Number and Operations

Multiplication and Division

Students consolidate their computational strategies for multiplication, building upon what they learned in Grades 3 and 4. Representations and story contexts help students to connect these strategies to the meaning of multiplication. By the end of Grade 5, all students should be able to carry out strategies based on the distributive property of multiplication over addition: specifically, those strategies that involve breaking one or both factors apart, multiplying each part of one factor by each part of the other factor, and then combining the partial products. They also practice notating their solutions clearly. Students learn the steps of the U.S. standard algorithm for multiplication, discuss the meaning of its notation, and practice using it.

Some students may also use other strategies, such as changing one factor and adjusting, or creating an equivalent problem by multiplying one factor by some number and dividing the other factor by the same number. As they solve computation problems with larger numbers, students are expected to use mental arithmetic, estimation (to determine if a result is reasonable), and a sound understanding of the operation of multiplication.

Examples of multiplication strategies

Breaking numbers apart by addition

Solution 1

148	
× 42	
4,000	40 × 100
1,600	40 × 40
320	40 × 8
200	2 × 100
80	2 × 40
16	2 × 8
6,216	

Solution 2

```
  1 3
   1
  148
×  42
  296
5,920
6,216
```

Solution 3

$148 \times 42 =$

$100 \times 42 = 4{,}200$

$48 \times 40 = 1{,}920$

$48 \times 2 = 96$

$4{,}200 + 1{,}920 + 96 = 6{,}216$

Changing one factor and adjusting

$148 \times 42 =$

$150 \times 42 = 6{,}300$ $(100 \times 42 + \frac{1}{2}$ of $100 \times 42)$

$2 \times 42 = 84$

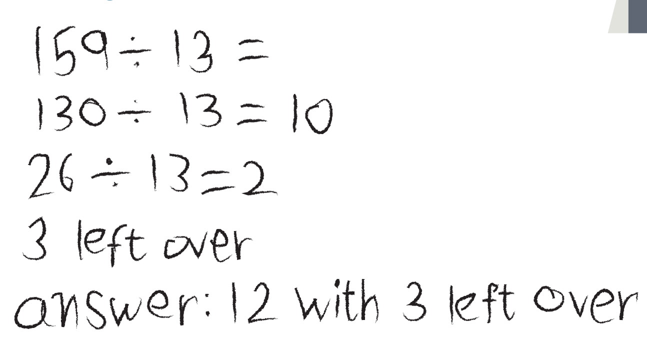
$6{,}300 - 84 = 6{,}216$

Students continue to develop ways to solve division problems fluently, focusing on the relationship between multiplication and division. They usually solve division problems either by building up groups of the divisor, or by breaking the dividend into convenient parts. As students refine their computation strategies for division, they find ways to use what they already know and understand well (familiar factor pairs, multiples of 10, relationships between numbers, etc.) to break apart the harder problems into easier problems. They also work on notating their solutions clearly and concisely.

Examples of division strategies for 159 ÷ 13

Using groups of the divisor

10 × 13 = 130
159 − 130 = 29
2 × 13 = 26
29 − 26 = 3
2 + 10 = 12
12 R3



Breaking the dividend into parts

Students are introduced to the order of operations and practice writing and interpreting expressions with grouping symbols (parentheses and brackets).

The **Algebra Connections in This Unit** Teacher Notes in Units 1 and 4 show how the distributive property and the relationship between multiplication and division are implicit in students' solution strategies. These pages also highlight what happens to the product when a factor is doubled, and how students use this idea when solving division problems.

MAIN MATH IDEAS

- Solving multiplication problems with 2-digit numbers
- Understanding and using the relationship between multiplication and division to solve division problems
- Writing and interpreting numerical expressions
- Solving multiplication problems fluently
- Solving division problems efficiently

Benchmarks

- Solve 2-digit by 2-digit multiplication problems efficiently. **(Unit 1)**
- Solve division problems with 1-digit and 2-digit divisors. **(Unit 1)**
- Use order of operations to solve computation problems. **(Unit 1)**
- Fluently solve multidigit multiplication problems using a variety of strategies including the U.S. standard algorithm. **(Unit 4)**
- Solve division problems with up to 4-digit dividends and 2-digit divisors efficiently. **(Unit 4)**

Rational Numbers

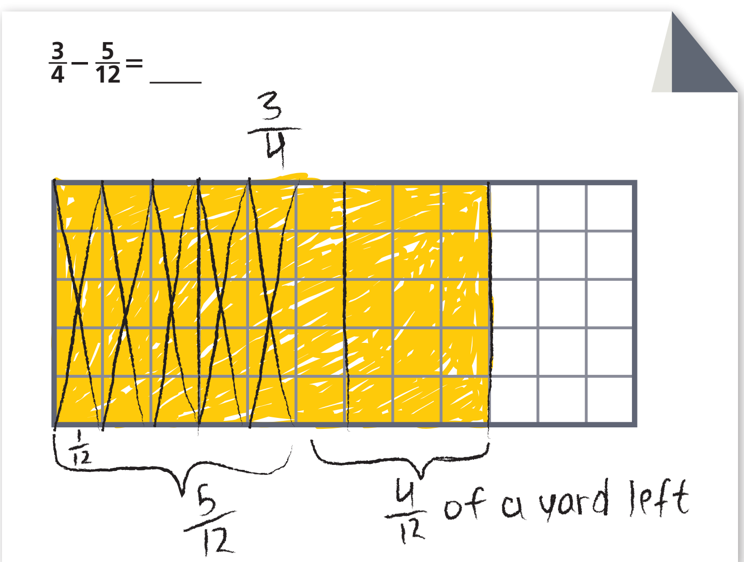
In Grade 5, there are three units in the Rational Numbers strand that focus on further developing and expanding students' understanding of the meanings of fractions and decimals, on the relationship between them, and on doing computation with all four operations. Students use various contexts and representations to compare fractions or decimals and to find equivalents. As students solve problems with fractions and decimals, they work on making sense of the magnitude of the numbers and of the operations of addition, subtraction, multiplication, and division. Their work with representations develops their number sense and supports their work with computation. This includes considering the reasonableness of their answers.

Addition and Subtraction

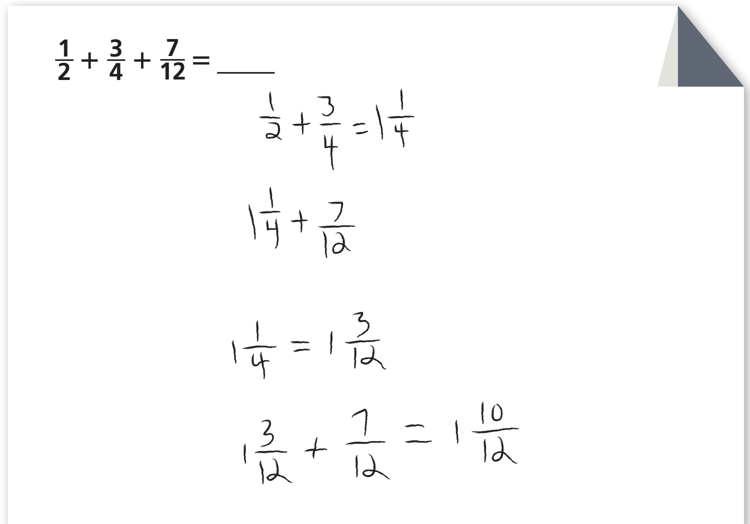
Fractions

Students compare fractions, identify equivalent fractions, and add and subtract fractions and mixed numbers. They compare fractions using fraction equivalents and the relationship of fractions to landmarks, such as $\frac{1}{2}$, 1, and 2, to decide which of two fractions is greater. Understanding equivalent fractions and how to find them is the basis for the approaches students develop for adding and subtracting fractions with unlike denominators.

Students use representations such as rectangles, rotations on a clock, and a number line, to visualize and reason about fraction equivalents and relationships. They use these same representations as the basis of students' initial work on adding and subtracting fractions.



Based on what they understand about adding and subtracting whole numbers, as well as what they see in their representations, students develop procedures for adding and subtracting fractions and mixed numbers without the use of representations. Their procedures involve finding equivalent fractions with common denominators and adding or subtracting the numerators.



MAIN MATH IDEAS

- Finding equivalents and comparing fractions
- Adding and subtracting fractions

Benchmarks

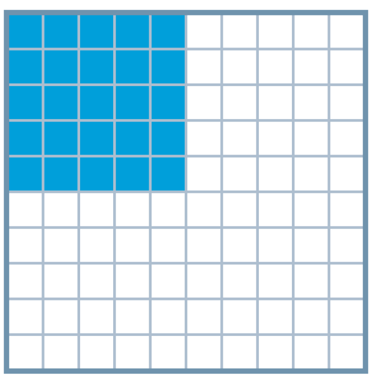
- Add fractions with unlike denominators. **(Unit 3)**
- Subtract fractions with unlike denominators. **(Unit 3)**
- Represent data including fractions on a line plot and solve addition and subtraction problems about the data. **(Unit 3)**

Decimals

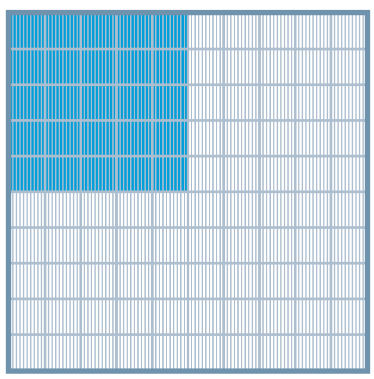
Students represent, write, read, compare, and add and subtract decimals to thousandths.

Students represent tenths, hundredths, and thousandths on hundredths or thousandths grids (rectangles divided into 100 or 1,000 squares) and on number lines. These two forms of representation, and connecting them to fraction equivalents, allow students to visualize the relationships among decimals and to see how these numbers extend the structure of our base-10 number system to include places with values less than 1. Students also use hundredths grids to help them compare decimals and to see equivalencies among tenths, hundredths, and thousandths. Doing this work with representations is an important foundation as students begin to add and subtract decimals.

As students represent decimals on the grids, they also write them with numerals. Students learn how to interpret the zeroes in decimals and learn that the way decimal numbers are said and written is related to the meaning of the numbers.



Fractions: $\frac{25}{100}$ or $\frac{1}{4}$
Decimal: 0.25



Fractions: $\frac{250}{1000}$ or $\frac{1}{4}$
Decimals: 0.250 or 0.25

To add and subtract decimals, students apply their understanding of the operations of addition and subtraction with whole numbers, their understanding of place value, and their understanding of decimals. Students begin by representing the numbers on the hundredths grids in order to visualize the operation with decimals and to help them carefully identify the values of the digits in each number.

As students internalize the images, they move to adding and subtracting decimals without using representations, applying strategies similar to those they use with whole numbers, such as keeping one number whole and adding or subtracting the other number in parts.

2 In Darston it rained 2.26 inches on Monday and 0.33 inch on Tuesday. How much more did it rain on Monday than on Tuesday?

.33 + .07 = .4
.4 + .6 = 1
1 + 1.26 = 2.26
.07 + .6 + 1.26
1.26 + .6 = 1.86
1.86 + .07 = 1.93 inches

MAIN MATH IDEAS

- Understanding the meaning of decimals
- Comparing decimals
- Adding and subtracting decimals

Benchmarks

- Write, compare, and round decimals to thousandths. **(Unit 6)**
- Add and subtract decimals. **(Unit 6)**

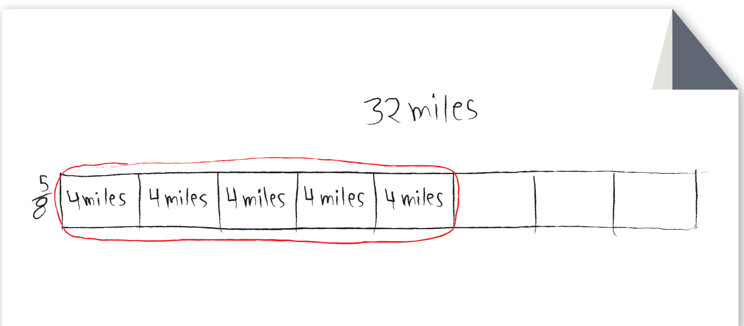
Rational Numbers, *continued*

Multiplication and Division

Fractions

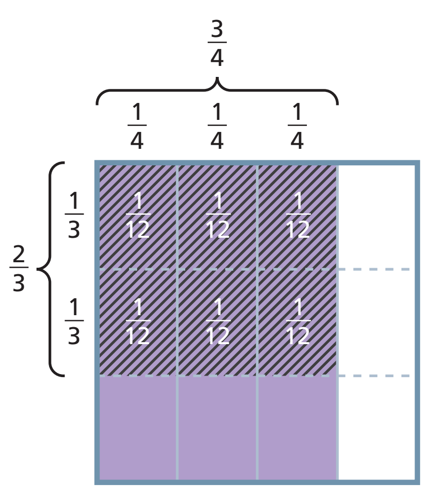
Students build on their understanding of the operations of multiplication and division and their understanding of fractions to multiply and divide with fractions. The emphasis is on using contexts and representations to solve multiplication and division problems that involve fractions and mixed numbers, and on helping students expand their understanding of multiplication and division to include fractions.

On Tuesday, Margaret biked $\frac{5}{8}$ of a 32-mile bike path. How many miles did she bike?



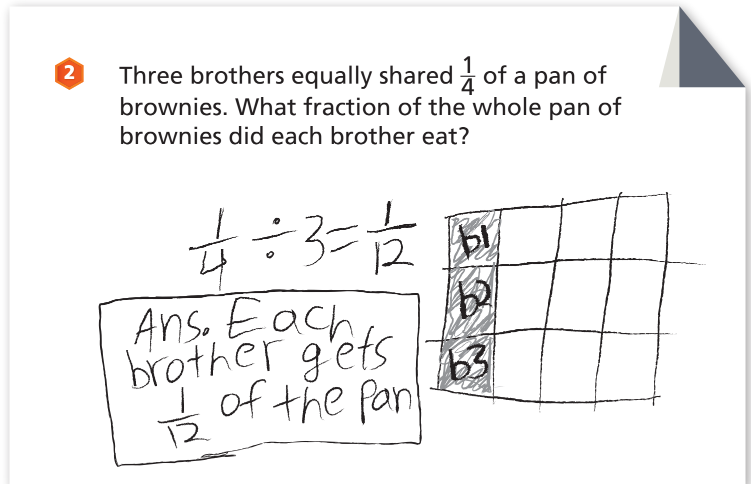
$\frac{5}{8} \times 32 = 20$

Students solve problems that involve multiplying a whole number by a fraction or a mixed number, as well as problems that involve multiplying two fractions. They use representations (especially fraction bars and arrays) and equations to relate problems with fractions or mixed numbers to whole-number problems situated in the same context. Study of the relationship between the size of the factors and the size of the product—when a problem includes fractions less than 1, fractions greater than 1, or mixed numbers—helps students evaluate the reasonableness of their answers.



$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

Division problems are limited to dividing a whole number by a unit fraction and dividing a unit fraction by a whole number. To solve problems that involve dividing a whole number by a unit fraction, students draw on their understanding of division as how many of a number fit into another number. Solving problems that involve dividing a unit fraction by a whole number fits more readily into students' ideas of breaking the fraction into equal parts. Students represent these problems with equations and solve them by drawing representations.



Students are also introduced to the idea that fraction notation ($\frac{6}{5}$) can also be interpreted as division notation ($6 \div 5$).

MAIN MATH IDEAS

- Multiplying and dividing fractions, mixed numbers, and whole numbers
- Interpreting fractions as division
- Converting measurements

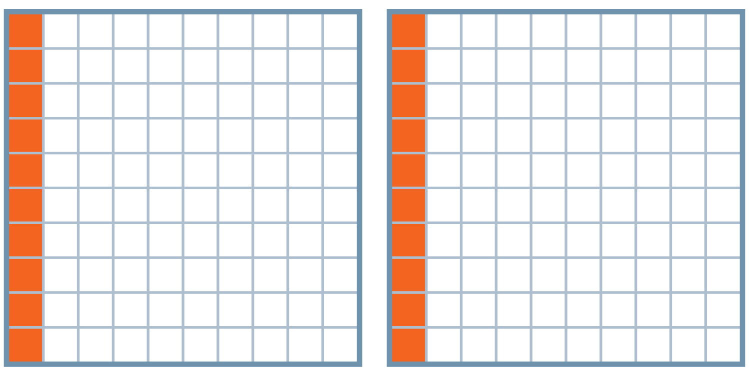
Benchmarks

- Multiply fractions, mixed numbers, and whole numbers. **(Unit 7)**
- Compare the size of the factors and the size of the product and explain their relationship. **(Unit 7)**
- Divide a unit fraction by a whole number and a whole number by a unit fraction. **(Unit 7)**
- Solve division problems with two whole numbers resulting in a fraction or a mixed number. **(Unit 7)**
- Solve measurement conversion problems including multi-step word problems. **(Unit 7)**

Decimals

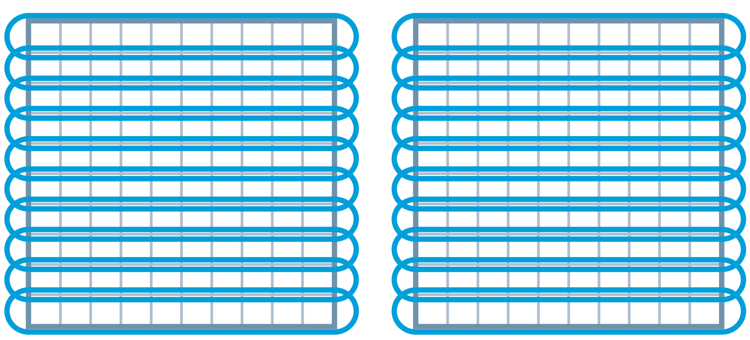
Students extend their understandings of place value and whole-number multiplication and division to solve multiplication and division problems with decimals. Similar to their work with fractions, as students multiply and divide with decimals, they realize that multiplication does not always make things "bigger," and division does not always make things "smaller."

Students use hundredths grids and number lines to make sense of what is happening when they multiply or divide with numbers less than 1. Similar to whole number multiplication, when students represent multiplying 2×0.1 or 2×0.01, they make 2 copies of one-tenth or of one-hundredth.



$2 \times 0.1 = 0.2$

When dividing $2 \div 0.1$ or $2 \div 0.01$, students find how many one-tenths or one-hundredths there are in 2.



$2 \div 0.1 = 20$
$20 \times 0.1 = 2$

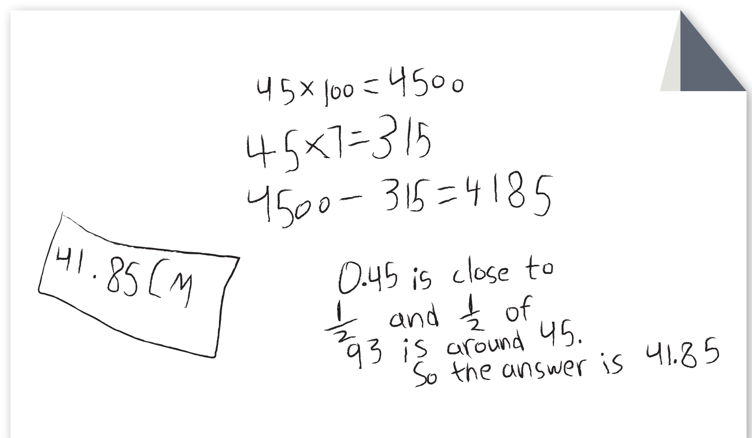
Students solve a series of problems where they multiply and divide whole numbers by powers of 10 (0.01, 0.1, 1, 10, 100) to examine and discuss the patterns they see in the location of the decimal point, how the magnitude of the answer changes, and how the place value of digits in the problems change.

As students work with these representations and patterns, they more easily recognize place-value relationships—that a digit in one place represents 10 times as much as it represents in the place to its right, and $\frac{1}{10}$ of what it represents in the place to its left.

Students solve multiplication and division problems involving decimals mostly by operating on the numbers as if they were whole numbers, and then reasoning about the magnitude of the answer to determine the correct product or quotient. By using this method to multiply and divide with decimals, students apply their understanding of place value, the magnitude of numbers, and the operations of multiplication and division to solve problems.

See the **Measurement and Data** section for more information on how students use knowledge of multiplication and division of fractions and decimals to solve measurement conversion problems.

The Flickerbill makes jumps of 0.45 centimeter.
How far does the Flickerbill go if it makes 93 jumps?



MAIN MATH IDEAS

- Multiplying with decimals
- Dividing with decimals
- Converting measurements

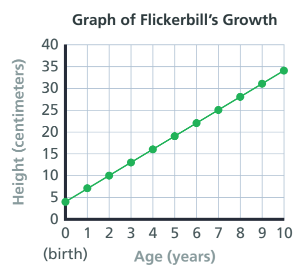
Benchmarks

- Recognize and use place-value relationships to explain patterns when multiplying or dividing by powers of 10 including placement of the decimal point. **(Unit 7)**
- Multiply and divide decimals to hundredths. **(Unit 7)**
- Solve measurement conversion problems including multi-step word problems. **(Unit 7)**

Analyzing Patterns and Rules

Students use coordinate graphs, tables, and symbolic notation to model real-world and mathematical situations. They analyze arithmetic patterns in tables and the shapes of the corresponding graphs to describe and compare the situations.

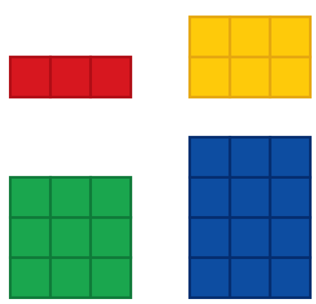
Age	Height (cm)
0 (birth)	4
1	7
2	10
3	13
4	16
5	19
6	22
7	25
8	28
9	31
10	34



Students work with situations in which one quantity varies in relation to another. Some situations are based on data, such as temperature changing over time, or height varying with age. Other situations are based on rules that determine the correspondence of two quantities, such as the heights of fictional animals that grow a certain amount each year, or geometric relationships, including how the area of a square changes in relation to the length of a side. For example, students use tables and graphs to consider how the perimeters change in a set of rectangles that are made from rows of tiles with three squares in each row.

Perimeter for 3-Across Rectangles

Number of Rows (n)	Perimeter (cm)
1	8
2	10
3	12
4	14
5	16
6	18
10	
15	
20	
100	
n	



As a class, they use words to express variations of a general rule for the perimeter of a rectangle, no matter how many rows of 3 squares it contains. For each variation, they represent the rule using symbolic notation. In this way, students become familiar with equivalent expressions with variables.

Double 3, double the number of rows, and add all that together.
$(2 \times 3) + (2 \times n)$
Double the number of rows, and then add 6.
$2 \times n + 6$
Add 3 to the number of rows, and then double that.
$(3 + n) \times 2$

Throughout this work, students move among the situation, tables, graphs, equations, expression of the rule in words, and other representations they have created. Their work with symbolic notation is closely connected to the context with which they are working. By moving back and forth across the context, the variety of representations, their own ways of describing general rules in words, and symbolic notation, students learn how notation with variables carries mathematical meaning.

For students to be able to create and use coordinate graphs, they must understand the structure and conventions of using a coordinate grid. See the **Geometry** section for more information about coordinate grids.

MAIN MATH IDEAS

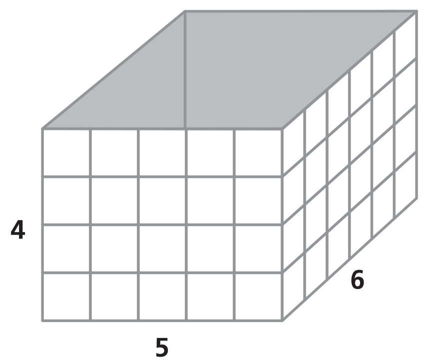
- Reading and constructing coordinate graphs
- Modeling situations with mathematics: graphs, ordered pairs, tables, and symbolic notation
- Analyzing and comparing mathematical patterns and relationships
- Analyzing numerical patterns in the perimeters and areas of related rectangles

Benchmarks

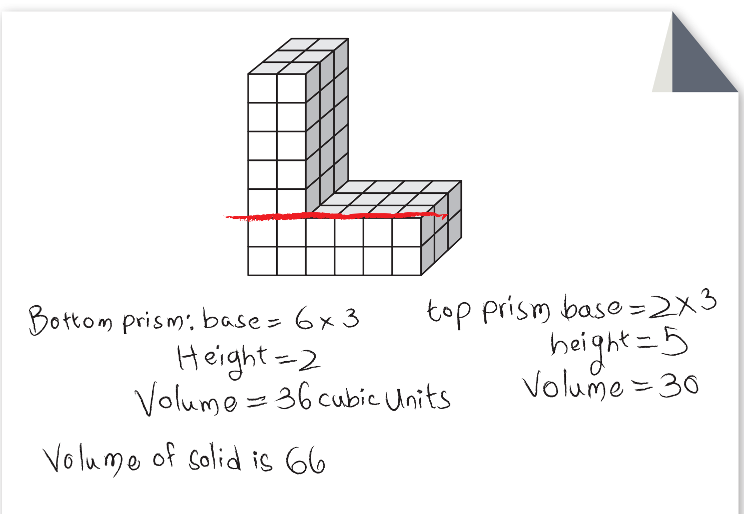
- Use tables to record ordered pairs and construct coordinate graphs to represent the relationship between *x*-coordinates and *y*-coordinates. **(Unit 5)**
- Determine what values are represented by points on a coordinate grid. **(Unit 5)**
- Represent real world and mathematical problems by graphing points in the coordinate plane and interpret the graph in the context of the situation. **(Unit 5)**
- Use tables and graphs to compare two situations governed by rules that generate numerical patterns. **(Unit 5)**
- Identify and explain numerical patterns when comparing perimeters or areas of related rectangles. **(Unit 8)**

Measurement and Data

A major focus of measurement in Grade 5 involves the structure and volume of three-dimensional (3-D) shapes: specifically, rectangular prisms and solids composed of rectangular prisms. Students create and determine the volume of boxes made from two-dimensional (2-D) patterns and create box patterns to hold a given number of cubes. By doing this work, students learn that the volume of a solid is the space that the solid occupies and can be measured in cubic units. They learn how the volume of a rectangular prism can be found by multiplying length by width by height, or multiplying the area of the base of the prism (length × width) by the height.

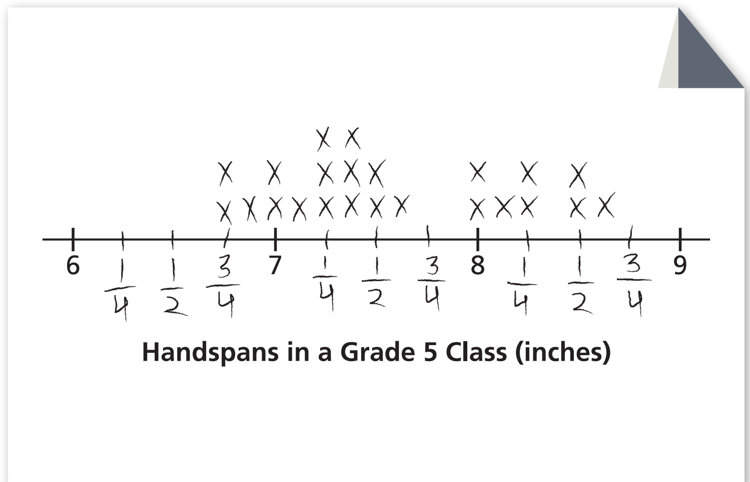


Students also learn that volume is additive. In activities with solids composed of rectangular prisms, students must be able to visualize the prisms that make up the solid, find the volume of each prism, and add these volumes to find the volume of the whole solid.



In the context of their work with multiplication and division, students apply what they have learned about multiplying and dividing rational numbers to convert measurements within a given measurement system, including solving multi-step word problems. Building on measurement ideas from earlier grades, students recognize that converting a larger unit of measure to a smaller unit of measure (e.g., meters to centimeters, yards to feet) results in more of the smaller units; and converting from a smaller unit of measure to a larger unit of measure (e.g., grams to kilograms, quarts to gallons) results in fewer of the larger units.

Students measure lengths to the nearest $\frac{1}{4}$ inch and display their data on a line plot. They solve problems about these lengths.



MAIN MATH IDEAS

- Translating between two-dimensional and three-dimensional shapes
- Structuring rectangular prisms and determining their volume
- Converting measurements

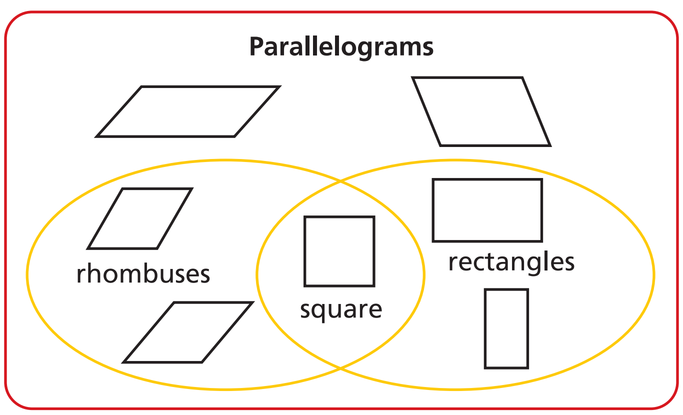
Benchmarks

- Find the volume of rectangular prisms, including the use of volume formulas. **(Unit 2)**
- Find the volume of a solid composed of two rectangular prisms. **(Unit 2)**
- Use standard units to measure volume. **(Unit 2)**
- Represent data including fractions on a line plot and solve addition and subtraction problems about the data. **(Unit 3)**
- Solve measurement conversion problems including multi-step word problems. **(Unit 7)**

Geometry

Students sort shapes based on attributes such as number of sides, relative lengths of sides, and sizes of angles. By finding shapes that share certain attributes, students determine and name properties and categories of triangles and quadrilaterals.

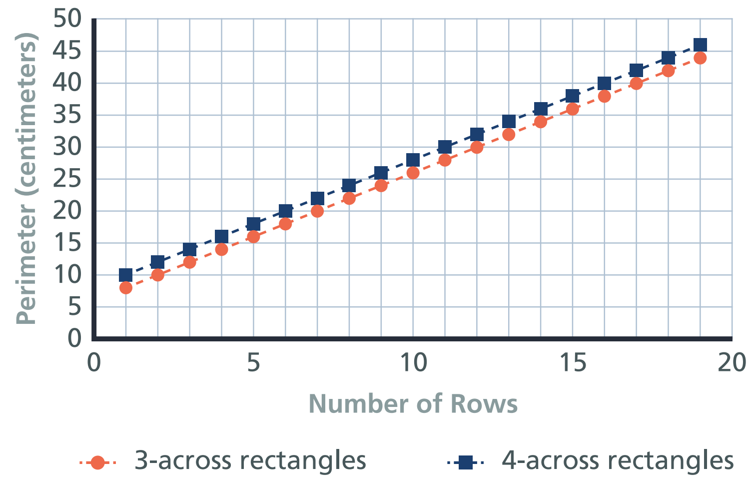
Students learn that some triangles or quadrilaterals fit more than one category as they consider problems and questions such as: *Draw a rectangle. Is what you drew a parallelogram? Are all squares rectangles? Are all rectangles parallelograms? Can a scalene triangle also be a right triangle?* Students make representations of the relationships among the different types of quadrilaterals, and see that not only can a figure belong to multiple categories, but that categories are subcategories of other categories. For example, squares are a subcategory of rectangles, which are a subcategory of parallelograms, which are a subcategory of quadrilaterals. The categories of rhombuses and rectangles overlap, but neither is a subcategory of the other.



In Unit 5, students use coordinate graphs to represent the correspondence between two quantities. Students learn that the two values represented by a point on a coordinate grid can be shown as an ordered pair—and that the order of the numbers matter: the first number is the value of the *x*-coordinate (the horizontal axis), and the second number is the value of the *y*-coordinate (the vertical axis).

Students plot ordered pairs as points on a grid using their *x*- and *y*- coordinates. See the **Analyzing Patterns and Rules** section for more information on how students use coordinate graphs, tables, and symbolic notation to solve mathematical and real-world problems.

Perimeter of 3-Across and 4-Across Rectangles



MAIN MATH IDEAS

- Classifying two-dimensional figures
- Finding perimeter and area of related rectangles
- Modeling situations with mathematics: graphs, ordered pairs, tables, and symbolic notation

Benchmarks

- Use tables to record ordered pairs and construct coordinate graphs to represent the relationship between *x*-coordinates and *y*-coordinates. **(Unit 5)**
- Classify polygons by their attributes, and know that some quadrilaterals can be classified in more than one way. **(Unit 8)**
- Identify and explain numerical patterns when comparing perimeters or areas of related rectangles. **(Unit 8)**