Number and Operations

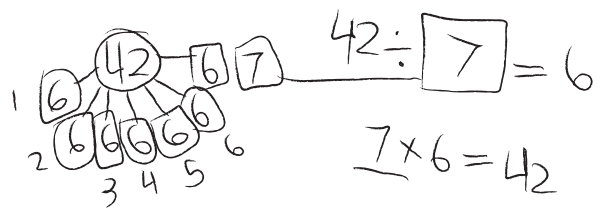
Multiplication and Division

Students develop an understanding of the operations of multiplication and division, particularly focusing on multiplication and division as involving equal groups. Given problems in familiar contexts, students identify the components of these problems—the number of groups, the number in each group, and the number in all the groups. They identify where these components appear in multiplication and division equations. Division contexts involve two types of division situations:

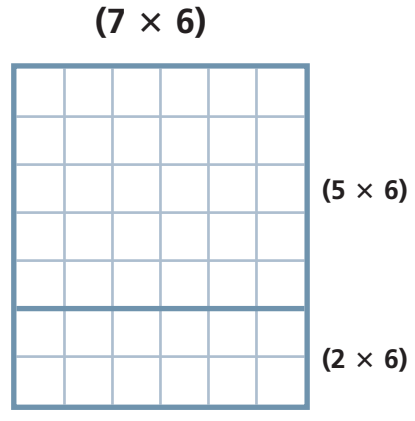
- Sharing (e.g., I have 18 balloons. I'm going to divide them evenly between my sister and me. How many balloons will each of us get?);
- Grouping (e.g., I have 18 balloons. I'm going to tie them together in bunches of 2 to give to my friends. How many bunches can I make?).

Creating a variety of physical models and other representations supports students' visualization of multiplicative relationships. Through working with such representations, students come to recognize the commutative and distributive properties and the relationship between multiplication and division.

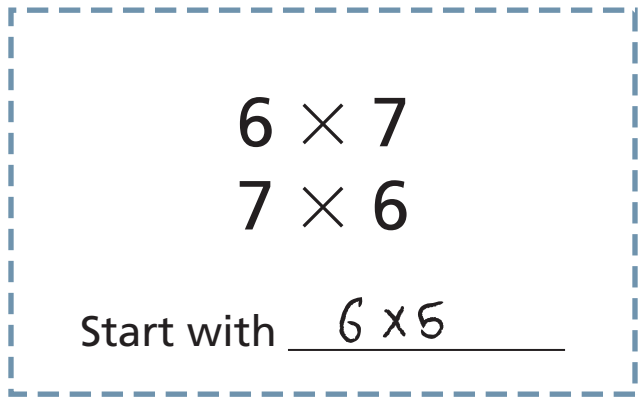
1 Mr. James bought 42 toy animals for his students to count and sort. How many 6-packs did he buy?



Key among the representations is the array—a rectangular arrangement of objects in equal rows and columns. Work with arrays also lays an important foundation for understanding why the dimensions of a rectangle can be multiplied to find the area of the rectangle.



Students develop fluency with multiplication facts to 10 × 10 and related division facts, using multiplication facts they already know to help them learn facts they are still working on. Visual images, including arrays and story contexts, help them see the relationships between known facts and unknown facts.



Multiplication with multiples of 10 is the focus of story problems set in the context of a toy factory that sells packages of toys in quantities of 1 through 9 and in multiples of 10. Students use representations to identify, express, and explain how an expression with single-digit factors, such as 3×4, is related to a problem involving a multiple of 10, such as 3×40. This concept, along with students developing fluency in multiplying in chunks (the distributive property), is the basis for solving multiplication and division problems within 100.

Students apply their growing understanding of multiplication and division to solve multi-step problems that involve more than one operation.

The **Algebra Connections in This Unit** Teacher Notes in Units 1, 5, and 8 show how the commutative, distributive, and associative properties of multiplication and the inverse relationship between multiplication and division are implicit in students' solution strategies. They also highlight how students identify and explain arithmetic patterns.

MAIN MATH IDEAS

- Understanding the meaning of multiplication
- Understanding and working with an array/area model of multiplication
- Learning the multiplication facts
- Developing strategies for division based on understanding the inverse relationship between multiplication and division
- Understanding the meaning and structure of multiplication and division and the relationship between them
- Solving multiplication and division problems, including multi-step problems and problems with multiple solutions
- Making sense of multiplying multiples of 10 by one-digit numbers
- Solving multiplication problems with 2-digit numbers
- Solving division problems
- Learning division facts
- Identifying arithmetic patterns and solving multi-step problems

Benchmarks

- Demonstrate an understanding of multiplication and division as involving equal groups. **(Unit 1)**
- Solve multiplication and related division problems by using skip counting or known multiplication facts. **(Unit 1)**
- Interpret and use multiplication and division notation. **(Unit 1)**
- Demonstrate fluency with multiplication facts $\times$ 1, $\times$ 2, $\times$ 5, and $\times$ 10. **(Unit 1)**
- Represent and explain the relationship between multiplication and division. **(Unit 5)**
- Solve multiplication and division word problems and write equations to represent the problems. **(Unit 5)**
- Solve division problems (2-digit number divided by single-digit number). **(Unit 5)**
- Demonstrate fluency with multiplication facts to 10 $\times$ 10. **(Unit 5)**
- Multiply a single-digit number by a multiple of 10, up to 90. **(Unit 5)**
- Solve multi-step problems involving multiplication and addition. **(Unit 5)**
- Solve multiplication and division problems within 100. **(Unit 8)**
- Demonstrate fluency with the division facts. **(Unit 8)**
- Solve multi-step problems involving more than one operation. **(Unit 8)**
- Solve multiplication and division problems involving masses and volumes. **(Unit 8)**

Number and Operations, *continued*

Addition, Subtraction, and the Number System

Students focus on understanding and extending knowledge of place value and the number system and on adding and subtracting fluently within 1,000. Students use their deepening understanding of place value to build and refine strategies for fluently solving addition and subtraction problems with whole numbers. This includes decomposing numbers by place, rounding numbers to the nearest 10 or 100 as a way to gauge the reasonableness of their solutions, and accurately adding and subtracting multiples of 10 and 100.

Students deepen their understanding of the meaning of addition and subtraction and the relationship between the operations through visualizing and solving story problems. Among the types of problems students encounter are adding to, taking from, putting together/taking apart, and comparison problems. Contexts include liquid volume, mass, money, and distance. Problems may involve more than two addends or situations that require more than one step.

In order to solve addition and subtraction problems, students must understand the meaning of the operations and have a good mental model of what is happening in the problem. The ability to visualize what is happening in these different types of story problems is an important foundation for students in understanding the operations and for making good decisions about choosing strategies for computation.

As students solve a variety of addition and subtraction problems, they discuss, refine, and compare strategies. Two addition strategies receive emphasis: 1) breaking apart both numbers by place and adding like places, and 2) breaking apart one number and adding it in parts. Students also examine the strategy of changing the numbers (either changing both numbers and adjusting or creating an equivalent problem) to make them easier to add.

Breaking the numbers apart

Adding by place

$$349 + 175 =$$

Solution 1	Solution 2
$300 + 100 = 400$	$300 + 100 = 400$
$40 + 70 = 110$	$(30 + 70 = 100)$
$9 + 5 = 14$	$10 + 100 = 110$
$400 + 110 + 14 = 524$	$9 + 5 = 14$
	$400 + 110 + 14 = 524$

Adding one number in parts

$$349 + 175 =$$

Solution 1	Solution 2
$349 + 100 = 449$	$349 + 100 = 449$
$449 + 70 = 519$	$449 + 50 = 499$
$519 + 5 = 524$	$499 + 25 = 524$

Changing the numbers

Changing the numbers and adjusting

$$349 + 175 =$$

Solution 1	Solution 2
$350 + 175 = 525$	$349 + 200 = 549$
$525 - 1 = 524$	$549 - 25 = 524$

Creating an equivalent problem

$$349 + 175 =$$

Solution 1	Solution 2
$324 + 200 = 524$	$400 + 124 = 524$

Subtraction strategies that are emphasized are subtracting a number in parts, adding up, and subtracting back.

Subtracting in parts

$251 - 187 =$

Solution 1

$251 - 187 =$

$251 - 100 = 151$

$151 - 80 = 71$

$71 - 7 = 64$

Solution 2

$$\begin{array}{r} 251 \\ -\ 100 \\ \hline 151 \\ -\ 50 \\ \hline 101 \\ -\ 30 \\ \hline 71 \\ -\ 7 \\ \hline 64 \end{array}$$

Adding up or subtracting back

Adding up

$251 - 187 =$

Solution 1

$187 + \underline{13} = 200$

$200 + \underline{51} = 251$

$13 + 51 = 64$

Solution 2

$187 + \underline{20} = 207$

$207 + \underline{40} = 247$

$247 + \underline{4} = 251$

$20 + 40 + 4 = 64$

Subtracting Back

$251 - 187 =$

Solution 1

$251 - \underline{51} = 200$

$200 - \underline{13} = 187$

$51 + 13 = 64$

Solution 2

$$\begin{array}{r} 251 \\ -\ 51 \\ \hline 200 \\ -\ 10 \\ \hline 190 \\ -\ 3 \\ \hline 187 \end{array}$$

$51 + 10 + 3 = 64$

The **Algebra Connections in This Unit** Teacher Notes in Units 3 and 7 show how the associative and commutative properties of addition and the relationship between addition and subtraction are implicit in students' work.

MAIN MATH IDEAS

- Using knowledge of place value to add and subtract
- Adding and subtracting fluently
- Understanding different types of addition and subtraction problems
- Solving problems involving measurement of liquid volume and mass
- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers

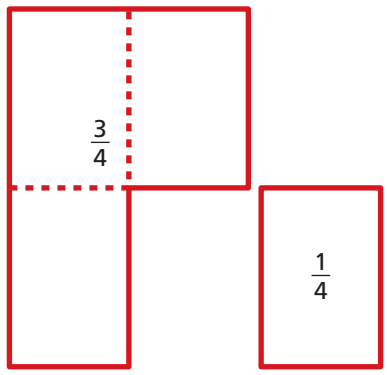
Benchmarks

- Use knowledge of place value to read, write, sequence, and round numbers up to 1,000. **(Unit 3)**
- Solve addition problems with 3-digit numbers (up to 400) by using strategies that involve breaking each number apart by place, or by adding on one number in parts. **(Unit 3)**
- Solve subtraction problems with 2- and 3-digit numbers (up to 300) by using strategies that involve either subtracting one number in parts, adding up, or subtracting back. **(Unit 3)**
- Solve addition and subtraction problems involving masses or volumes. **(Unit 7)**
- Solve 3-digit addition problems using at least one strategy fluently. **(Unit 7)**
- Solve 3-digit subtraction problems fluently. **(Unit 7)**

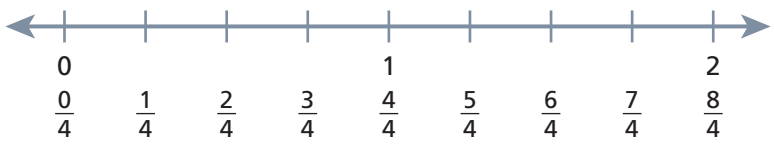
Rational Numbers

Fractions

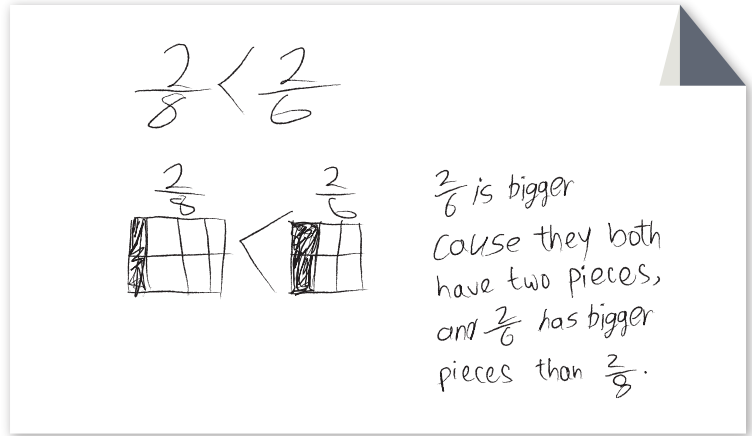
Students extend their understanding of the number system to include fractions. Students learn that when a whole is divided into equal parts, one of those parts is a unit fraction, and they come to recognize multiple equal parts as multiples of a unit fraction. For example, $\frac{1}{4}$ is a unit fraction—it is one part of a whole divided into fourths—and $\frac{3}{4}$ is three $\frac{1}{4}$s. The denominator in a fraction represents the number of equal parts a whole is divided into, and the numerator is the number of parts being used or identified.



Students work with fractions using an area model, representing halves, thirds, fourths, sixths, and eighths with a variety of contexts and representations, such as "brownies" (rectangles of a fixed size), fraction sets (made from sheets of $8\frac{1}{2}$ in. $\times$ 11 in. paper), and pattern blocks. They also represent fractions on number lines, identifying how fractions relate to whole numbers and other fractions, thereby supporting their understanding of fractions as numbers. A story about ants walking city blocks provides a context that helps students think in terms of iterating unit fractions along a number line, naming fractional points between the whole numbers ($\frac{3}{8}$, $\frac{7}{4}$), and also naming whole numbers as fractions ($1 = \frac{2}{2} = \frac{3}{3}$, $2 = \frac{4}{2} = \frac{6}{3}$).



Students compare fractions and reason about their sizes, using both area models and number lines to explain how they know which is greater or if the fractions are equivalent. After comparing a number of fractions, students think through and explain how to determine which fraction is greater when two fractions have the same numerator or denominator. As students represent and compare fractions, they identify a number of common fraction equivalents.



MAIN MATH IDEAS

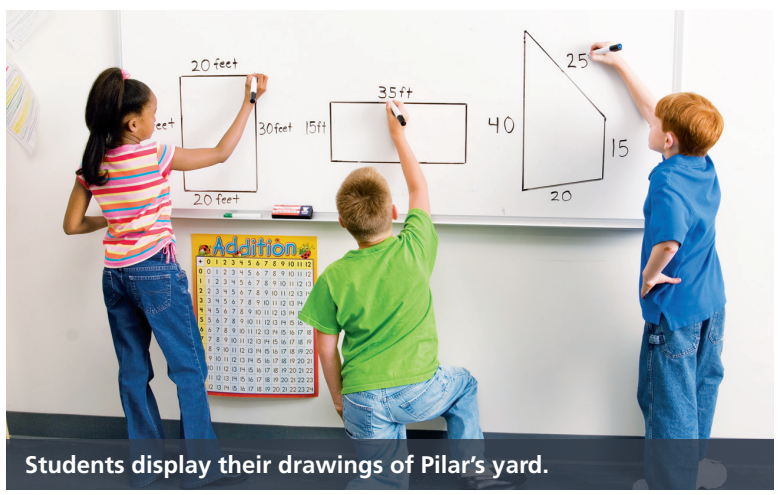
- Understanding the meaning of fractions as equal parts of a whole
- Understanding the meaning of fractions as numbers
- Comparing fractions and reasoning about fraction equivalencies with representations
- Modeling with fraction notation

Benchmarks

- Partition a quantity into equal parts, and name those parts as fractions. **(Unit 6)**
- Represent fractions as numbers on a number line. **(Unit 6)**
- Compare fractions with the same numerator or same denominator by reasoning about their size. **(Unit 6)**
- Identify equivalent fractions. **(Unit 6)**

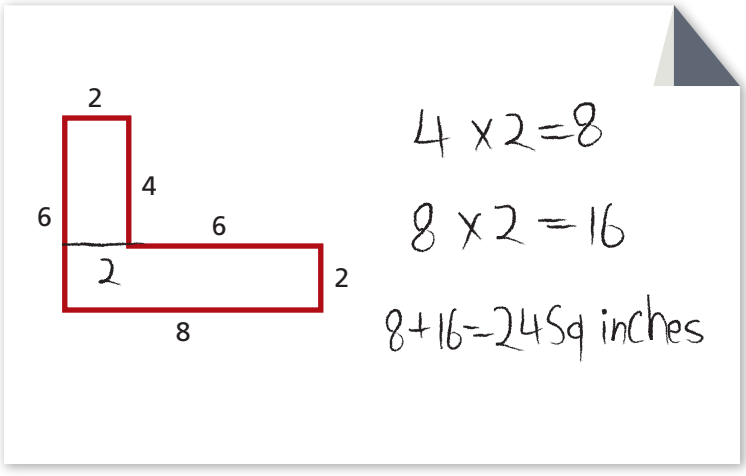
Measurement

The main focus of students' measurement work in Grade 3 is understanding and finding perimeter and area. Perimeter is the distance around the outside edges of a two-dimensional (2-D) shape. Students measure length and calculate perimeter with both U.S. standard units and metric units. They also consider how different shapes can have the same perimeter.



Students display their drawings of Pilar's yard.

Area is the amount of 2-D space a given shape covers and is usually measured in square units. Students first calculate area of rectangles in terms of arrays, skip counting by rows or columns. Then, building on the work they have done with arrays and multiplication, students see that the area of a rectangle can be found by multiplying the length and width. Students also recognize area as additive: a shape can be decomposed and its area found by adding the areas of the parts. Students consider how different shapes can have the same area.



Other measurement work in Grade 3 includes generating linear measurement data to the nearest fourth inch and representing those data on a line plot. Students also work with time, liquid volume, and mass. Their work focuses on using measurement tools accurately (rulers, clocks, graduated cylinders, pan balances), establishing measurement benchmarks, and solving word problems that involve measurement.

MAIN MATH IDEAS

- Generating measurement data
- Understanding and finding perimeter
- Understanding and finding area
- Solving problems involving measurement of liquid volume and mass

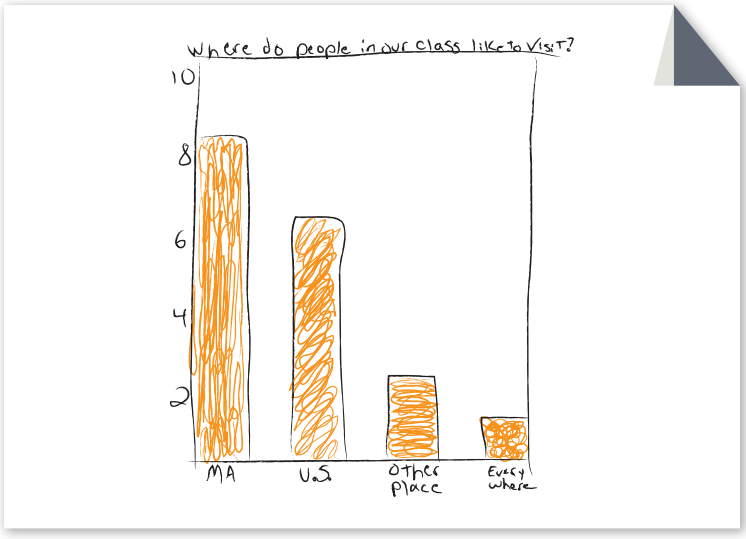
Benchmarks

- Generate measurement data by measuring lengths to the half inch. **(Unit 2)**
- Tell time to the nearest minute. **(Unit 3)**
- Measure and find the perimeter of 2-D figures using U.S. standard and metric units. **(Unit 4)**
- Find the area of 2-D figures using U.S. standard and metric units. **(Unit 4)**
- Measure to the nearest fourth inch and represent measurement data to the nearest fourth inch on a line plot. **(Unit 6)**
- Estimate and measure liquid volume and mass using standard units. **(Unit 7)**
- Find the area of a rectangular array by breaking it apart (using the distributive property). **(Unit 8)**
- Solve multiplication and division problems involving masses and volumes. **(Unit 8)**

Data

Students collect, represent, describe, and interpret both categorical and numerical data. They consider how to examine a data set as a whole and make statements about the whole group. Students also solve "How many more?" and "How many less?" questions about the data.

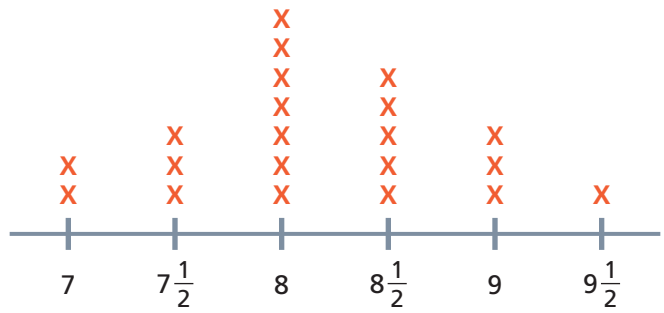
To make sensible statements about a categorical data set that has many different values, students group the data into categories that help them see the data as a whole. They represent sets of categorical data using bar graphs and pictographs. This work includes bar graphs in which the scales have intervals greater than 1 and pictographs in which each picture represents more than one piece of data. When students organize and represent the data in different ways, they are creating different models of the data and can learn different things about the data set.



Paper Airplanes Made by Students	
Dart	
Delta Wing	
Fighter	
Glider	
Flying Wing	

Each = 2 planes

Students use line plots to order numerical data by value so that they can see the shape of the data—where the data are concentrated and where they are spread out; which intervals have many pieces of data and which have very few. They describe what values would be typical or atypical based on the data, and they compare data sets to develop a sense of how data can be useful in describing and comparing characteristics of a group. Students interpret line plots and create their own line plots to represent numerical data, including measurement data in inches and half inches.



Length of Our Feet in Inches

MAIN MATH IDEAS

- Describing, summarizing, and comparing data
- Representing data
- Generating measurement data

Benchmarks

- Organize, represent, and describe categorical data, choosing categories that help make sense of the data. **(Unit 2)**
- Make and interpret a bar graph and a pictograph, including use of scales greater than 1. **(Unit 2)**
- Make a line plot for a set of measurement data, with a scale that includes inches and half inches. **(Unit 2)**
- Describe and summarize a set of data, describing concentrations of data and what those concentrations mean in terms of the situation the data represent. **(Unit 2)**
- Generate measurement data by measuring lengths to the half inch. **(Unit 2)**
- Measure to the nearest fourth inch and represent measurement data to the nearest fourth inch on a line plot. **(Unit 6)**

Geometry

Students study the attributes of 2-D shapes and how these attributes determine their classification. For example, a polygon is classified as a triangle or a quadrilateral based on the number of its sides.

Number of sides	Name of polygon	Examples
3	triangle	
4	quadrilateral	

Students also work with the idea that shapes in different categories may share attributes. Students compare the attributes of quadrilaterals, rectangles, rhombuses, and squares and identify examples and non-examples of these different shapes. Students learn that a given shape may fall into more than one category.

All Quadrilaterals

- Have 4 straight sides
- Are closed shapes
- Have 4 angles
- Have 4 vertices

All Squares

- Have 4 straight sides
- Have 4 equal sides
- Have 4 right angles
- Have 4 vertices
- Are quadrilaterals
- Are rectangles

All Rectangles

- Have 4 straight sides
- Have 4 right angles
- Have 4 vertices
- Have opposite sides that are equal
- Are quadrilaterals

All Rhombuses

- Have 4 straight sides
- Have 4 angles
- Have 4 vertices
- Have 4 equal sides

MAIN MATH IDEA

- Describing and classifying 2-D figures

Benchmark

- Categorize quadrilaterals, including squares, rhombuses, and rectangles, based on their attributes. **(Unit 4)**