Learning Multiplication Facts

Students are expected to know the multiplication facts to 10×10 fluently by the end of Grade 3. They are assessed on the $\times 1, \times 2, \times 5$, and $\times 10$ facts in this unit, and they are assessed on the remaining facts in Unit 5.

Fluency means that facts are quickly accessible mentally, either because they are immediately known or because the calculation that is used is so effortless as to be essentially automatic (in the way that some adults quickly derive one fact from another).

Students use Multiplication Cards (see more below) to practice and learn the multiplication facts in this unit and in Unit 5, and some may need additional practice after these units are completed. With your guidance about which multiplication facts to work on, students use their Multiplication Cards to continue learning these facts at home or outside of math time. Enlist parents or other family members in helping with practice. This Teacher Note provides recommendations for supporting students in this ongoing practice.

More than Just "Facts"

The pairs of factors from 1×1 through 10×10 are traditionally referred to as "multiplication facts," and that terminology is used in this curriculum. However, "fact" often implies something that can only be memorized (such as the first president of the United States, or the capital of Nebraska) and not learned through reasoning. Students learn multiplication facts best by using what they already know about numbers and number relationships and about multiplication and its properties. Relying on memory alone is not sufficient. If students forget answers they are trying to memorize, they are left with no way to think about the problem. If, however, their learning of the facts is based on reasoning, they have a way to determine the answer. This reasoning is built on students' understanding of the meaning and properties of multiplication that they have developed through using different models and representations for multiplication, including skip counting, story contexts, and arrays.

For example, the product of 6×7 can be determined in many ways that are logically connected to our system of numbers and operations. If a student knows that $5 \times 7 = 35$, and interprets 5×7 as five groups of 7, the student can reason that adding on one more 7 will result in the product of 6×7. If a student knows that $3 \times 7 = 21$, and understands that 6×7 is twice as many groups of 7, the student can reason that the product will be twice 21, $2 \times (3 \times 7) = 42$. Relating one fact to another in this way is based on the distributive and associative properties of multiplication. Students apply these properties as they use reasoning strategies to learn their facts.

Knowing that multiplication is commutative is also crucial for learning the multiplication facts. The work with Array Cards supports this understanding (see **Teacher Note 6:** Representing Multiplication with Arrays). Once students have developed a strong visual image of how one Array Card represents two facts (e.g., 6×7 and 7×6), they learn that using this understanding quickly narrows down the number of facts still to be learned.

Helping Students Learn the Multiplication Facts

For many students starting out in Grade 3, learning multiplication facts may seem overwhelming—an endless mass of facts with no order and reason. Therefore, bringing order and reason to students' learning of multiplication facts in a way that lets them have control over their progress is essential. Traditionally, students learned one "table" at a time—first the $\times 2$ facts, then the $\times 3$ facts, the $\times 4$ facts, and so on. However, the multiplication facts can be grouped in ways that better support reasoning about related facts.

Learning the $\times 1, \times 2, \times 5$, and $\times 10$ facts

In this unit, students are assessed on the multiplication facts that involve $\times 1, \times 2, \times 5$, and $\times 10$ (up to 10×10). The remaining facts are assessed in Unit 5.

In Grades K–2, students have had a great deal of practice in counting by 1s, 2s, 5s, and 10s. Because of this, most students either already know, or easily learn, the $\times 1$ (the number itself) or $\times 2$ (the number doubled) facts. They also recognize that the $\times 5$ facts end with 5 or 0, and the $\times 10$ facts end with 0. However, although most third graders can easily count by 2, 5, and 10, the student who is fluent does not skip count to determine the product of multiplication facts involving these numbers. Some students only need practice in connecting the products in the well-known skip counting sequence to its factors in order to learn the facts. To learn what some students consider the more difficult $\times 5$ facts (e.g. 7×5), some students think of the relationship between 5 and 10. They know 7×10 is 70, and because 5 is $\frac{1}{2}$ of 10, the product of 7×5 is half of 70, or 35. Other students learn these more difficult facts by relating them to known facts, for example, 7×5 is 5 more than 6×5.

In general, most students easily learn these sets of facts.

Once the ×1, ×2, ×5, and ×10 facts are known, there are 21 facts left to be learned. (This assumes students are using the commutative property.) Students learn these remaining facts in different ways; the following presents one sequence that many students find useful.

The ×3 and ×4 facts

Many students know a number of the remaining ×3 facts: 3 × 3, 4 × 3, 6 × 3, 7 × 3, 8 × 3, 9 × 3. For the ones they do not know, encourage students to think about these facts as one more group than the related ×2 facts. For example, if a student knows 8 × 2, and understands that 8 × 2 = 2 × 8 and 8 × 3 = 3 × 8, the student can think of 8 × 3 as one more 8. Some students use doubling for some of these facts, "4 × 3 = 12, so 8 × 3 would be double that, or 24."

Thinking about doubling also helps students learn the remaining ×4 facts: 3 × 4, 4 × 4, 6 × 4, 7 × 4, 8 × 4, 9 × 4. Help students think of these as doubling the ×2 facts—6 × 4 = (6 × 2) + (6 × 2) or 6 × 4 = (6 × 2) × 2. Students may verbalize this idea as "6 times 4 is 6 times 2 and another 6 times 2," or "to get 6 times 4, I double 6 × 2." Doubling is also useful within the ×4 facts; for example, when students know that 3 × 4 = 12, then that fact can be used to solve 6 × 4: 6 × 4 = (3 × 4) + (3 × 4). Getting used to thinking about doubling with smaller numbers also prepares students for using this approach with some of the harder facts.

Strategies for the remaining "hard" facts

Assuming students have learned all of the ×1, ×2, ×3, ×4, ×5, and × 10 facts, only 10 facts remain to be learned. (Some students may still be learning a few of the other facts.) Those 10 facts tend to be the most difficult for students (and adults): 6 × 6, 7 × 6, 8 × 6, 9 × 6, 7 × 7, 8 × 7, 9 × 7, 8 × 8, 8 × 9, and 9 × 9.

Once students realize that they only have a small list of facts to learn, it is easier for them to focus on one or two of them at a time, and think through strategies that help them practice them. Some students use doubling to help them learn 6 × 6, 8 × 6, 8 × 8, 9 × 6, and 8 × 9. Some students use related facts close to the fact they are working on ("I know 10 × 7 is 70, so 9 × 7 is one less 7"). Some students are intrigued by square numbers, and learn these easily. They then use those to help them learn other "hard facts." ("I always remember that 8 × 8 is 64, so 9 × 8 is just one more 8, that's 72.")

Using the Multiplication Cards

Students use a set of Multiplication Cards, which they make in Investigation 3, to identify the facts they know and the ones they are working on. They sort the cards in order to keep track of which facts remain to be learned. For each fact they do not yet know fluently, they come up with a fact they do know that will help them, and write that fact as a clue on the card. For example, to solve 8 × 7, some students use 8 × 5 as the clue (and then add 16), some use 7 × 7 (and then add 7), and still others use 4 × 7 (and then double the product). These ideas are more than just "tricks" to help students learn the facts; they are based on the properties and meaning of multiplication. As students apply these properties to help them learn facts they do not yet know, they are also deepening their understanding of how multiplication works.

Once students have identified the facts they are still learning, use the suggestions above, or other practice methods, to help them become fluent with all the facts. Depending on how many facts students are working on, consider helping them focus on a few facts each week until they are fluent. Students who are not fluent with the ×2, ×5, and ×10 facts at the end of this unit, should continue practicing those facts so they are fluent before starting Unit 5.