Number and Operations

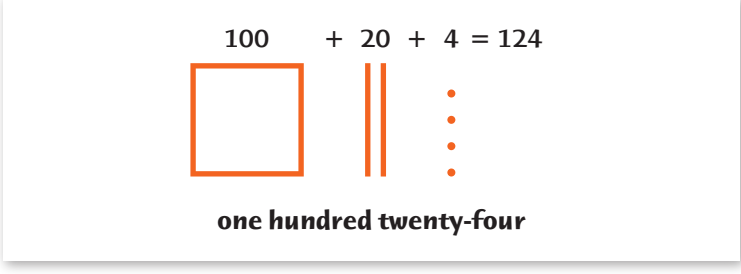
The Number System

Students begin the year briefly reviewing the counting of large quantities and the number sequence. They learn to recognize and identify coins and their values, and use equivalencies among them. Such models support students in thinking about and working with groups. As a review of place value, they investigate what happens when they count a set of objects by groups of 2, 5, and 10, focusing specifically on how the written numeral is another representation of the tens and ones structure of numbers in the base-10 number system.

Grouping by 10s		
Total Number of Cubes	Number of Towers of 10	Number of Leftovers
13	1	3
17	1	7
23	2	3

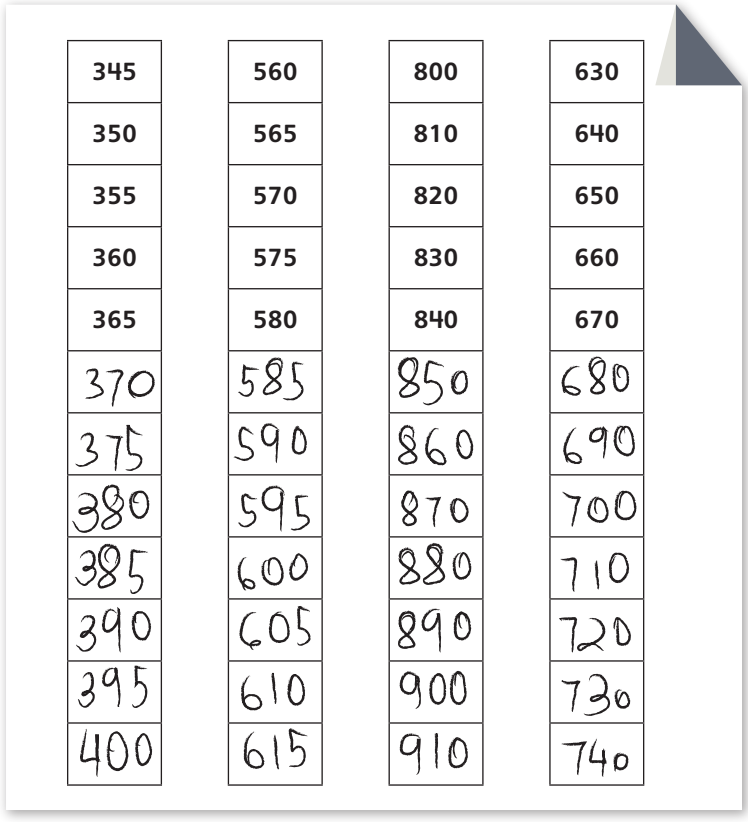
[Students count sets of cubes into groups of 10 and record the results.]

In addition to money and cubes in groups of 10, students work with a model for place value, Sticker Station, which sells stickers singly, in strips of 10, and in sheets of 100. This context—and the notation developed to represent it—is used to represent and visualize the composition of 2- and 3-digit numbers. Students come to see that 100 can be made from 100 ones, 10 tens, or 1 hundred and that numbers like 300 are made of 3 hundreds, or 100 + 100 + 100. This culminates in students being able to represent 3-digit numbers in sticker notation and in expanded form. They think about how the number of sheets, strips, and singles connect to the number of 100s, 10s, and 1s, and to the way a number is written in words, with numbers, and in expanded form.



[Students use expanded form, sticker notation, and words to represent the number 124.]

Students extend the rote sequence of numbers to 500, and then 1,000. They find and use patterns in the sequence of numbers in a 500 Book (made from five 100 charts) and a 1,000 Book (made from five 200 charts) as they read, write, and compare numbers, and as they practice counting within this range, by 1s, 5s, 10s, and 100s.



[Students skip count by 5s and 10s and record the sequences on Counting Strips.]

MAIN MATH IDEAS

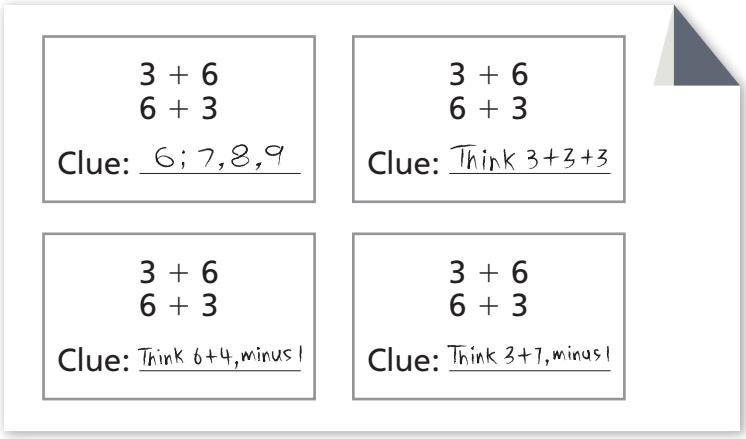
- Understanding and extending the counting sequence
- Understanding place value

Benchmarks

- Recognize and identify coins and their values. **(Unit 1)**
- Understand that 100 can be seen as 1 hundred, as 10 tens, and as 100 ones. **(Unit 3)**
- Understand that multiples of 100 (e.g., 200, 300, 400, etc.) are made up of a number (2, 3, 4, etc.) of hundreds. **(Unit 3)**
- Understand that 3-digit numbers represent amounts of hundreds, tens, and ones. **(Unit 5)**
- Read, write, count, and compare numbers to 1,000. **(Unit 5)**
- Count by 5s, 10s, and 100s within 1,000. **(Unit 5)**

Addition and Subtraction

Students develop fluency with addition facts to 10 + 10 and the associated subtraction facts over the course of the year. Games, activities, and a Classroom Routine provide repeated practice, as do Fact Cards, which students get in related sets and sort into envelopes of facts they know and facts they're still working on. Throughout, this work focuses on using known facts as "clues" to learn other facts (e.g., 6 + 8 = 6 + 6 + 2 or 14 − 6 = 14 − 4 − 2; or, "I know 8 + 3 = 11, so 11 − 3 = 8."). The development of fluency is based on what students know and understand, rather than on memorization.



[Four students' clues for remembering 3 + 6 and 6 + 3.]

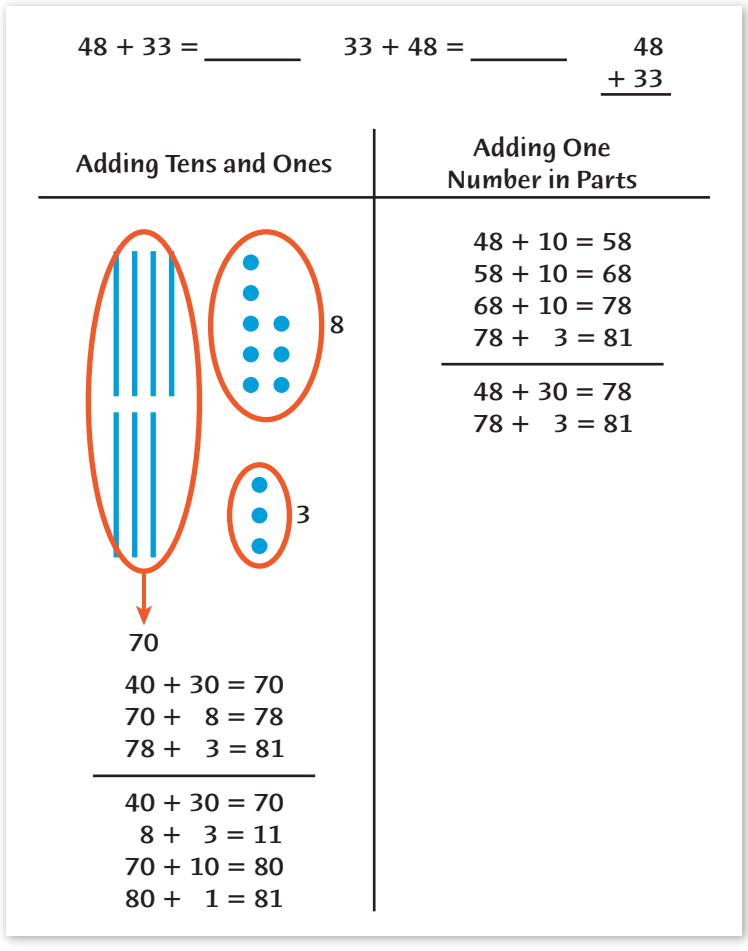
As in Grade 1, students make sense of and solve a variety of story problem types involving addition and subtraction, using a variety of contexts (objects, stickers, money, measurements), with unknowns in all positions.

Problem Type	Description and Location
Add To	In these problems, two amounts are joined. The unknown can be the result (U1), the amount added (U3), or the starting amount (U3).
Take From	In these problems, one amount is removed from another. The unknown can be the amount remaining (U1), the amount removed (U3), or the initial amount (U3).
Put Together/ Take Apart	These problems involve two groups, but no action (e.g., joining or removing). The unknown can be the total (U1), the number in one group (U3), or the number in both groups (U3).
Comparison	These problems involve comparing two amounts. The unknown can be the difference (U1), or either of the amounts (bigger or smaller) (U5, U8).

[For more information, see Teacher Note 10: Types of Story Problems, Unit 1.]

Students use mathematical tools and representations (e.g., number lines,* sticker notation) to model and solve problems and to clarify and communicate their thinking.** They are encouraged to show their mathematics work on paper in ways that make sense to them; many use a combination of pictures, words, numbers, and equations. The focus is on developing ever more efficient strategies for solving problems and for recording their work.

While students see and discuss a range of methods, there is a focus on two particular strategies for addition: adding tens and ones and adding one number in parts.



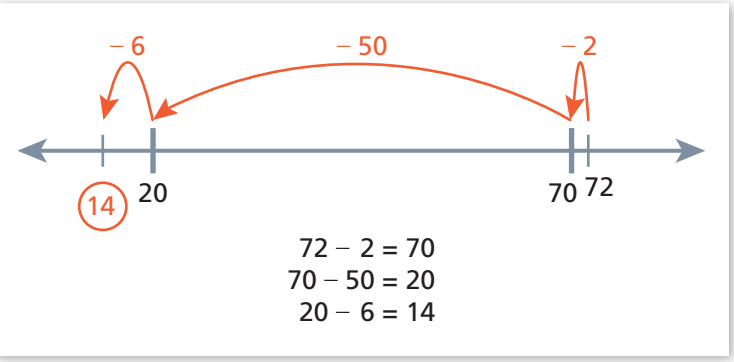
[The teacher records students' strategies for solving a story problem about 48 + 33.]

* When students solve problems about combining, comparing, or removing, some use the number line to represent the problem, solve the problem, and/or show their work. While not a benchmark in a specific unit, teachers are encouraged to look for and track evidence of students' understanding of the number line throughout the year.

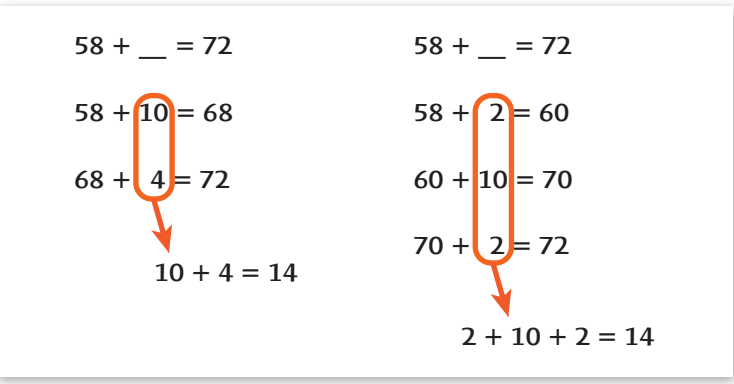
** Throughout *Investigations*, students are asked to show their work. As they describe their strategies, they are often asked to consider and explain why their strategies work. While not a benchmark in a specific unit, there are many identified opportunities to look for and track evidence of students' abilities throughout the year.

Number and Operations, *continued*

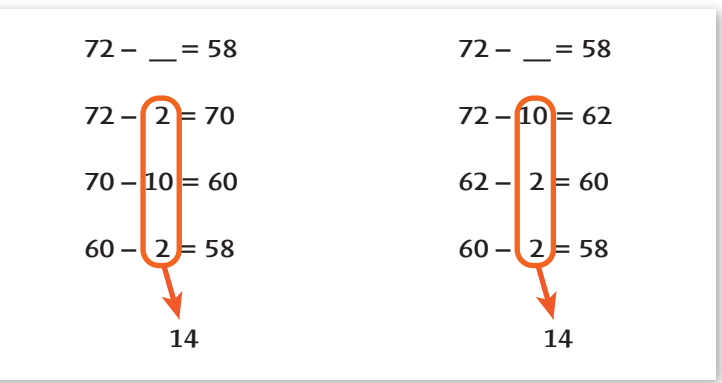
Similarly, students focus on two main strategies for subtraction: subtracting in parts and adding up/subtracting back.



[A student solves 72–58 by subtracting the 58 in parts.]



[These students add up to find the difference between 58 and 72.]



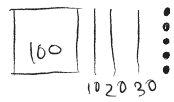
[These students subtract back to find the difference between 72 and 58.]

All of these strategies depend on an understanding of place value. Students end the year with at least one strategy for accurately and efficiently adding and subtracting within 100.

Students apply their understanding of place value and the operations, as well as their strategies for adding and subtracting 2-digit numbers, as they work with larger numbers. They use place-value notation to model and solve problems about adding and subtracting 3-digit numbers. They encounter addition problems where there are more than 9 ones (and/or 9 tens), and subtraction problems where a hundred (and/or a ten) needs to be broken apart; and they reflect on how the digits in the numbers change in those situations. Class discussions focus on using equations to model the steps of students' strategies and on relating the numbers and symbols to the place-value representation (e.g., sticker notation or towers of 10 cubes) and to components of the problem.

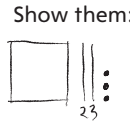
Problem 1

Kira has 135 stickers.
Show them:



Equation:
100+30+5 = 135

Jake has 123 stickers.
Show them:



100+100=200
30+20=50
5+3= 8
250+8=258

Equation:
100 + 20+3 = 123

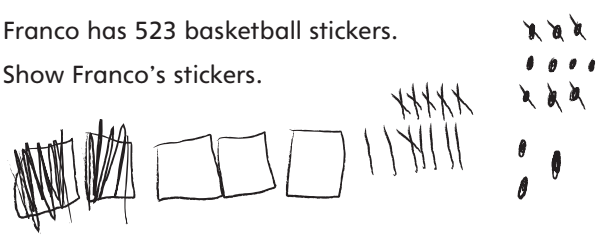
If Kira and Jake combine their sets, how many stickers will they have? Use equations to show your work.

135+123=258

Problem 2

Franco has 523 basketball stickers.

Show Franco's stickers.



He gives 156 of these stickers to Kira.
Write an equation that represents the problem:

523 – 156 = ?

How many does Franco have left?
Solve the problem. You can use your sticker drawing to help you. Use equations to show your work.

523-156=367 stickers

In the curriculum units that focus on addition and subtraction, the **Algebra Connections in This Unit** Teacher Notes illustrate the kinds of generalizations students use as they add and subtract, show how students are applying the commutative and associative properties of addition, and highlight how students study the relationship between addition and subtraction.

MAIN MATH IDEAS

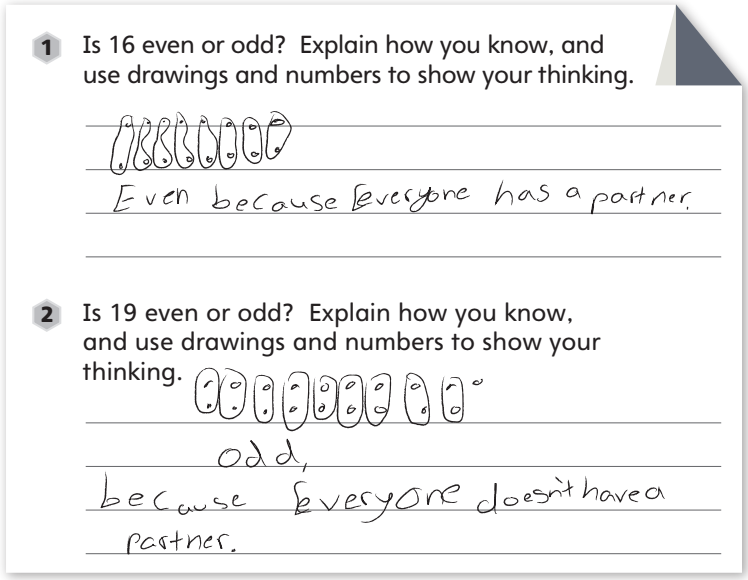
- Fluency within 20
- Understanding, representing, and solving problems involving addition and subtraction
- Using knowledge of place value to add and subtract

Benchmarks

- Use known combinations to add several numbers in any order. **(Unit 1)**
- Solve a comparison story problem with the difference unknown. **(Unit 1)**
- Solve put together/take apart story problems with the total unknown, and add to and take from story problems with the result unknown. **(Unit 1)**
- Solve a put together/take apart story problem with both addends unknown, and find all the possible combinations. **(Unit 3)**
- Solve a put together/take apart story problem with one addend unknown. **(Unit 3)**
- Solve two-step story problems about money. **(Unit 3)**
- Solve story problems with an unknown change. **(Unit 3)**
- Solve story problems with an unknown start. **(Unit 3)**
- Solve a 2-step story problem that involves finding the difference between a 2-digit number and 100. **(Unit 5)**
- Add/subtract 10 or 100 to/from numbers within 1,000. **(Unit 5)**
- Add fluently within 100. **(Unit 5)**
- Solve comparison story problems with a bigger unknown. **(Unit 5)**
- Solve a comparison story problem with a smaller unknown. **(Unit 8)**
- Fluently subtract two 2-digit numbers. **(Unit 8)**
- Fluently add and subtract within 20. **(Unit 8)**
- Represent and solve addition and subtraction problems with 3-digit numbers. **(Unit 8)**

Foundations of Multiplication

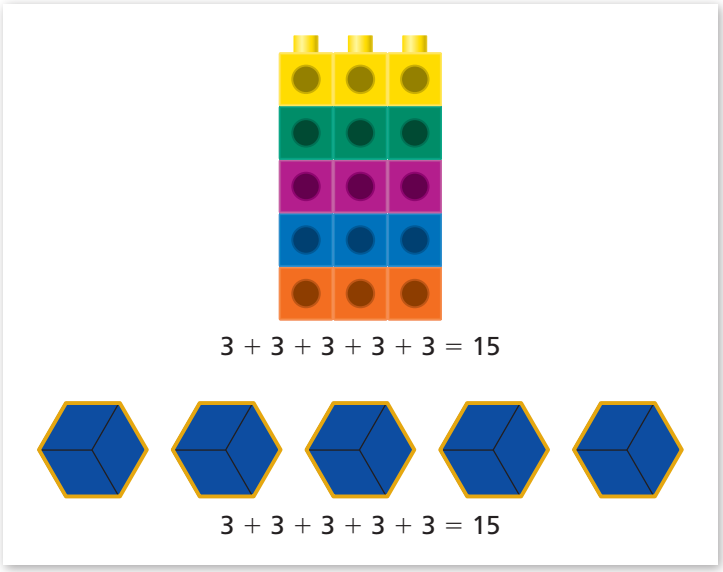
To lay the foundation for the study of multiplication, students build and model multiplicative situations that involve the accumulation of equal groups. They investigate odd and even numbers and multiplicative relationships in contexts where there are two equal groups (teams) and groups of 2 (partners). They use equations to represent even numbers as the sum of equal addends, and odd numbers as the sum of equal addends plus 1.



[A student explains how he knows whether numbers are odd or even.]

Number and Operations, *continued*

Work with multiplicative relationships also extends to adding multiple groups of 2 through 6 in two additional contexts. In the first context, students make cube buildings in the form of arrays, with an equal number of rooms per floor. They use equations to model their buildings, representing the total number of rooms as the sum of equal addends. In the second context, students investigate equal groups as they cover a certain number of a pattern block shape (e.g., hexagons) with another pattern block shape (e.g., rhombuses). Again, they use an equation to show the total number of blocks as the sum of equal addends. Students make observations about and compare the mathematical structure of these different-looking contexts and notice how different situations can have the same underlying mathematical relationship.



["Three rooms on each floor is the same as three rhombuses for one hexagon. There are 5 floors and 5 hexagons, so they both use the same equation."]

In all of these contexts, students explore multiplicative structures as they solve problems about adding equal groups, laying the foundation for later work with multiplication.

MAIN MATH IDEAS

- Investigating odd and even numbers
- Visualizing equal groups in the structure of arrays
- Describing and representing equal groups as the foundation of multiplication

Benchmarks

- Define even and odd numbers in terms of numbers that can/cannot be organized into groups of two or two equal groups. **(Unit 7)**
- Write an equation to express an even number as a sum of two equal addends. **(Unit 7)**
- Solve problems that involve equal groups. **(Unit 7)**
- Write an addition equation to express the total number of objects in a rectangular array. **(Unit 7)**

Geometry

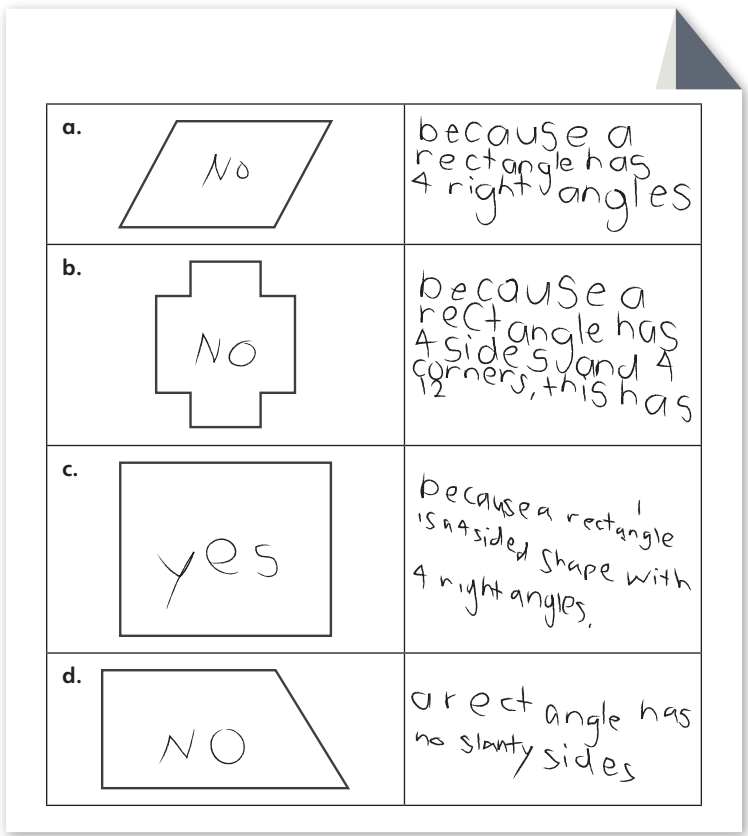
Students observe and describe the defining attributes of two-dimensional (2-D) and three-dimensional (3-D) shapes. These attributes include, for 2-D shapes, number and lengths of sides, and number of angles and vertices, and for 3-D shapes, the number and shape(s) of faces. As students sort, compare, construct, and draw 2-D shapes and 3-D objects, they develop visual images and geometric language for describing them.

Describing GeoBlocks

General Features	Geometric Features
Some look like real objects: a box, a ramp	Faces are square
Made of wood	Has 6 faces (sides)
Smooth	Cube
	3-D shape
	Has vertices (corners, points)
	Faces are flat
	Rectangular Prism: 6 faces: 2 square faces, 4 rectangles
	Triangular Prism: 5 faces: 2 triangles, 3 rectangles

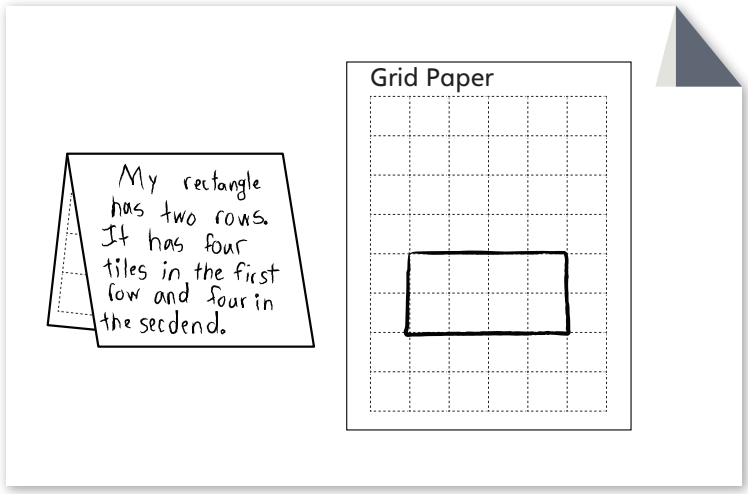
[Students identify the geometric features of GeoBlocks.]

As part of this work, students consider the defining attributes of polygons, with a particular focus on quadrilaterals. They explore the properties of rectangles and squares, and they look particularly at the defining attributes of relative side lengths (e.g., Are all four sides of equal length?) and the type of angles as they consider questions such as, "What makes a rectangle a rectangle?"



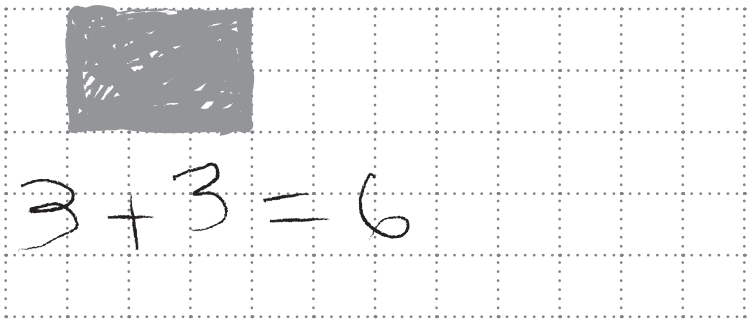
[A student explains why shapes are (or are not) rectangles.]

Students work with rectangles from two perspectives: as 4-sided shapes with 4 right angles and as arrays made of equal rows and columns of squares. Students investigate the structure of arrays as they partition rectangles into rows and columns and as they compose rectangles with square tiles.



[Students build and describe rectangles in terms of the number of columns and rows.]

The study of arrays supports students' work with number and operations. As students make and describe rectangular arrays, they use various counting strategies to determine the total number of squares in an array and use an equation to represent this total as the sum of equal addends. This work with equal groups contributes to laying the foundation for multiplication.



[As students play *Double Arrays,* they practice the doubles facts and learn about the structure of an array.]

MAIN MATH IDEAS

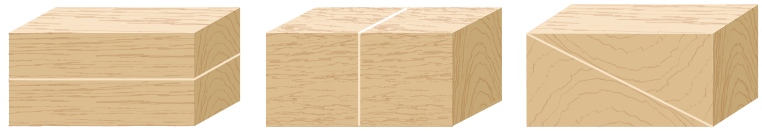
- Describing, identifying, and comparing attributes of 2-D and 3-D shapes
- Visualizing the structure of arrays
- Visualizing equal groups in the structure of arrays

Benchmarks

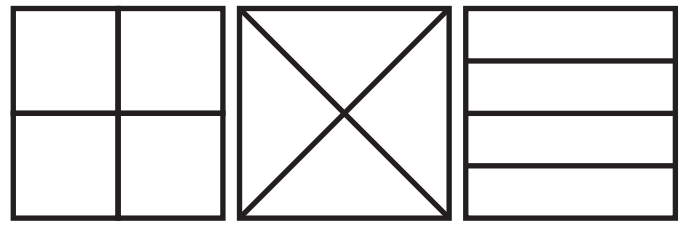
- Identify defining attributes of 2-D and 3-D shapes (number and shape of faces, number and length of sides, number of angles and vertices) and draw shapes with those attributes. **(Unit 2)**
- Make a rectangle out of same size squares and specify the number of rows and the number of squares in each row. **(Unit 2)**
- Write an addition equation to express the total number of objects in a rectangular array. **(Unit 7)**

Fractions

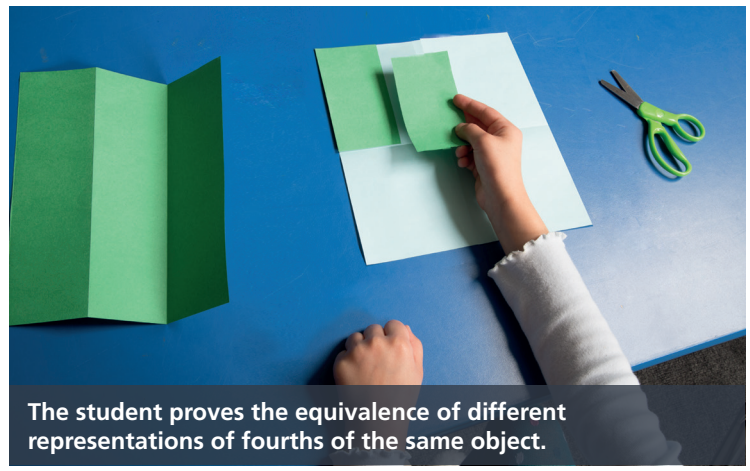
Work with fractions is directly related to geometry. As students fold paper squares, partition geometric shapes into equal parts on geoboards and dot paper, or compose and decompose GeoBlocks, they investigate halves, thirds, and fourths of squares, rectangles, circles, and prisms. They consider whether or not equal parts of the same whole (i.e., parts of equal area in 2-D shapes or equal volume in 3-D shapes) must look the same. They find ways to show that different-looking halves of a rectangular prism or fourths of a paper square are equal.



[Students investigate halves of a rectangular prism.]

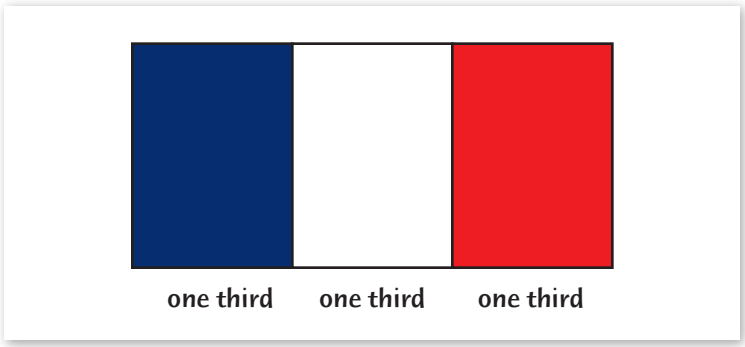


[Students investigate different-looking fourths.]



The student proves the equivalence of different representations of fourths of the same object.

Students use fractional language to identify regions of geometric shapes that are partitioned into equal parts. In addition to the terms *one half, one third,* and *one fourth,* they also hear and use language such *as half of, halves, one third of, thirds, fourths,* and *quarters* to describe these regions, as well as *three thirds* and *four fourths* to describe one whole. Like fraction words, fraction symbols and notation are used in many everyday situations and are therefore familiar to, but not necessarily understood by, many students. They see but are not expected to use such notation, though many choose to do so.



[Students color, discuss, and identify the three equal regions of a fraction flag.]

MAIN MATH IDEA

- Understanding equal parts of a whole

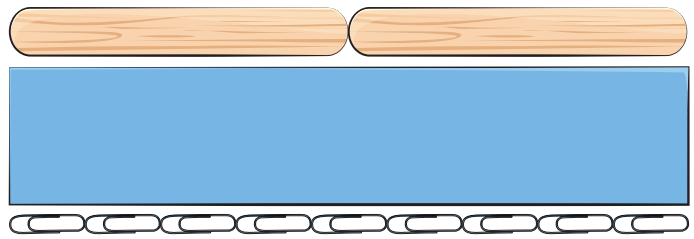
Benchmarks

- Recognize that [halves, thirds, fourths] of the same whole can look different. **(Unit 2)**
- Partition 2-D shapes into halves, thirds, and fourths and name the regions. **(Unit 2)**

Measurement

Students study measurement in a variety of contexts. They work with time and money, develop their understanding of length and how it is measured, and solve problems in which they relate addition and subtraction to length.

Students use linear units to measure objects and compare measurements. They learn about iterating a unit and about the relationship between the size of a unit and the results of measuring: the smaller the unit, the greater the number of units for the same length.



[Students iterate a unit to measure length and consider the relationship between the sizes of units and the results of measuring.]

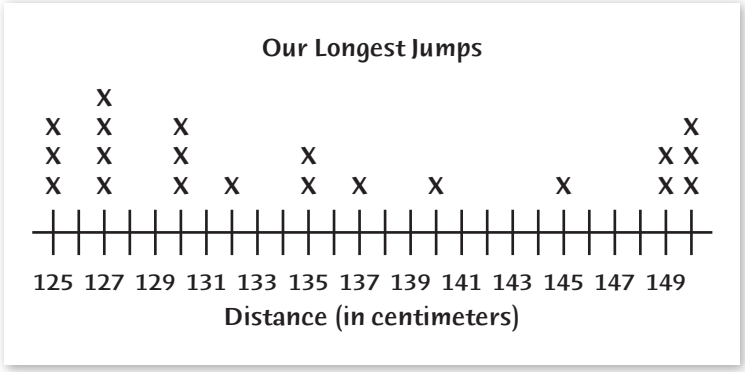
Students first use non-standard units, such as shoe-lengths, craft sticks, cubes, and paper clips, to measure lengths. By discussing their methods for measuring, students learn that agreeing on a common unit is critical for communicating measurement information to others and comparing results. This leads to work with standard measures.

The experience of iterating a single unit (or multiple units) over a length helps students see the advantage of measurement tools that iterate some number of units in one stable tool. Using individual "inch-bricks," students construct and use their own inch rulers, which fosters not only an understanding of the conventional units, but also the process of measuring with a tool and the principles that underlie the design and use of the tool.

Students make their own 12-inch measuring tool by gluing inch-bricks onto the template.

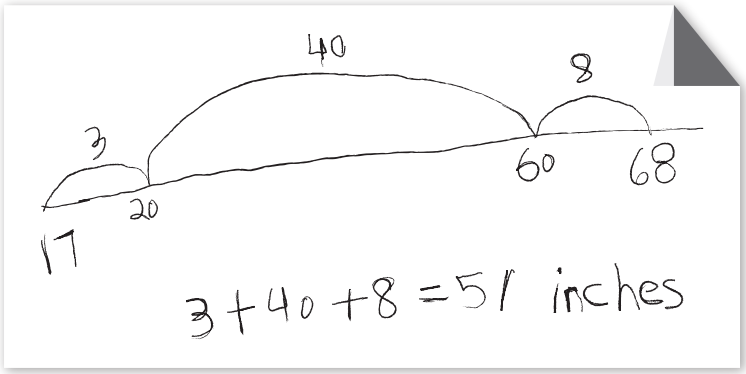
Measurement, *continued*

Students use rulers, yardsticks, and metersticks to measure lengths in U.S. standard units and metric units. They collect data about how far they can jump in inches and centimeters, and represent the data on a line plot.



[Students' measurement data is represented on a line plot.]

Measuring and comparing lengths provides a particularly good context for modeling and solving comparison problems (e.g., comparing the length of the longest and shortest jumps and determining the difference).



[A student compares two lengths (17 in. and 68 in.) and finds that the difference is 51 inches.]

Understanding that time can be measured is an important concept that is worked on across all Grade 2 units. After reviewing how to tell time to the hour and half hour, students look at fractional parts of a whole (60 min), partitioning clocks into 2 and then 4 equal parts as an introduction to telling time to the quarter hour. Students come to see those sections as made up of 5-minute intervals, and practice counting around the clock by 5s as a way into telling time to the nearest 5 minutes. Throughout, practice with telling time involves naming times displayed on both digital and analog clocks. Students notate times in digital form. They set analog clocks to times that are read aloud or presented in digital format. They think about what time it will be in one hour, in one half hour, or in 15 minutes. They also think about when daily activities happen, and learn the convention of labeling such times with A.M. or P.M.

MAIN MATH IDEAS

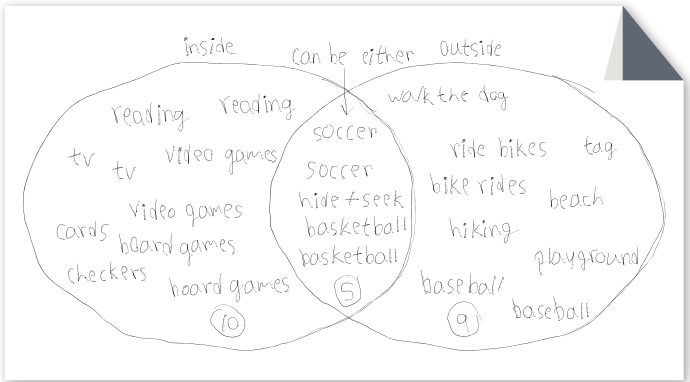
- Using linear units
- Measuring with standard units
- Understanding, representing, and solving problems involving addition and subtraction
- Understanding time

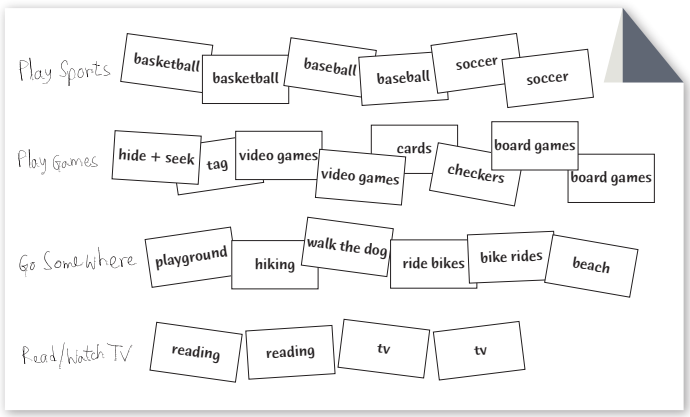
Benchmarks

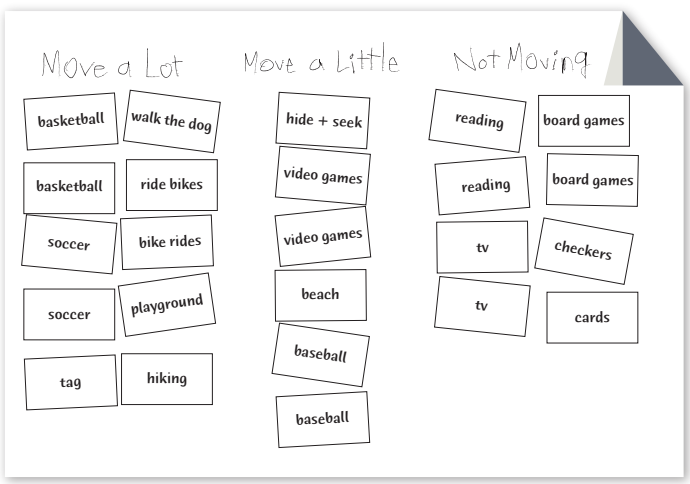
- Recognize that, when measuring the same length, larger units yield smaller counts (and vice versa). **(Unit 6)**
- Estimate and measure lengths in inches, feet, centimeters, and meters. **(Unit 6)**
- Solve comparison and other story problems about lengths. **(Unit 6)**
- Name, notate, and tell time to the nearest 5 minutes using analog and digital formats and associate A.M. and P.M. with time of day. **(Unit 8)**

Data

Students' study of data begins with sorting and classifying people and objects based on visible attributes. These activities provide experience noticing the similarities and differences among individuals and objects in a group—and deciding which attributes to attend to. This leads into work sorting and classifying data having many different values and considering how to group the data into different categories. For example, answers to the question "What is your favorite weekend activity?" can be classified in a variety of ways (outdoor and indoor activities; or things you do by yourself, things you do with one friend, and things you do with a group). By grouping the data in different ways, students can use the same data to draw different conclusions.



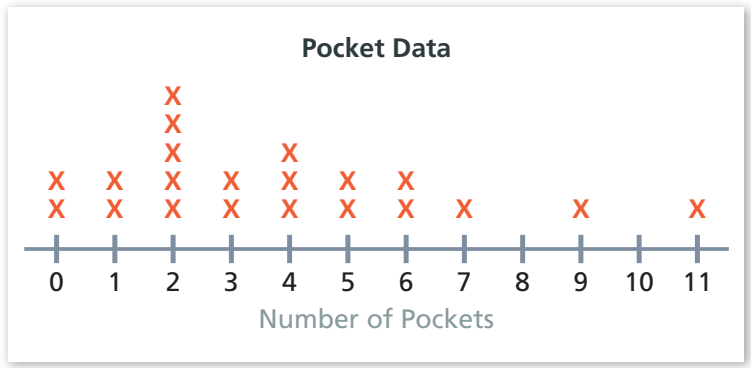




[Students find different ways to classify data about favorite weekend activities.]

Data investigations engage students in modeling with mathematics. By making predictions and then collecting their own data, students see how data are used to answer a real-world question or to give evidence about an issue. They use a variety of representations including picture graphs, Venn diagrams, bar graphs, cube towers, and line plots, as well as their own unique representations, to show the data that they collect. In comparing a variety of representations of the same data, students learn how different representations can make different aspects of the data set more visible. The goal is a representation that communicates a clear description of the data, an important aspect of mathematical modeling.

Students create line plots from data collected in response to questions with numerical answers such as, "How many teeth have you lost?" "How many pockets are you wearing?" and "What is the length of your longest jump?" In creating this kind of representation, students have to think through two ways numbers are used: to indicate the *value* of a piece of data (I have *6 pockets*); or to indicate *how often* a particular data value occurs (*2 students* have 6 pockets).



[Students represent Pocket Data on a line plot.]

Data, *continued*

Once data are classified, ordered, and represented, students describe and interpret them. They think about the counts at each value and consider the trends that these counts, considered together, reveal. Students are encouraged to make connections between the numerical information and the context, and to make observations about the data set as a whole, thinking about questions such as, "What do these data (or this model) tell us about the people in our class (or the group surveyed)?" Learning something about the context being modeled is the purpose of mathematical modeling.

By participating in an entire data investigation from start to finish, students encounter many of the same issues encountered by statisticians as they decide how to collect, keep track of, organize, represent, describe, and interpret their data.

MAIN MATH IDEAS

- Sorting and classifying
- Collecting and representing data
- Describing and interpreting data

Benchmarks

- Organize a set of data into up to four categories. **(Unit 4)**
- Create, describe, and interpret a variety of data representations, including picture graphs and bar graphs. **(Unit 4)**
- Order, represent, and describe a set of numerical data. **(Unit 4)**
- Represent measurement data on a line plot. **(Unit 6)**