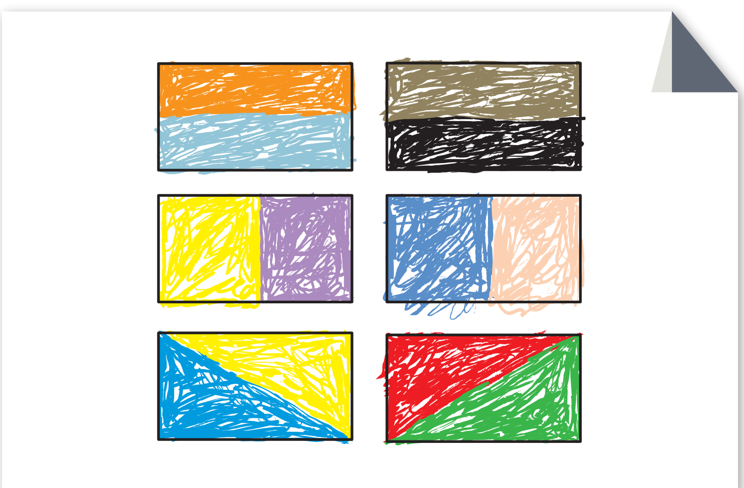
- Compare the lengths of two objects indirectly by using a third length. **(Unit 4)**
- Demonstrate accurate measuring techniques when measuring an object or distance with multiple units. These techniques include starting at the beginning, ending at the end, leaving no gaps or overlaps, measuring in a straight line, and keeping track of the number of units. **(Unit 4)**
- Tell time to the hour. **(Unit 4)**
- Tell time to the half hour. **(Unit 8)**

Fractions

As students learn to measure, they encounter lengths that are not a whole number of units long and times that are on the half hour. The fraction work connects to these experiences.

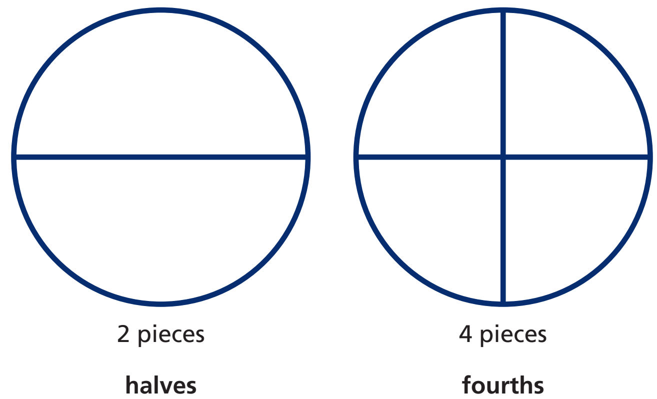
Using the context of designing "Fraction Rugs," students partition circles, rectangles, and squares into halves and fourths. The focus is on understanding that halves (or fourths) involve two (or four) *equal* parts. As they describe circular and rectangular rugs, they learn and use words like *half, one half, halves,* and *half of,* and the parallel language for fourths (e.g., *one fourth, one quarter*).

In their work with fractions, students partition familiar 2-D shapes into fractional parts, and in doing so, they decompose 2-D shapes into two or more smaller shapes. They attend to defining attributes of shapes as they notice the shapes that result when familiar 2-D shapes are divided into fractional pieces: half- and quarter-circles, triangles, squares, and rectangles.



[Students' half-and-half rugs]

Students come to see that the more parts a shape is partitioned into, the smaller the parts. In other words, one fourth of a shape is smaller than one half of the same shape, because fourths cut a shape into more, smaller pieces. Using the context of favorite kinds of pizza, they explain which piece they would rather have, one half or one fourth, and why.



[Would you rather have one half or one fourth of a pizza?]

MAIN MATH IDEA

- Understanding halves and fourths

Benchmark

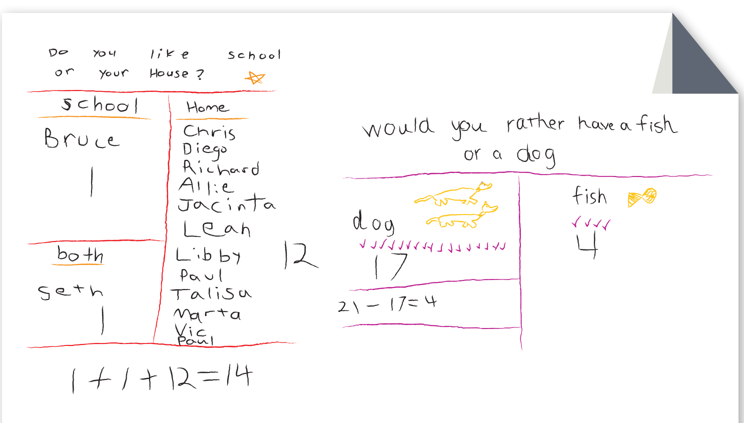
- Understand that halves or fourths (quarters) apply to wholes divided into two (four) equal parts; partition circles and rectangles into two and four equal parts. **(Unit 4)**

Data

Data investigations engage students in modeling with mathematics. Students collect, represent, describe, and interpret categorical data with up to three categories. As they create representations, students work on keeping track of the data and on finding ways to represent the data that help others see what the data show.

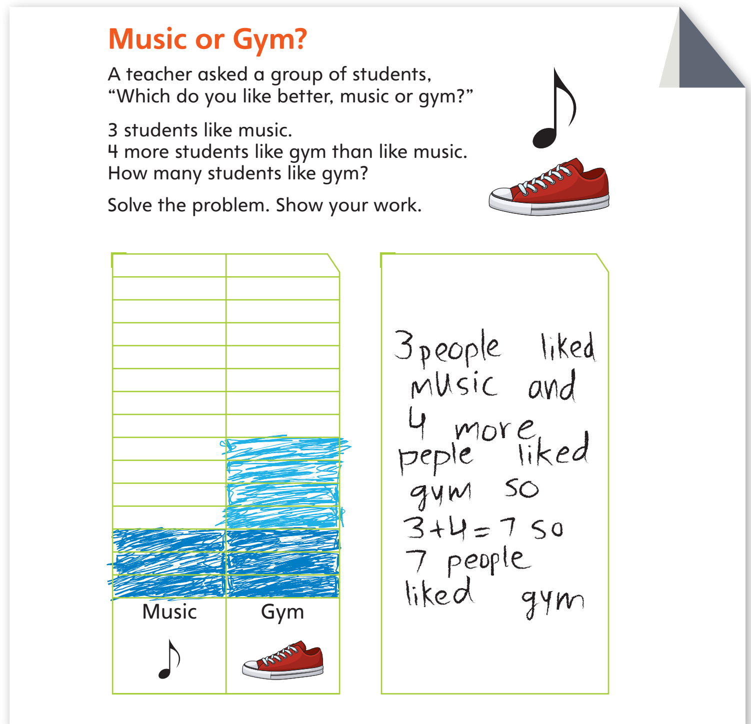
As part of this work, students are introduced to several standard forms of representation, including picture graphs and horizontal and vertical tables. Guided by the question "What do these data tell us about [our class]?", students discuss and compare representations—and consider what features of a representation help communicate a clear description of the data, an important aspect of mathematical modeling. In the context of this overall question, their descriptions focus on quantitative characteristics of the data such as, "How many are in each group?"; and "Which category has more data? How many more?" Students also determine the total number of responses in all categories and think about the relationship of that total to the total number of people who responded.

Students carry out their own data investigation. They develop a question, collect the data, represent it, and describe and interpret it, which may, in turn, bring up more questions. Once data are collected, they are represented, examined, and analyzed to find out what information the data provide about the original question. By carrying out their own investigations, students encounter the real and challenging issues that are involved in data collection and analysis.



[Students represent and analyze the data from their own surveys.]

Data collection and representation provide a useful context for revisiting comparison story problems with the difference unknown and for exploring comparison problems in which information about one group, and the difference between that group and another group of unknown size, is provided. Students use models such as vertical tables and cubes to represent and solve these problems.



[Sample student work for a comparison problem with bigger unknown]

MAIN MATH IDEA

- Collecting, representing, describing, and interpreting data

Benchmark

- Represent and describe a set of data with two or three categories (e.g., how many are in each group, which group has more/how many more, and how many people responded to the survey). **(Unit 6)**