



A Framework for Reflecting about Equity in the *Investigations* Mathematics Classroom

For the past two years, the project staff of the *Investigations* Center for Curriculum and Professional Development has been studying literature that addresses issues of equity, access, identity, and agency in mathematics education (for example, see Aguirre et. al., 2013; Ball, 2018; Gutiérrez, 2007; Hammond, 2015; Kay, 2018; Ladson-Billings, 2006; Nasir, 2016; NCSM & TODOS, 2016). During this study, we have been asking ourselves:

1. How can a mathematics curriculum be a tool for anti-racist work?
2. How can *Investigations* better support students who have been historically marginalized, especially Black and brown students, to be doers of mathematics?

Based on our reading and discussions among ourselves and with others who are engaged in equity work in different contexts, we developed this framework for our own work as we select and develop enhancements to the curriculum and to professional development that supports the curriculum. We are now making this working draft available for school-based educators who might also find it useful as a tool for reflecting on issues of equity, identity, and agency in mathematics learning and teaching.

Because we have always viewed curriculum as a tool for both teacher and student learning, we are considering both how the teacher-facing materials in the curriculum can best support teachers to address issues of identity and agency in the mathematics classroom and how the student-facing materials can best affirm students' "intellectual, cultural, racial, ethnic, and linguistic background [Aguirre et al., 2013]."

The Curriculum-Teacher Partnership

The curriculum-teacher partnership is critical. While we, the *Investigations* staff, working with consultants whose work focuses on issues of equity in mathematics, are creating guidance and tools for supporting equitable teaching and learning that will become part of the curriculum itself, the implementation of these elements will depend on teachers' awareness and reflection. As the article on "The Teacher-Student-Curriculum Partnership" in the *Investigations Implementation Guide* at each grade level states, the lengthy development process of the curriculum, in close collaboration with teachers, "has resulted in a focused, rigorous, coherent core curriculum that is based on the real needs of real students and real teachers. ... [but] Only the teacher is present every day in the classroom, observing students' work, listening to their discourse, and developing an understanding of their mathematical ideas ... [p. 5]."

With the importance of this partnership in mind, and with the caveat that this document is very much a work in progress, we share this framework for using *Investigations* to promote equity in the mathematics classroom and to support the identity and agency of students who have been historically marginalized in mathematics, including Black, Latinx, emergent bilingual, and gender- and neurologically-diverse learners.

Four Categories to Guide Equitable Mathematics Education

We have identified four categories of possible focus for reflection and action. As we develop material to help users focus on equity while using *Investigations*, we are working to select efforts in these four categories that we think have the potential to impact: 1) teachers' reflection about equity in their classrooms, as the materials are used as a tool for teacher learning, and 2) equitable opportunities for participation in mathematical thinking, as the materials are used as a tool for student learning.

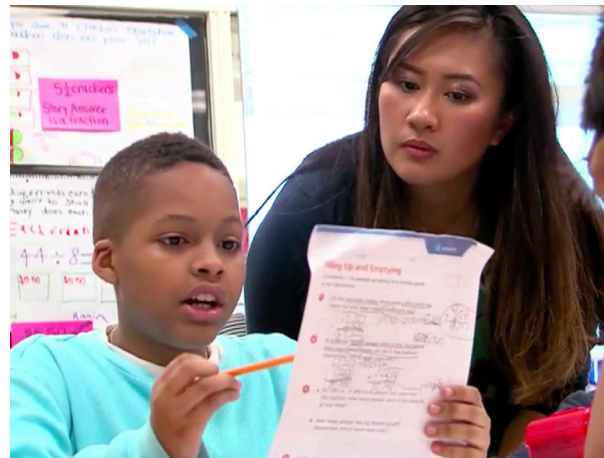
1. Deep and Rigorous Mathematics
2. Equitable Participation in a Collaborative Mathematical Community
3. Strength-based Assessment and Accommodation
4. Connections to Students, Their Families and Communities

We invite you to consider the questions we've listed under each of the four categories below and to choose one or several as a focus for your own reflection this year. Just as we've found it helpful to engage with colleagues for support and challenge in our own discussions, this document might be more generative if you can use it with a group of colleagues. We are very interested in how you use this framework, whether you find it helpful, what questions are most relevant to your own context, and to what actions your reflection leads.

We acknowledge that these categories are not completely distinct, but we have found them helpful in trying to describe for ourselves the territory for potential action. We have also identified two frameworks to guide our work: Aguirre et al.'s (2013) equity-based practices, and the implementation steps from the NCSM/TODOS joint position paper, *Mathematics Education Through the Lens of Social Justice* (2016). In the descriptions below, we cite elements from these two frameworks that seem most relevant to each category, with the understanding that many of these could apply to multiple categories.

1 Deep and Rigorous Mathematics

Equitable teaching and learning of mathematics can only proceed in an environment where students engage deeply with significant mathematical ideas, develop conceptual understanding of those ideas, participate in mathematics activities with high cognitive demand, and experience the joy and beauty of mathematics. Development of students' identity and agency as mathematicians requires consistent, ongoing opportunities to think through unfamiliar problems, build and use a repertoire of representations, recognize connections among mathematical ideas, look for patterns and regularities, make conjectures and mathematical arguments, and ask new questions, as well as confidently apply known procedures to familiar problems.



Connections to Key Frameworks

EQUITY-BASED PRACTICES (AGUIRRE ET AL., 2013)

- › Going deep with mathematics: Presents tasks that have high cognitive demand and include multiple solution strategies and representations
- › Going deep with mathematics: Supports students in analyzing, comparing, justifying, and proving their solutions
- › Drawing on multiple resources of knowledge: Uses previous mathematics knowledge as a bridge to promote new mathematics understanding

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- › Cultivate and sustain a positive mathematics identity and affect in students as doers of mathematics
- › Analyze curriculum for access to high cognitive demand tasks that are meaningful and connected to children's lived experiences

Why consider this topic in relation to issues of equity?

The first guiding principle of the *Investigations* curriculum, which has remained our primary principle since the first edition was published in the 1990s, is: "Students have mathematical ideas." In part of the description of that principle (*Implementing Investigations*, p. 4, at each grade level), we state, "if given the opportunity to learn in an environment that stresses making sense of mathematics, students build on the ideas they already have and learn about new mathematics they have never encountered." From the beginning, the curriculum has been developed on the premise that every student—students of different ethnicities, language, gender, and a range of backgrounds and experiences—can be engaged in mathematical thinking, not just parroting. As

Duckworth (1996) puts it, if we listen to students, we will “give them reason.” In other words, we will seek to “understand the way in which what a child says or does ... make[s] sense [p. 87].” Without the opportunity to engage with interesting, deep, and rigorous mathematics content and an orientation in instruction to building on how students make sense of that content, there can be no equity.

We intend the curriculum to support students’ engagement in rich, rigorous, and deep mathematics. However, mathematics is often assumed to be a field for only select people who are born with a “math gene.” In particular, Black and brown students have been historically excluded from engaging in rigorous mathematics. Rochelle Gutiérrez notes how a focus on an “achievement gap” rather than a focus on excellence and advancement results in “a static notion of student identity (as quantifiable in terms of race, class, gender, language, etc.) and ignores the multiple identities and agency of students [2008, p. 359].” As Lisa Delpit says in *Multiplication is for White People* [2012], “What happens when we assume that certain children are less than brilliant? Our tendency is to teach less, to teach down, to teach for remediation [p. 6].”



To engage all students in deep mathematics, how do we—as curriculum developers, coaches, and teachers—

- › highlight, build on, and connect students’ ideas and representations, with emphasis on participation of students with a range of understanding, fluency, and confidence?
- › make use of partial understanding and incorrect ideas to move the mathematics forward and to support students in developing their confidence to contribute these important ideas to the class’s mathematics work?
- › focus core activities so that students are making sense of complex mathematical ideas?
- › develop opportunities for students to experience the joy and beauty of mathematics by investigating mathematical patterns and relationships in numerical and geometric contexts?
- › make sure the math is deep and rigorous for students at different places in their understanding?
- › make accommodations for students with different needs and strengths, while making sure they remain engaged in challenging tasks that involve making sense of mathematics?
- › make use of students’ multiple representations and modes of communication in order to emphasize their different ways of seeing, expressing, and investigating mathematics?
- › build new mathematics on what students know, always presuming that students are bringing significant ideas to the table?

2 Equitable Participation in a Collaborative Mathematical Community

A strong collaborative mathematics community gives students the opportunity to work together to solve problems, to learn from each other, to take risks, to support and encourage others, and to be supported and encouraged by others in their growth as math learners. In a collaborative learning environment, students play an active role in their learning. Students have opportunities to decide how they are going to solve problems, to select tools and materials, and to choose which activities they work on. They are invited to keep thinking about mathematical ideas that come up in class and to ask their own mathematical questions.



Equitable participation in this community means that all students have the opportunity to express their ideas, to revise and build on them, to have others respectfully listen to and engage with their ideas, and to be seen as mathematical thinkers who make meaningful contributions to classroom conversations. This is true whether they are working with a partner, in a small group, or as part of a whole class discussion, and regardless of their perceived math status or first language. All students' ideas should be a part of the public discourse, "including those who do not feel confident about their math or communication skills and those who opt to stay on the periphery [*Implementing Investigations in Grade X*, p. 52]." Having one's ideas taken seriously and taken up publicly supports students' identity and agency as mathematicians.

Connections to Key Frameworks

EQUITY-BASED PRACTICES (AGUIRRE ET AL., 2013)

- ▶ Challenging spaces of marginality: Encourages student-to-student interaction and broad-based participation
- ▶ Leveraging multiple mathematical competencies: Structure student collaboration to use varying math knowledge and skills to solve complex problems.

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- ▶ Interrogate individual and societal beliefs underlying the deficit views about mathematics learning and children with specific attention to race/ethnicity, class, gender, culture, and language
- ▶ Cultivate and sustain a positive mathematics identity and affect in students as doers of mathematics
- ▶ Increase use of participation structures that maximize mathematical discourse and student contributions while minimizing status issues in the classroom

Why consider this topic in relation to issues of equity?

Central to the structure and pedagogy of the *Investigations* curriculum is a collaborative and supportive learning environment. This community is essential to creating a classroom where students solve problems based on their own understanding, share their ideas and thinking, learn from and build on each other's ideas, and view the doing of mathematics as a collaborative effort. "Teachers and students need to work together to build a mathematical community in which students are prepared to listen actively and contribute ideas [*Implementing Investigations in Grade X*, p. 30]." Such a learning community requires "a focus on respect for students' ideas and acceptance of differences," a belief "that mistakes are important opportunities to learn," and a plan for encouraging students to disagree respectfully with each other (*Ibid.*, p. 30). The structures and discussions throughout the *Investigations* curriculum support the building of mathematical ideas as a collaborative effort.

However, students who have been historically marginalized in mathematics may not see themselves as powerful in relationship to mathematics and may not be perceived as having mathematical ideas that contribute to the community. The classroom environment can have a particularly large impact on these students: "Students experience mathematics classrooms as racialized spaces, where Black and Latina/o students are subject to negative stereotypes about their ability to do mathematics [Nasir, 2016, p. 11]." Teachers and students alike may have unexamined, implicit biases about whether students of color, emergent bilingual students, girls, or neurologically diverse students can be "good at math." This can impact whose ideas are shared, taken up, and treated as mathematically powerful. Therefore, we need to consider how to create a classroom community where students who have been subjected to these stereotypes feel their ideas are respected, are comfortable sharing their ideas, and feel safe taking risks.

The curriculum emphasizes to teachers that "it is important to consider that students in an *Investigations* classroom work individually, in pairs or small groups, and as a whole class. When working in pairs or small groups, students need to be able to listen to one another's ideas

and work together, sharing materials or reviewing each other's work when necessary [*Implementing Investigations in Grade X*, p. 29]." Because discussions are an opportunity to learn, the curriculum helps teachers focus on the main math ideas, "carefully select and scaffold problems, representations, and strategies for the whole class to consider," and predict and guide discussions. It supports teachers in thinking of ways to invite all students to participate (*Ibid.*, p. 30). Critical to this is "a classroom culture that allows all students to share their ideas, listen to, and learn from each other [*Ibid.*, p. 52]."



However, as Deborah Ball cautions, a curriculum that focuses on discourse presents both opportunities and risks.



Classrooms that are rich in mathematical discourse and discussion are crucial for empowering young people. They are also high-risk for reproducing patterns of racism and marginalization. (Ball, 2019, slide 11).

For example, partnerships or small groups can be taken over by one member and whole class discussions can be dominated by a small group of eager participants. Ideas about students' math abilities may impact whose work is publicly shared and in what ways, and who is called on and for what kind of contribution.

To support equitable participation in a collaborative mathematical community, how do we—as curriculum developers, coaches, and teachers—

- › explicitly emphasize the development of this community from the beginning of the school year as students engage in the first mathematics unit?
- › build and maintain a collaborative learning community throughout the year, returning frequently to how the class is working together in mathematics and how students are viewing themselves and each other as participants in that community?
- › create small groups in ways that best support students' learning and then help pairs and small groups work together equitably?
- › choose which students' strategies and ideas to share in group discussions? For what reasons do we center certain students' work and in what order?
- › encourage and prepare reticent students to share their ideas in whole group discussions?
- › decide who we are calling on? How do we keep track of what kinds of questions or opportunities are offered to which students?
- › connect different students' ideas to each other and treat all contributions as valuable?
- › keep track of student participation over time?
- › reflect on different norms for participation and the impact of those norms? Are we reproducing patterns of racism, sexism, and marginalization?
- › support and encourage students to see their home and community languages as assets that connect and contribute to the classroom discourse?

3 Strength-based Assessment and Accommodation

All students come to the classroom knowing and understanding some aspects of mathematics. Work with students begins with what they know and understand, with what makes sense to them, with what they are able to do. Having someone recognize and appreciate one's knowledge and ability as a math thinker develops and builds one's mathematical identity. In addition, this information is what enables teachers to support the range of learners in the classroom, to provide responsive accommodations that are truly (and not overly) helpful. Such accommodations help students engage deeply with the mathematics. Because students come to any classroom with a range of different experiences, needs, strengths, and challenges, such accommodations are designed to support and extend students' competencies, contributions, and modes of communication.



Connections to Key Frameworks

EQUITY-BASED PRACTICES (AGUIRRE ET AL., 2013)

- › Leveraging multiple mathematical competencies: Presents tasks that offer multiple entry points, allowing students with varying skills, knowledge, and levels of confidence to engage with the problem and make valuable contributions
- › Challenging spaces of marginality: Positions students as sources of expertise for solving complex mathematical problems
- › Affirming mathematics learners' identities: Recognizes mathematical identities as multifaceted, with contributions of various kind illustrating competence

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- › Focus on mathematical strengths and areas of growth with targeted meaningful feedback that promotes learning, not labeling
- › Refrain from using deficit discourse in professional learning communities and instructional decision making

Why consider this topic in relation to issues of equity?

The *Investigations* curriculum is designed to help the teacher “learn what each student knows and understands and to use that information to inform your practice [*Implementing Investigations in Grade X*, p. 52].” Every session includes activity-specific questions to consider as teachers

formatively assess students at work. Support for teachers is embedded in each Investigation to help them provide differentiation for students who could benefit from an intervention or extension, and for students who are English language learners. These differentiation suggestions are focused on the math at hand, and fall under one of eight instructional strategies, which are designed to be useful across activities and instructional situations.

Strength-based approaches to student learning build on students' knowledge and experience. Labels, such as "low group," "high flyers," and "gifted," support a deficit view of certain students and "perpetuate static views about children and their mathematics competencies. Eliminating the deficit discourse by focusing on learning rather than labels is a key step toward a more just and equitable mathematics education [NCSM/TODOS, 2016, p. 2]." Students who have been historically marginalized are more often deemed in need of interventions that focus only on facts and memorization rather than deep mathematics. Over-scaffolding for these students results in tasks with low cognitive demand and a lack of opportunity to learn rigorous mathematics. Aguirre et al. (2013) point out that "Equity does not mean that every student should receive identical instruction. Instead, equity demands that responsive accommodations be made as needed to promote equitable access, attainment, and advancement in mathematics education for each student [p. 9]."



In order to build on competencies and provide strength-based assessment and accommodations, how do we—as curriculum developers, coaches, and teachers—

- › adopt a strength-based approach to assessment that starts with describing what students know and partially understand with as much specificity as possible?
- › determine what students know and use that information as evidence to inform instruction?
- › support students in using (and learning to use) what they know?
- › help students develop tools, structures, and habits that support them to solve problems, work independently, and persist and persevere?
- › analyze assessments to identify with specificity what students do know and understand and use this evidence to plan next steps for their work?
- › build in ways for students to assess their own mathematics progress?
- › see students' first language as a competency and use it as a resource?

4 Connections to Students, Their Families and Communities

Students need to see themselves, their families and people in their communities—as well as those from other communities—as doers of mathematics. In other words, math class should provide students with mirrors for seeing themselves in the mathematics, and windows for seeing the lives of others (Gutiérrez, 2007). When students feel connected to and invested in the mathematics they are engaging in, this personal connection positively impacts their mathematical identities and sense of agency.



This sense of agency is critical in order “to nurture a democratic society where all can use, know, and understand mathematics to comprehend and critique the world through mathematics and to experience its wonder, joy, and beauty [NCTM, 2020, p. xiv].”

One way to achieve this is to help students see their own experiences as part of the mathematics they are doing and to see mathematics as useful to their lives. For example, working on mathematics set in contexts that are familiar or of special interest can help students engage with and make sense of the math and see math as useful and powerful. As Gutiérrez (2007) writes, “students [should] find mathematics not just ‘real world’ as defined by textbooks or teachers, but also meaningful to their lives [p 3].”

Connections to Key Frameworks

EQUITY-BASED PRACTICES (AGUIRRE ET AL., 2013)

- › Drawing on multiple resources of knowledge: Taps mathematics knowledge and experiences related to students’ culture, community, family, and history as resources

NCSM/TODOS POSITION STATEMENT

- › Adapt instruction that routinely connects children’s mathematical thinking and students’ mathematical, cultural, linguistic, and community-based funds of knowledge

Why consider this topic in relation to issues of equity?

In *Investigations* students are expected to use their ideas and knowledge to solve problems and make sense of new ideas. As we field-tested the curriculum in a variety of classroom settings, we developed and tried out problem contexts, looking for ones that were familiar and engaging to a wide range of students and were not exclusionary. We also developed tools to inform and involve students’ families in their mathematics work, believing that “families are critical partners with schools in developing students who are powerful and confident mathematical thinkers [Implementing Investigations in Grade X, p. 40].” *Investigations* helps teachers connect to students’

lives through family letters and homework that 1) communicate to families the work that is being done in the classroom, 2) provide opportunities for families to engage together in math at home and 3) bring parts of home life into the math they are doing in school.

While these efforts are a beginning, the joint statement from NCTM and TODOS (2016) challenges us to think more deeply about how to make these connections: “A social justice approach to mathematics education assumes students bring knowledge and experiences from their homes and communities that can be leveraged as resources for mathematics teaching and learning [p. 2].” Students who have been historically marginalized in mathematics often do not have opportunities to see themselves in the mathematics they are engaging in and therefore do not see connections between the math and their lives. They are not encouraged to bring their full selves and the experiences of their families and communities into school and do not have the opportunity to develop what Su (2020) calls “mathematical affection”: “To miss out on mathematics is to live without an opportunity to play with beautiful ideas and see the world in a new light. To grasp mathematical beauty is a unique and sublime experience that everyone should demand [p. 8].”



To help students see themselves, their families and their communities in the math, how do we—as curriculum developers, coaches, and teachers—

- › find ways to use mathematics to connect with and get to know students and their families?
- › use problem contexts that are familiar and connect to the particular group of students?
- › encourage students to come up with their own mathematics contexts that connect to their daily lives and interests to help them solve problems and make sense of the math?
- › include projects in which students can use math to solve problems and/or address issues in their school or community?
- › help students investigate mathematics in other areas of interest, for example, in nature, art, games, or music?
- › find opportunities to use mathematics to investigate community issues and issues of social justice?
- › include problems with a range of cultures/people/settings so that students have a window into other people’s lives (for example, problems based on children’s literature)?
- › include opportunities for students to learn about a wide range of contributions to mathematics, by people from a broad range of cultures and identities?

The *Investigations* Center for Curriculum and Professional Development

The *Investigations* staff, together with consultants whose work focuses on equity, are developing resources designed to support teachers and educators in thinking about issues of equity, access, identity and agency in the *Investigations* classroom. The Center is dedicated to advancing the teaching and learning of mathematics for all students and teachers.



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