Number and Operations

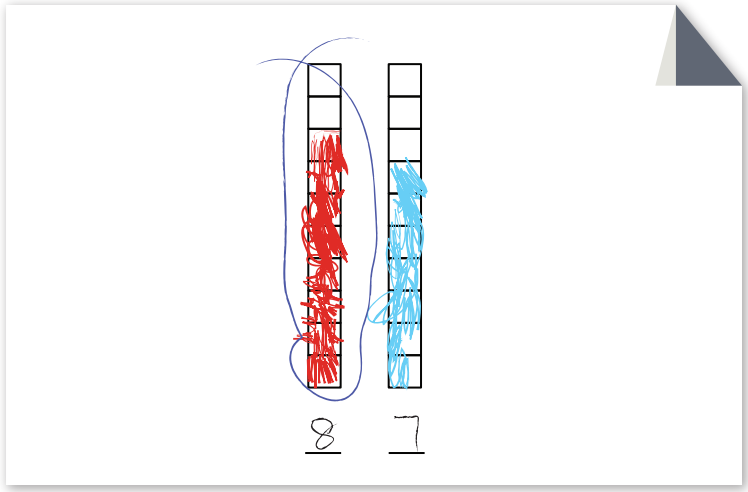
Counting

Counting is a central focus, as it is the basis for understanding the number system and for almost all the number work in the primary grades. Students hear and use the counting sequence (the number names, in order) in a variety of contexts. They have many opportunities to connect the number names (e.g., "five") with the written numbers (e.g., 5) and with the quantities they represent. They have repeated experiences counting sets of objects (e.g., "How many bears are there?"), making sets of a given size (e.g., "Can you show me 8 bears?"), and using spoken and written numbers to label those sets. Ten Frames offer students a tool for organizing sets of up to 10 objects and provide them with a structure for thinking about a 10 as two groups of 5 and, later, as a group of 10 ones—an idea critical in their work with teen numbers.

[This student used pictures, numbers, and words to show that he counted 13 nuts. He drew a circle for each nut and wrote both the number and the unit he counted—"13 nts." When he recounted his circles to check, he realized he had one too many, so he scribbled out one circle.]

As students count sets of objects and make sets, they begin to see the importance of counting each object once and only once and of having a system for keeping track of what has been counted and what still remains to be counted. Students encounter increasing amounts over the course of the year, beginning with sets of up to 10, then 15, and ending the year fluently counting to—and counting out sets of—20. Because counting is a part of activities such as taking attendance, collecting data via *Today's Question,* and solving problems about the class, students also have repeated experiences with an amount that may be greater than 20 (e.g., the number of students in their class). Similarly, they may encounter numbers in this range as they count to determine the lengths of objects and paths.

As students develop accurate counting strategies, they also build an understanding of how numbers in the counting sequence are related: Each number is one more (or one less) than the number before (or after) it. Students develop an understanding of the concepts of greater than, fewer than, and equal to; and they develop language for describing quantitative comparisons (e.g., *bigger, more, smaller, fewer, less, same, larger, greater, equal*) as they count and compare quantities.



[Students grab and then count the number of cubes in each of two handfuls. They represent the handfuls, use numbers to record the totals, and circle the amount with more.]

In addition to counting quantities, students practice writing the numbers to 20 and reciting the rote counting sequence to 100. They count by 1s from 1—and from numbers other than 1—using the number line to keep track of the numbers they are saying. They also practice the counting-by-10s sequences to 100, tying it to the context of counting students' fingers.

The Teacher Note **Algebra Connections in This Unit** in Unit 2 examines how students grapple with the idea that the number of objects in a set is fixed no matter how it is arranged or counted as they think about the question, "Does order matter when counting?"

MAIN MATH IDEAS

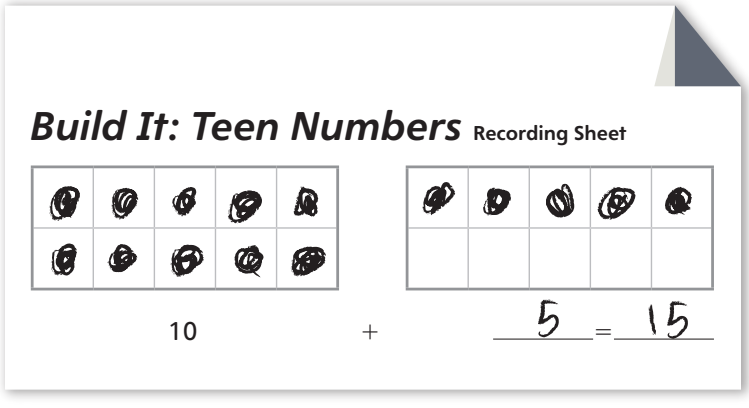
- Counting and representing quantities
- Comparing and ordering quantities
- Collecting, representing, describing, and interpreting data

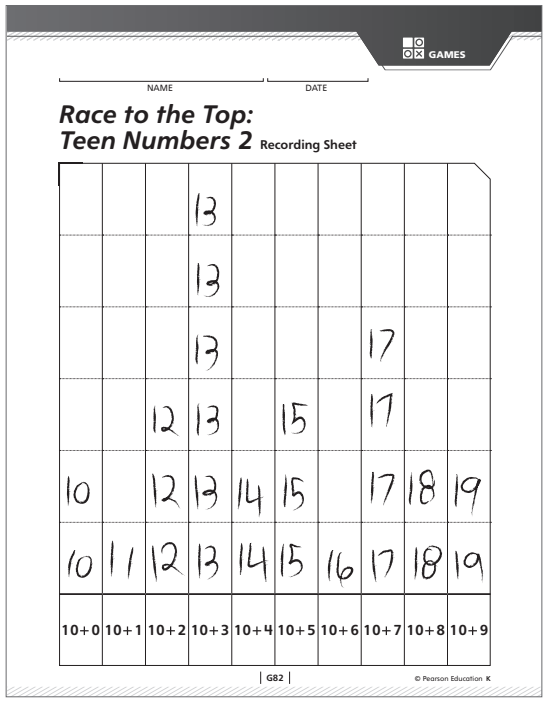
Benchmarks

- Count and count out a set of up to 10 objects. **(Unit 2)**
- Compare two quantities up to 10 to determine which is greater. **(Unit 2)**
- Count and count out a set of up to 15 objects. **(Unit 4)**
- Count and count out a set of up to 20 objects. **(Unit 6)**
- Write the numbers to 10. **(Unit 6)**
- Write the numbers to 20. **(Unit 8)**
- Rote count by 1s and 10s to 100; when counting by 1s, start from a number other than 1. **(Unit 8)**

The Number System

Students' work composing and decomposing numbers in different ways culminates in a focus on ideas that lay the foundation for understanding place value, the base-10 nature of our number system. Students build on their familiarity with Ten Frames and use their structure to explore the two-addend combinations that make 10. This sense of 10 as an important number in our number system supports students in making sense of the teen numbers as ten 1s and some number of leftover 1s. Using equations to record such information (e.g., $15 = 10 + 5$, $16 = 10 + 6$) helps students begin to notice important regularities in the structure of these numbers and the written numerals: The 1 in 15 refers to the group of ten 1s and the 5 refers to the number of leftover 1s.





MAIN MATH IDEA

- Understanding place value

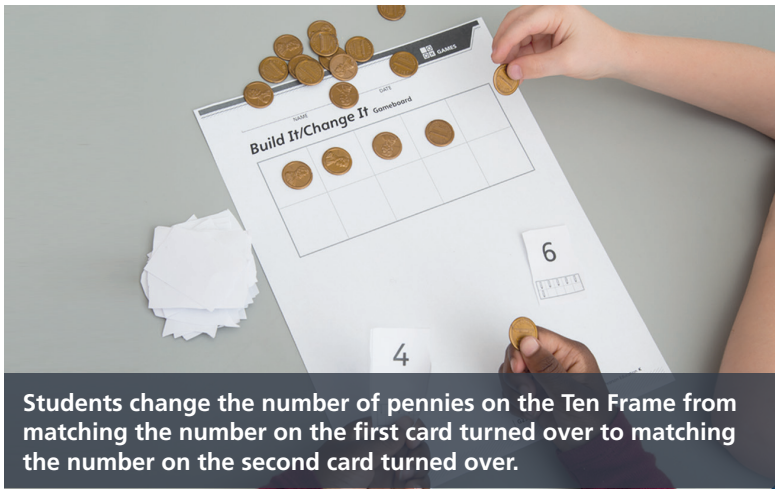
Benchmarks

- Figure out a missing addend when the sum is 10. **(Unit 8)**
- Represent the teen numbers as ten 1s and some number of 1s. **(Unit 8)**

Number and Operations, *continued*

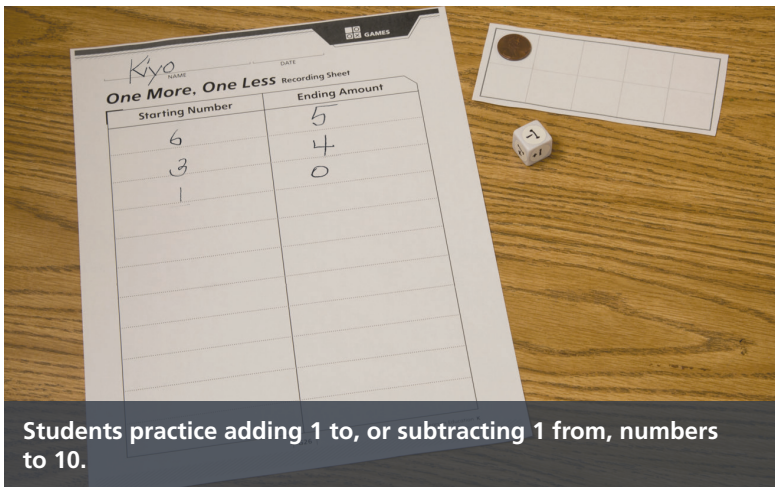
Addition and Subtraction

Young students develop their understanding of addition and subtraction by having many opportunities to count, visualize, model, solve, and discuss different types of problems. Many counting activities build a bridge to the operations of addition and subtraction, as students add a small amount to (or remove a small amount from) a set and figure out "How many now?"



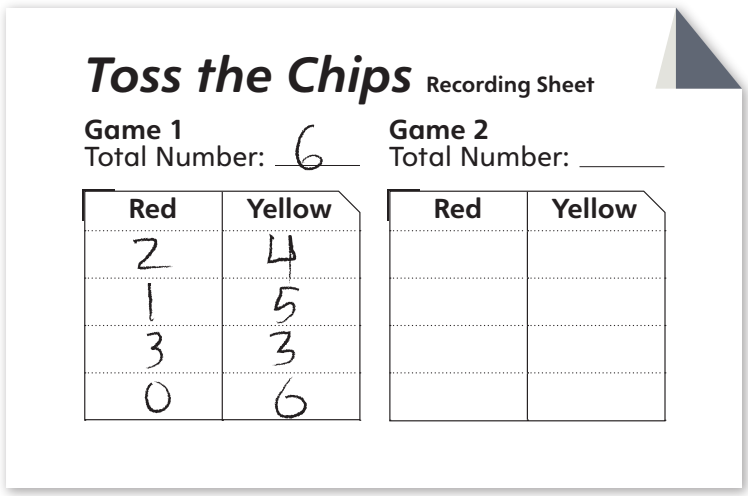
Students change the number of pennies on the Ten Frame from matching the number on the first card turned over to matching the number on the second card turned over.

Students play a variety of games that model both addition and subtraction. They have repeated experiences joining two or more amounts, removing an amount from a whole, and thinking of a number as being composed of two parts.



Students practice adding 1 to, or subtracting 1 from, numbers to 10.

As students record combinations of two-color counters or find ways to arrange and numerically describe sets of 5–10 square tiles, they begin to see that numbers can be composed in different ways. They work on activities that involve seeing and describing a given quantity (e.g., 6 tiles) as made up of groups (e.g., a group of 4 and a group of 2). They are also asked to decompose quantities (e.g., 6 can be split into 4 and 2) and to find one or more combinations of a quantity (e.g., 6 can also be decomposed into 6 and 0, 3 and 3, or 5 and 1). This work lays the foundation for making meaningful sense of $4 + 2 = 6$ and $6 - 4 = 2$.



Toss the Chips **Recording Sheet**

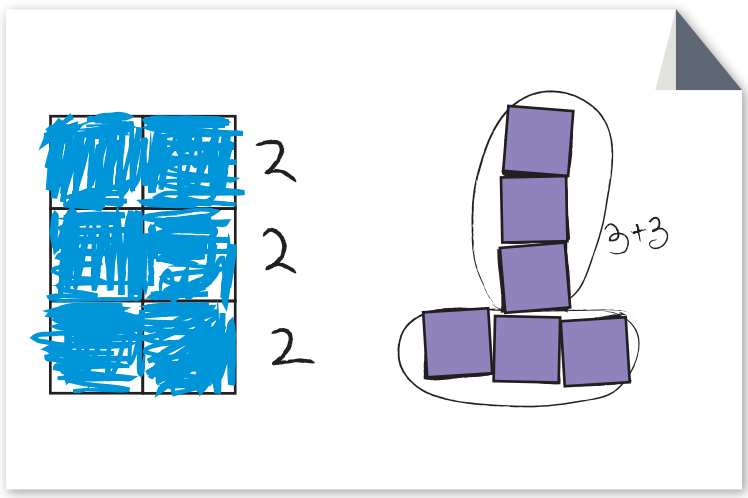
Game 1
Total Number: 6

Red	Yellow
2	4
1	5
3	3
0	6

Game 2
Total Number: _____

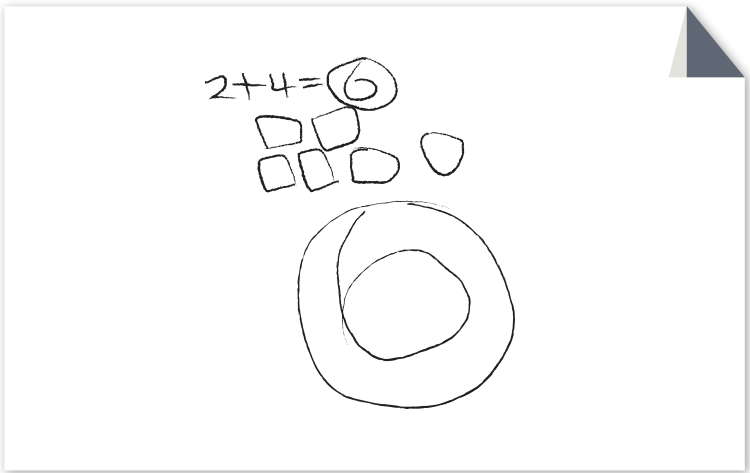
Red	Yellow

[Students think about different ways 6 can be composed of (or decomposed into) two amounts.]



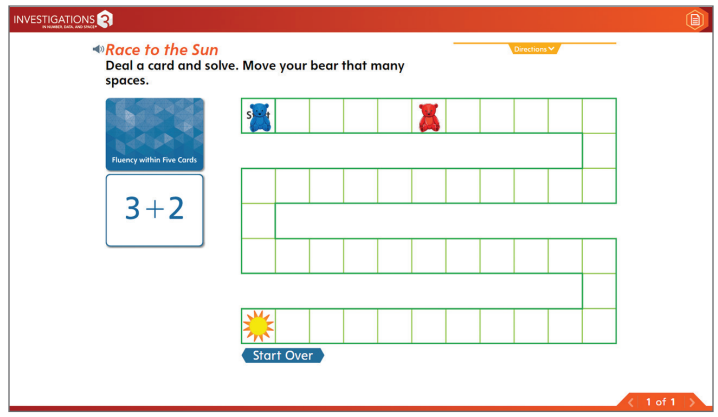
[Students record their arrangements of 6 tiles and indicate how they know there are 6 tiles in all.]

Students also encounter story problems about adding to, taking from, putting together, and taking apart. They retell the stories, act them out, model them with objects, solve them in the whole group and on their own, and eventually show their work on paper. They use mathematical tools and representations to model and solve problems and to clarify and communicate their thinking. They are just beginning to learn how to represent their mathematical work on paper and are encouraged to do so in ways that make sense to them. Many use a combination of pictures, words, numbers, and notation, building on the ways they have observed their teacher representing their strategies for the class throughout the year.



[This student draws a group of 2, a group of 4, and determines that she has 6 in all.]

Having had many opportunities to determine the expression that matches a given situation, students tell stories to match a given addition or subtraction expression. They play games where such expressions determine the number of spaces to move or counters to take. Such repeated practice leads to being able to fluently add and subtract within 5.



[Students turn over a card, determine the answer to the expression (e.g., 3 + 2), and move their bear that many spaces.]

In the units that focus on addition and subtraction (Units 4, 6, and 8), the **Algebra Connections in This Unit** Teacher Notes show how students apply the commutative property of addition, encounter the relationship between addition and subtraction, and make generalizations about the operations of addition and subtraction.

MAIN MATH IDEA

- Understanding, representing, and solving addition and subtraction problems

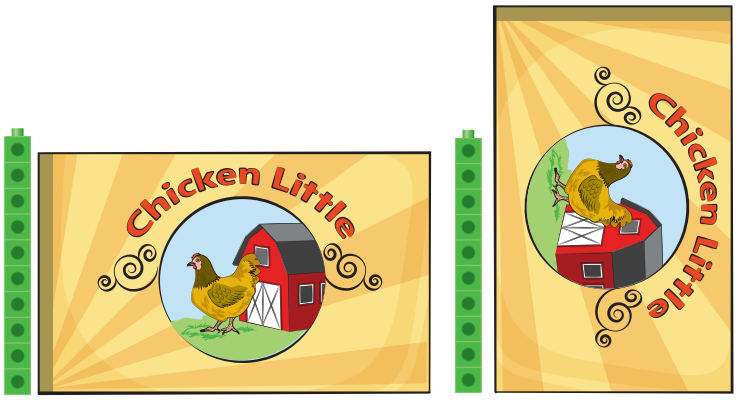
Benchmarks

- Figure out what is one more or one less than a number. **(Unit 4)**
- Represent and solve addition story problems within 10. **(Unit 6)**
- Decompose a number into two addends in more than one way. **(Unit 6)**
- Represent and solve subtraction story problems within 10, with result unknown. **(Unit 8)**
- Add and subtract fluently within 5. **(Unit 8)**

Measurement

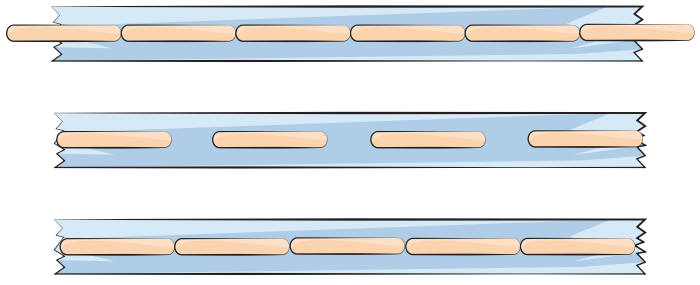
The measurement work begins with students' real-world experiences. They discuss what they know about measuring and think about what could be measured on a given object. For example, given a box that holds pencils or markers, one could measure how tall, wide, or long it is; the distance around it; how much it weighs; or how many pencils or markers fit in it. The measurement work in Kindergarten focuses on length and weight and offers a natural context for applying and practicing counting skills.

Young students' ideas about measuring grow out of a great deal of experience with informal measuring. Comparing is a natural activity. Even very young children spontaneously try to see who or what is bigger, taller, heavier, or smaller. Therefore, students' work with length (and weight) begins with direct comparison. As students directly compare the lengths of objects to determine which is longer, they discuss and make sense of important aspects of accurate measurement, such as choosing which dimension to measure and how to line up objects to compare them.



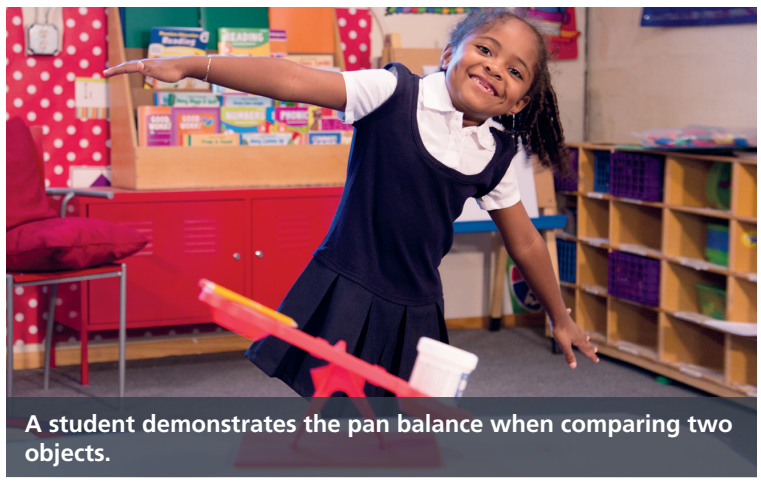
[Students begin to think about the different dimensions of objects.]

Through such activities and discussions, students learn and become comfortable with the language used to describe length—*long, short, wide, tall, high* (and their comparative forms—*longer, wider, as long as,* etc.). These qualitative comparisons inspire students' natural curiosity: "How long is it?" As they investigate questions—such as, "How many cubes long is my shoe?" or "How many craft sticks is the path from the window to the door?"—students apply their counting skills to measurement contexts and begin to think about important ideas in measurement, such as what happens if the units are (or are not) laid straight or if there are (or are not) gaps or overlaps between them. These ideas are explored in subsequent grades.



[Students begin to think about measuring accurately.]

Like the work with linear measurement, students' initial exploration of weight focuses on the direct comparison of objects. Because weight is not a visible attribute in the same way that length is, it can be a more challenging idea, but weight measurement is part of most students' real-world experiences. Students begin by comparing weights directly, first with their hands and then with a pan balance. They use comparative language such as *heavy, heavier, light,* and *lighter* to describe the weights of objects. They also quantify weight as they use the pan balance to determine the number of pennies or cubes that weigh the same as an object. This work provides another opportunity to apply and practice counting skills as students answer the question "How much does it weigh?"



A student demonstrates the pan balance when comparing two objects.

MAIN MATH IDEAS

- Understanding length
- Understanding weight
- Counting and representing quantities

Benchmark

- Describe length and decide which of two objects is longer. **(Unit 2)**

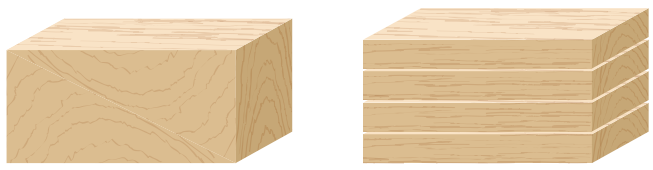
Geometry

The geometry work builds on students' experiences interacting with and observing shapes in their environment and daily lives. Students develop a deeper understanding of the two-dimensional (2-D) and three-dimensional (3-D) world in which they live by identifying, describing, and comparing shapes, as well as composing and decomposing shapes.

Students are encouraged to use their own words to describe shapes and to associate them with familiar objects as they learn the names used to identify them. They may describe a circle as being "round like a ball," or a rectangular prism as being "tall like a building." They describe 2-D shapes with words such as "flat" and think of 3-D shapes as "ones you can hold in your hand."

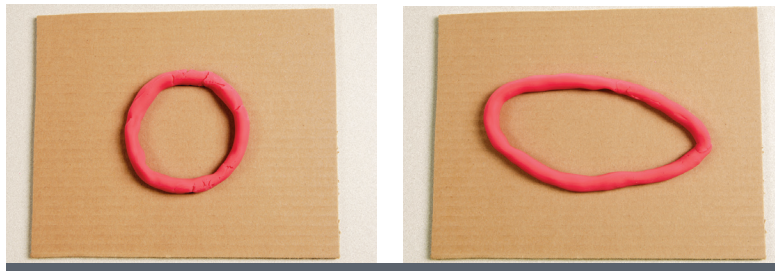
Students explore the idea that shapes can be combined to make other shapes. For example, they investigate the different combinations of pattern block shapes that can be combined to make a hexagon and compose a replica of a rectangular prism from smaller 3-D shapes.

[Ways to make a hexagon]



[One way to make a rectangular prism]

Students deepen their understanding of the attributes of 2-D and 3-D shapes and how different shapes relate to each other as they construct shapes with materials like clay and geoboards. Constructing shapes requires students to form and use mental images of the shapes and to think about the attributes of particular shapes.



When comparing shapes, students often use informal language to describe mathematical attributes. For example, "The oval and circle are both round, but the oval is longer;" and " The oval is stretched out like an egg, but is not as round as the circle."

As students are exposed to many different examples of shapes, and as they make and build various shapes, they develop and deepen their understanding of how certain attributes define a shape, and they are able to identify shapes regardless of size or orientation.

Work with geometric shapes and materials also provides many opportunities for students to practice and use words and phrases that describe the relative position of an object, such as *above, below, beside, in front of, behind,* or *next to*.

MAIN MATH IDEAS

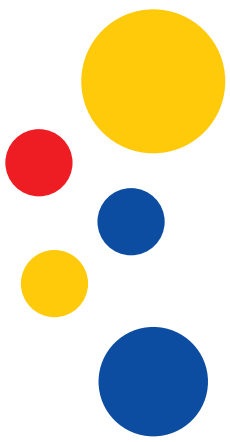
- Describing, identifying, and comparing 2-D shapes
- Composing and decomposing 2-D shapes
- Describing, identifying, and comparing 3-D shapes
- Composing and decomposing 3-D shapes

Benchmarks

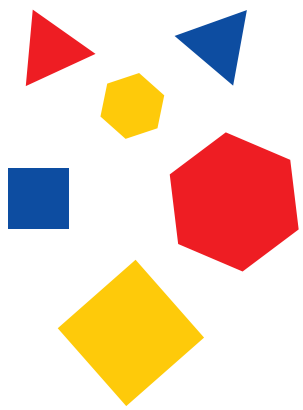
- Identify and describe the overall size, shape, and features of familiar 2-D shapes. **(Unit 3)**
- Make 2-D shapes. **(Unit 3)**
- Combine shapes to make 2-D shapes. **(Unit 3)**
- Identify and describe the overall size, shape, and features of familiar 3-D shapes. **(Unit 5)**
- Make 3-D shapes. **(Unit 5)**
- Combine shapes to make 3-D shapes. **(Unit 5)**
- Understand words that describe relative position. **(Unit 5)**

Data

Identifying attributes and using them to sort and classify are essential to the study of mathematics, particularly data and geometry. Students think about attributes of math manipulatives (e.g., attribute blocks, GeoBlocks), objects (e.g., buttons), and students in their class, and use the attributes to sort. This work builds on students' natural interest in thinking about how things are the same and different, and it helps students see how two objects can be the same in some ways and different in others.

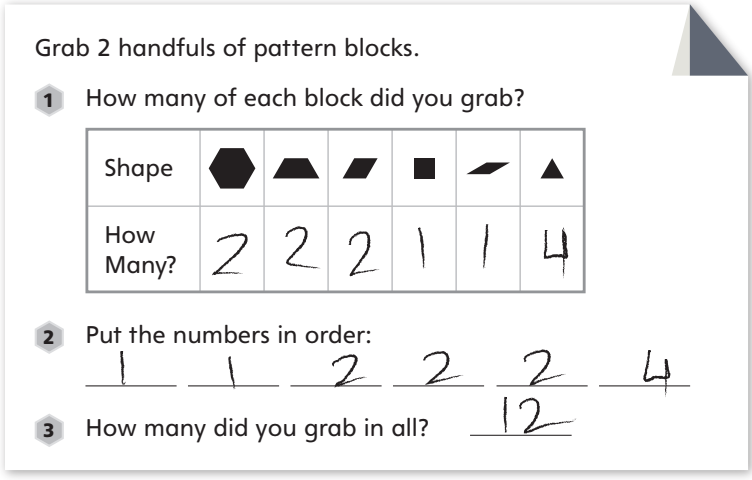


[5 shapes are circles.]



[6 shapes are not circles.]

In a connection to students' work with **Number**, students collect data about the number of pattern blocks they can grab with one or two hands. They classify the data according to shape, count the number in each group, order the groups from least to most, and then determine the total amount.



Grab 2 handfuls of pattern blocks.

1. How many of each block did you grab?

Shape	⬢	⏢	▰	■	▬	▲
How Many?	2	2	2	1	1	4

2. Put the numbers in order: 1, 1, 2, 2, 2, 4

3. How many did you grab in all? 12

[Students grab two handfuls of pattern blocks and record their data.]

Unlike materials such as attribute blocks and pattern blocks, which have a clearly defined set of attributes, data sets are often open-ended with less defined attributes. For example, when students collect data about their favorite lunch food, they must consider various attributes as they think about how to group and organize the data into categories.

Students sort their favorite lunch foods into categories.

Students gain experience with the many aspects of carrying out a data investigation—collecting, recording, representing, describing, and interpreting data. These experiences provide students with an opportunity to model real-world situations with mathematics. In the Classroom Routine *Today's Question*, students respond to a survey question and then look at the data and think about what the data tell them about their group. They count and compare the number in each group and think about how the sum of these two quantities is related to the total number of students who responded to the survey, identifying a one-to-one correspondence between number of responses and number of people surveyed. They eventually use an equation to model this information. As students encounter different ways to record and represent data, they see and discuss which ones allow for easier counting and comparing.

Are you the oldest child in your family?

11	8
Kyle Lisa Kiyo Dennis Mia Mary Yoshio Rebbecca raul Jenny Shavonne	Laquinta Tammy Emma Corey Ricardo Jae Carmen Manuel
YES	NO

19 students answered the survey.

[An example of one class's response to the survey question "Are you the oldest child in your family?"]

Would you rather have a dog or a cat?

Dog	Cat
Mary	
Jae	
Ricardo	
Latoya	
Kiyo	
Mia	
Yoshio	Lisa
Tammy	Russell
Hugo	Lionel
Cindy	Rebecca
Dennis	Sarah
Brad	Victor

12 people would rather have a dog.

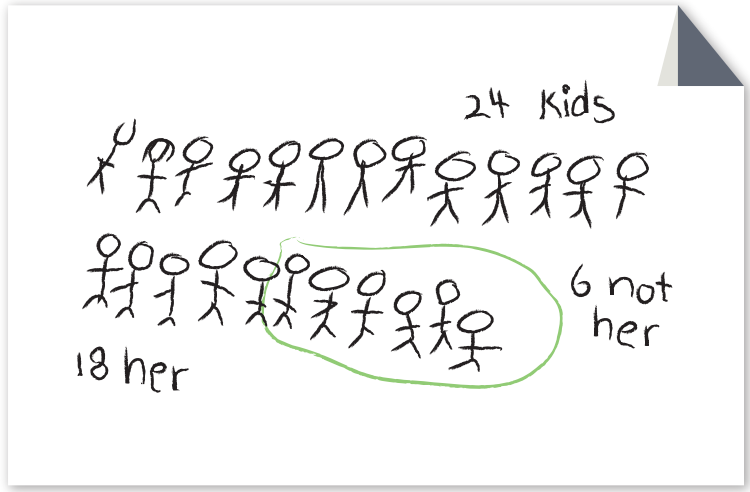
6 people would rather have a cat.

18 people answered *Today's Question.*

12 + 6 = 18

[An example of one class's response to the survey question "Would you rather have a dog or a cat?"]

Students' experiences with data are further developed as they conduct their own survey, choosing and posing a yes/no question that is of interest to them, deciding how to record and keep track of information, and then making a representation of their data. They analyze the data as they count and compare responses and make statements about what they learned. They also use data to solve mathematical problems about their class, such as, "Do we have enough chairs for our class?" or "If 6 students are absent, how many are present?"



[An example of one student's work using attendance data to solve the problem "If 6 students are absent, how many are present?"]

MAIN MATH IDEAS

- Sorting and classifying
- Collecting, representing, describing, and interpreting data
- Comparing and ordering quantities
- Counting and representing quantities

Benchmarks

- Sort a set of objects by a given attribute and order the groups based on the number in each. **(Unit 7)**
- Using data to represent and solve a real-world problem. **(Unit 7)**