# Number and Operations

Multiplication and Division

A major focus of the multiplication and division work in Grade 4 is developing strategies for operating with larger numbers: multiplying two 2-digit numbers and multiplying up to a 4-digit number by a 1-digit number; and dividing with up to a 4-digit dividend. This work builds on different aspects of understanding multiplication and division encountered in Grade 3, such as thinking in terms of equal groups, using arrays and area models to represent multiplication, multiplying by multiples of 10, and breaking multiplication or division problems apart to solve them.

Students continue to build on their understanding of multiplication and division as they solve story problems about equal groups, and they are introduced to, and solve, multiplicative comparison problems. For example, in Unit 1 students are asked to find the length of a python that is 4 times as long as a rattlesnake and to show their solutions using representations and equations.
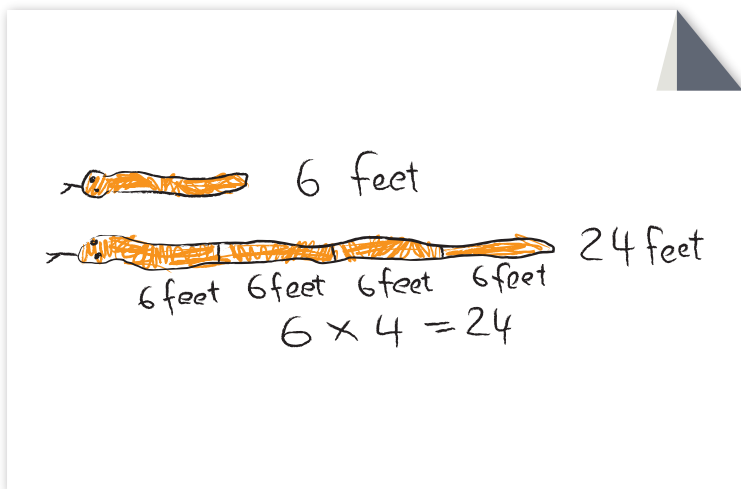
Students also solve division problems where they have to make sense of what to do with a remainder in the context of the problem.


Students find factors and multiples of numbers and identify prime and composite numbers. As students become fluent and flexible in understanding the relationships between numbers and their factors, they are able to apply this knowledge to solving multiplication and division problems.

As students develop and refine strategies for multiplying numbers with two or more digits, being able to visualize how multiplication works is critical in applying the distributive property to solve problems. Students use story contexts, pictures, and unmarked arrays to help them keep track of the parts of the problem as they break the numbers apart or change the numbers.
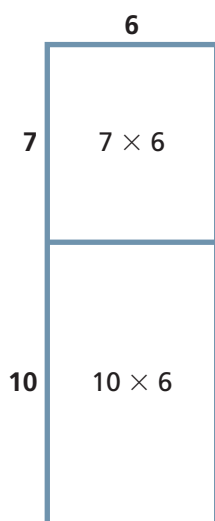
$$17 \times 6 = (10 \times 6) + (7 \times 6)$$

Students' strategies for solving multiplication problems include breaking the numbers apart, changing one factor and adjusting, and creating an equivalent problem.

Breaking the numbers apart by addition

$48 \times 42 =$

Solution 1	Solution 2
$40 \times 40 = 1{,}600$	$48 \times 40 = 1{,}920$
$40 \times 2 = \quad 80$	$48 \times 2 = \quad\ \ 96$
$8 \times 40 = \quad 320$	$\overline{ 2{,}016}$
$8 \times 2 = \quad\ \ 16$	
$\overline{ 2{,}016}$	

Changing one factor and adjusting

$48 \times 42 =$

$$50 \times 42 = 2{,}100$$
$$2 \times 42 = 84$$
$$2{,}100 - 84 = 2{,}016$$

Creating an equivalent problem

$48 \times 42 = 96 \times 21$

$$96 \times 20 = 1{,}920$$
$$96 \times 1 = 96$$
$$1{,}920 + 96 = 2{,}016$$

Students solve division problems with up to 4-digit dividends, including problems with quotients that have a remainder. To solve division problems, students use one of two basic strategies. They either use multiplication to build up groups of the divisor to the dividend, or they use division to break the dividend into parts.

Using groups of the divisor

$460 \div 8 =$

$$50 \times 8 = 400$$
$$460 - 400 = \quad 60$$
$$7 \times 8 = \quad\ \ 56$$
$$60 - 56 = \quad\quad 4$$
$$50 + 7 = \quad\ \ 57$$
$$57R4$$

Breaking the dividend into parts

$460 \div 8 =$

$$400 \div 8 = 50$$
$$\underline{60 \div 8 \text{ is } \ \ 7R4}$$
$$460 \div 8 \text{ is } 57R4$$

Whichever computation strategy students apply for any operation, using multiples of 10, 100, and 1,000 is key to efficiency.

The **Algebra Connections in This Unit** Teacher Notes in Units 1, 3, and 7 show how the distributive property of multiplication and the inverse relationship between multiplication and division are implicit in students' work as they solve problems. These pages also highlight two particular generalizations about multiplication that students work on in Grade 4: a factor of a number is also a factor of that number's multiples; and if one factor in a multiplication expression is halved and another factor is doubled, the product is unchanged.

Number and Operations, *continued*

MAIN MATH IDEAS

- Visualizing multiplication
- Reasoning about numbers and their multiples and factors
- Solving multiplicative comparison problems
- Solving multiplication problems with 2-digit numbers
- Understanding and using the relationship between multiplication and division to solve division problems
- Understanding the meaning and structure of multiplication and division
- Solving measurement problems
- Solving multiplication problems
- Solving division problems

☑ Benchmarks

- Use multiplication to solve multiplicative comparison problems. **(Unit 1)**
- Determine whether numbers up to 100 are prime or composite. **(Unit 1)**
- Find factors of numbers up to 100 and recognize multiples of 1-digit numbers. **(Unit 1)**
- Multiply 2-digit numbers by 1-digit and small 2-digit numbers (e.g., 12, 15, 20), using strategies that involve breaking the numbers apart. **(Unit 3)**
- Solve division problems (2-digit and small 3-digit numbers divided by 1-digit numbers), including some that result in a remainder. **(Unit 3)**
- Multiply a number by a multiple of 10. **(Unit 3)**
- Multiply two 2-digit numbers and up to a 4-digit number by a 1-digit number. **(Unit 7)**
- Solve division problems with up to 4-digit dividends and 1-digit divisors. **(Unit 7)**
- Solve measurement and conversion problems. **(Unit 7)**

Addition, Subtraction, and the Number System

Students extend their knowledge of the base-10 number system up to 1,000,000, examining how the value of any digit is determined by its place and that any digit is ten times as great as the same digit one place to its right. Students read numbers, write them in expanded form, and round them. Special focus is given to the structure of 10,000 as 10 thousands, 100 hundreds, 1,000 tens, and 10,000 ones. Understanding these place-value ideas is fundamental to solving addition and subtraction problems fluently.

Students practice a variety of addition and subtraction strategies, most of which should be familiar from Grade 3. Students discuss, refine, and compare strategies for addition and subtraction, including the U.S. standard algorithms. Students learn the steps of the U.S. standard algorithms for addition and subtraction, discuss the meaning of the notation, and practice the algorithms. By explaining, refining, and comparing the addition and subtraction strategies that they are using, students are adding to the repertoire of strategies they can use for flexible and fluent computation.

Solution 1	Solution 2
	$\overset{1\ 1}{}$
687	687
$+\ 215$	$+\ 215$
$\overline{800}$	$\overline{902}$
90	
$+\ 12$	
$\overline{902}$	

Students work with place value of large numbers, and their study of addition and subtraction strategies are combined as they add and subtract numbers in the thousands and ten thousands.

Students are encouraged to look carefully at the numbers in the problem, and then choose from a variety of strategies they can use confidently, including the U.S. standard algorithms, to solve problems such as the following:

Ursula traveled from San Francisco, California, to Cape Town, South Africa. It is 16,489 kilometers from San Francisco to Cape Town. She then traveled from Cape Town to Tokyo, Japan. It is 14,722 kilometers from Cape Town to Tokyo.

How much farther is it from San Francisco to Cape Town than from Cape Town to Tokyo? Show how you solved the problem.

Solution 1

$$16489 - 14722 = 16467 - 14700$$
$$16467 - 14000 = 2467$$
$$2467 - 700 = 1767 \text{ Kilometers}$$

Solution 2

$$\begin{array}{r} \overset{5\ \ 14}{16,\cancel{4}89} \\ -14,722 \\ \hline 1,767 \text{ km} \end{array}$$

The **Algebra Connections in This Unit** Teacher Note in Unit 5 shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic idea that given an addition expression, if one addend increases by some amount and the other addend decreases by the same amount, the sum is unchanged—a concept students may use to create equivalent addition problems that are easier to solve (e.g., $597 + 375 = 600 + 372$).

MAIN MATH IDEAS

○ Extending knowledge of the number system to 1,000,000

○ Adding and subtracting fluently

○ Describing, analyzing, and comparing strategies for adding and subtracting whole numbers
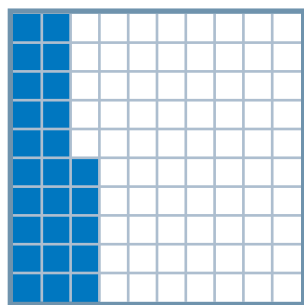
☑ Benchmarks

○ Read, write, and compare numbers up to 1,000,000 and round them to any place. **(Unit 5)**

○ Fluently solve multidigit addition and subtraction problems using a variety of strategies, including the U.S. standard algorithms. **(Unit 5)**

○ Use addition and subtraction to solve word problems involving measurement. **(Unit 5)**

Rational Numbers

Fractions and Decimals

Students build their understanding of the meaning of rational numbers as they work with halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths. They are introduced to decimals and learn that tenths and hundredths can be written with both fraction and decimal notation. Using two different models—an area model (rectangles) and a linear model (number lines)—students visualize rational numbers and see how the numbers are related to each other. They see that in a given context the same fraction can represent different quantities ($\frac{1}{2}$ of 24 square units is 12 square units; $\frac{1}{2}$ of 60 square units is 30 square units), but that *as a number*, a fraction always has the same relationship to 1 and to other numbers.

Students represent tenths and hundredths on rectangles divided into hundredths, name them as fractions and decimals, and recognize that quantities expressed as tenths can also be expressed as hundredths: 0.1 (or $\frac{1}{10}$) is equal to 0.10 (or $\frac{10}{100}$).


$$0.25 \qquad \frac{25}{100}$$

Students compare and order fractions using representations, their knowledge of fraction equivalents, and relationships to benchmarks such as $\frac{1}{2}$ and 1. Students compare decimals by representing them on rectangles divided into tenths and hundredths, and by placing them on a number line.

Less than one half	One half	Between one half and one whole	One whole	More than one whole
$\frac{1}{4}, \frac{1}{5}, \frac{2}{5},$ $\frac{3}{10}, \frac{2}{6}$	$\frac{3}{6}, \frac{2}{4}, \frac{4}{8},$ $\frac{5}{10}, \frac{6}{12}$	$\frac{2}{3}, \frac{3}{5}, \frac{5}{8},$ $\frac{9}{10}, \frac{4}{5}$	$\frac{5}{5}, \frac{2}{2}, \frac{3}{3},$ $\frac{10}{10}, \frac{8}{8}$	$\frac{5}{2}, \frac{6}{3}, \frac{10}{5},$ $\frac{3}{2}, \frac{6}{5}$

As students represent, compare, and order numbers, they find equivalent fractions or decimals, explain how they know the numbers are equivalent, and discuss how to generate equivalent fractions.

Students begin computing with rational numbers. They add and subtract fractions and mixed numbers (mostly with like denominators), add tenths and hundredths, and multiply whole numbers by fractions. In order to extend what they know about operations with whole numbers and to make sense of operations with rational numbers, students use contexts and representations such as rectangles (an area model) and number lines (a linear model), and they discuss what they understand about these operations and numbers.
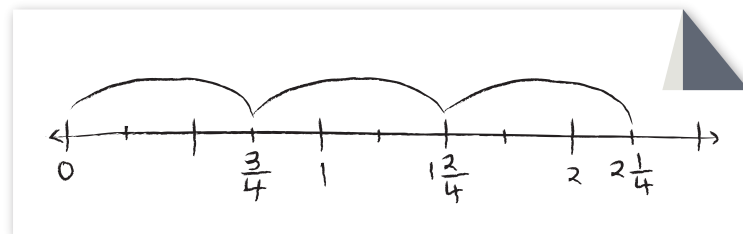
$$3 \times \frac{3}{4} = 2\frac{1}{4}$$

MAIN MATH IDEAS

- Understanding the meaning of fractions and decimals
- Comparing the values of fractions and decimals
- Computing with rational numbers

☑ Benchmarks

- Identify equivalent fractions and explain why they are equivalent. **(Unit 6)**
- Compare fractions with like and unlike denominators. **(Unit 6)**
- Add and subtract fractions and mixed numbers with like denominators. **(Unit 6)**
- Multiply a fraction by a whole number. **(Unit 6)**
- Read, write, and compare decimals in tenths and hundredths. **(Unit 6)**
- Add tenths and hundredths. **(Unit 6)**
- Represent data on a line plot including fourths and eighths. **(Unit 6)**

Analyzing Patterns and Rules

Students model mathematical relationships associated with two different contexts (Penny Jars and Windows and Towers) where two quantities are related in predictable ways. To show how the quantities in these contexts are related, students create their own pictures and learn to use tables and symbolic notation. These contexts, which have additive and multiplicative components, help students distinguish between additive and multiplicative relationships. Within each context, students solve multi-step problems that involve both operations.

Students generate number patterns in the contexts of Penny Jars and Windows and Towers. They learn to analyze these patterns, and recognize how the additive part of the context (the starting amount) and the multiplicative part (the regular increase) define the relationship between the two quantities.

Using words and symbolic notation, students develop rules for finding the total number of pennies after any round in a Penny Jar situation, and for finding the number of windows in a tower of a certain height. For example, students write a rule for a Penny Jar that has a starting amount of 1, and 3 pennies are added each day.

> **Rules for Penny Jar A**
>
> Number of rounds times 3. Add 1.
>
> Start with 1. Add on a 3 for each round.
>
> $(3 \times N) + 1$

MAIN MATH IDEAS

○ Modeling situations with mathematics

○ Generating and analyzing patterns

○ Solving multi-step problems

☑ Benchmarks

○ Generate a number pattern that follows a given rule and analyze features of the pattern in order to solve problems. **(Unit 8)**

○ Model the mathematics of a situation with tables and with mathematical notation, including using letters to represent unspecified quantities. **(Unit 8)**

○ Solve multi-step word problems using the four operations. **(Unit 8)**

Penny Jar Table

Number of Rounds	Total Number of Pennies
Start	6
1	11
2	16
3	21
4	26
5	31

Windows and Towers Table

| Number of Floors	Total Number of Windows
Start (skylights)	4
1	12
2	20
3	28
4	36
5	44

Measurement

Students continue to build on measurement work from earlier grades, which includes linear measurement, area, angle measurement, and measurement conversions.

Students deepen their understanding of perimeter as the distance around a 2-D shape by finding the perimeter of the 2-D faces of objects around their classroom. Students also determine generalizable methods for finding the perimeter of rectangles.

> side 1 length + side 2 length + side 3 length + side 4 length
>
> 2 × long side length + 2 × short side length
>
> (short side + long side) × 2

Students deepen their understanding of area as the amount of space a shape covers by finding the area of different polygons by using symmetry and non-standard units. Based on their work with arrays in their study of multiplication, students explain and apply a generalizable method for finding the area of a rectangle (multiplying the length by the width).

Students learn that angles are measured in terms of an amount of turn or rotation that is part of a circular arc and are measured in degrees. Students also find that angle measure is additive. For instance, if three of the same angles fit together to form a right angle, then each of the smaller angles must be $\frac{1}{3}$ of 90°, or 30° (30° + 30° + 30° = 90°). Using this additive property, students combine and decompose angles whose measures they know, in order to make new angles or to find the measure of other angles. Students also learn to measure angles using a protractor. They choose which scale on the protractor to use by recognizing whether the angle is acute or obtuse.
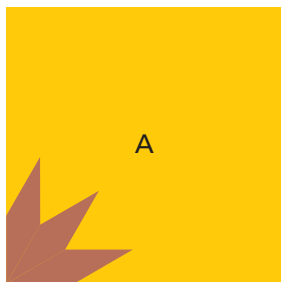
STUDENTS MIGHT SAY

"All of these small angles are the same size so they each have to be 30 to add up to 90 degrees."

Students generate measurement data by measuring lengths to the nearest $\frac{1}{4}$ inch and display their data on a line plot. They use these data to solve addition and subtraction measurement problems involving fractions. See the **Data** and **Rational Numbers** sections for more information.

Students learn measurement equivalents within a single measurement system and convert measurements (from a larger unit to a smaller unit) using these equivalents. They solve word problems involving time, money, length, weight, mass, liquid volume, and capacity, including multi-step problems involving more than one operation.

MAIN MATH IDEAS

- Generating measurement data
- Solving measurement problems
- Describing and measuring angles
- Understanding and determining area
- Solving measurement problems

☑ Benchmarks

- Convert linear measurements from a larger unit to a smaller unit. **(Unit 4)**
- Determine the perimeter and area of rectangles, including using generalizable methods. **(Unit 4)**
- Add or subtract angles to determine the size of angles. **(Unit 4)**
- Use a protractor to measure angles and sketch angles of specific sizes. **(Unit 4)**
- Use addition and subtraction to solve word problems involving measurement. **(Unit 5)**
- Solve measurement and conversion problems. **(Unit 7)**

Data

Students model real-world situations with mathematics as they collect, represent, describe, and analyze data in order to compare two groups. They generate measurement data about two groups that include fractions and represent the two data sets. Students first choose their own way to represent the data sets, and then they use line plots.

Group 3: Time Balanced on Right Foot (seconds)

Group 3: Time Balanced on Left Foot (seconds)
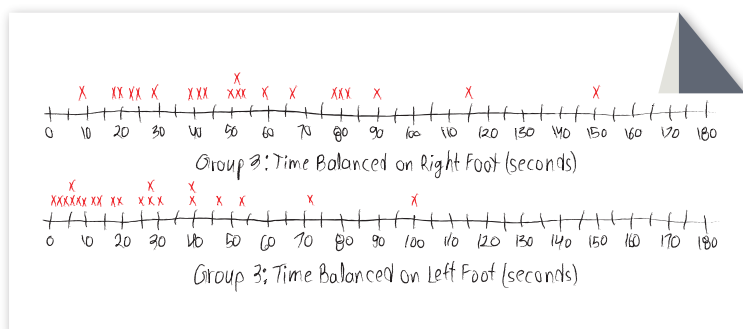
Based on their representations, students describe and summarize the data—choosing the most important features that characterize a data set—and compare how the two sets are similar or different. They return to the context from which the data were taken to draw conclusions about the phenomenon, and refer to the data to defend their claims.

Philadelphia is hotter because in Philadelphia they have 13 days 90° or above. In Boston they have 3 in 90° or above. Also the high in Philadelphia is 95 and in Boston is 93.

MAIN MATH IDEAS

○ Representing data

○ Describing, summarizing, and comparing data

○ Analyzing and interpreting data

○ Generating measurement data

☑ Benchmarks

○ Use a line plot to organize, represent, and analyze measurement data about two groups in order to compare the two groups. **(Unit 2)**

○ Design a data question that involves measurement to compare two groups. **(Unit 2)**

○ Use a line plot to represent measurement data that includes fractions. **(Unit 2)**

○ Represent data on a line plot including fourths and eighths. **(Unit 6)**

Geometry

Students define polygons as closed shapes with straight sides and become familiar with a polygon's components—line segments, points, vertices, and angles. They study different attributes and consider how those attributes determine classifications. Polygons can be classified by their number of sides: triangles have 3 sides, quadrilaterals have 4 sides, pentagons have 5 sides, and so on. Triangles can be classified by the size of their angles (acute, obtuse, or right).


Students classify quadrilaterals by such features as the relative length of sides (rhombuses have four equal sides), the number of pairs of parallel sides (parallelograms have two pairs of parallel sides and trapezoids have one pair), and the number of pairs of perpendicular sides (the number of right angles.)

Students' work with these geometric shapes is essential for work they do in measurement—finding angle size through reasoning and learning to use a protractor—and in finding perimeter and area of rectangles. See the **Measurement** section for more information.

In their work with polygons, students learn about mirror symmetry. They identify lines of symmetry and use lines of symmetry to help them determine the area of shapes.
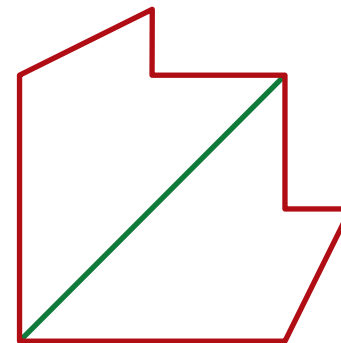
MAIN MATH IDEAS

○ Describing and classifying 2-dimensional figures

○ Identifying mirror symmetry in shapes

☑ Benchmarks

○ Draw and identify lines and angles, including parallel and perpendicular lines, and classify polygons by properties of their sides and angles. **(Unit 4)**

○ Identify lines of symmetry in polygons. **(Unit 4)**