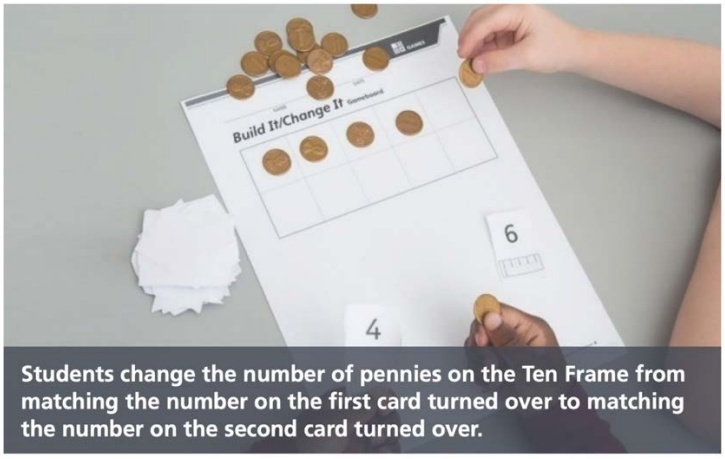
Math Content by Strand

Addition and Subtraction

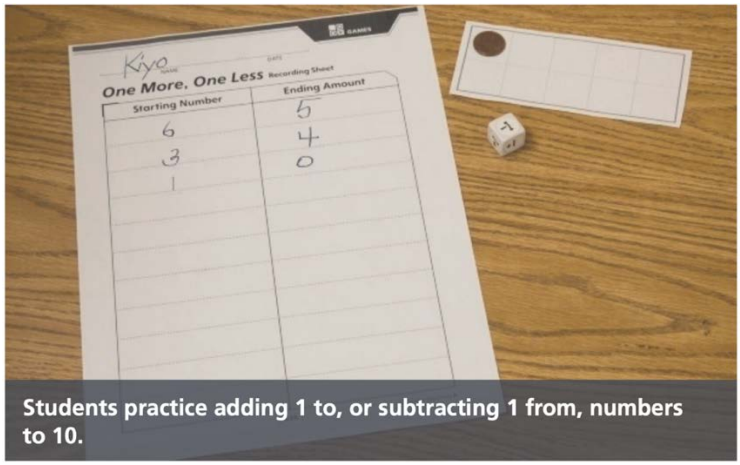
Kindergarten

Young students develop their understanding of addition and subtraction by having many opportunities to count, visualize, model, solve, and discuss different types of problems. Many counting activities build a bridge to the operations of addition and subtraction, as students add a small amount to (or remove a small amount from) a set and figure out "How many now?"



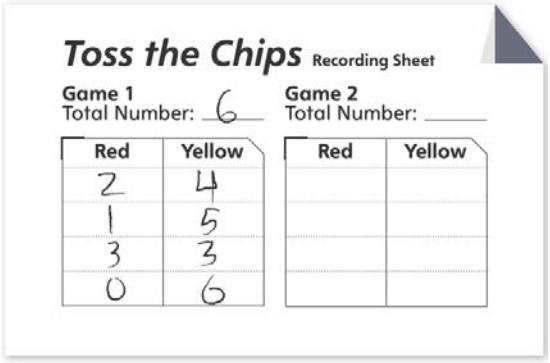
Students change the number of pennies on the Ten Frame from matching the number on the first card turned over to matching the number on the second card turned over.

Students play a variety of games that model both addition and subtraction. They have repeated experiences joining two or more amounts, removing an amount from a whole, and thinking of a number as being composed of two parts.



Students practice adding 1 to, or subtracting 1 from, numbers to 10.

As students record combinations of two-color counters or find ways to arrange and numerically describe sets of 5–10 square tiles, they begin to see that numbers can be composed in different ways. They work on activities that involve seeing and describing a given quantity (e.g., 6 tiles) as made up of groups (e.g., a group of 4 and a group of 2). They are also asked to decompose quantities (e.g., 6 can be split into 4 and 2) and to find one or more combinations of a quantity (e.g., 6 can also be decomposed into 6 and 0, 3 and 3, or 5 and 1). This work lays the foundation for making meaningful sense of 4 + 2 = 6 and 6 - 4 = 2.



Toss the Chips Recording Sheet

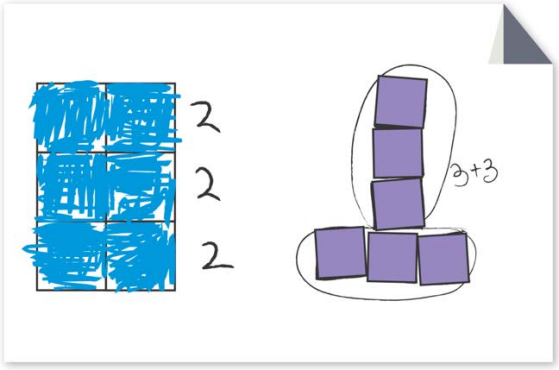
Game 1 Total Number: 6

Red	Yellow
2	4
1	5
3	3
0	6

Game 2 Total Number: _____

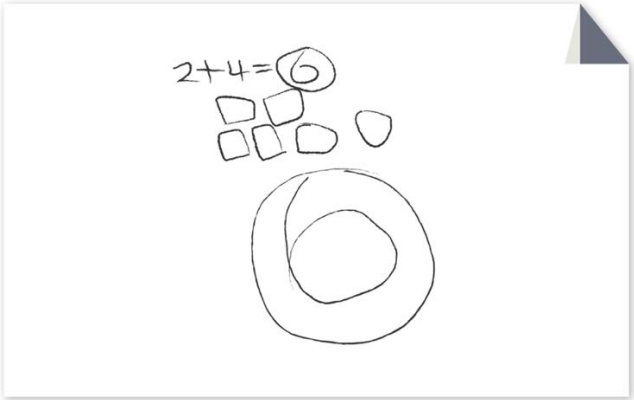
Red	Yellow

[Students think about different ways 6 can be composed of (or decomposed into) two amounts.]



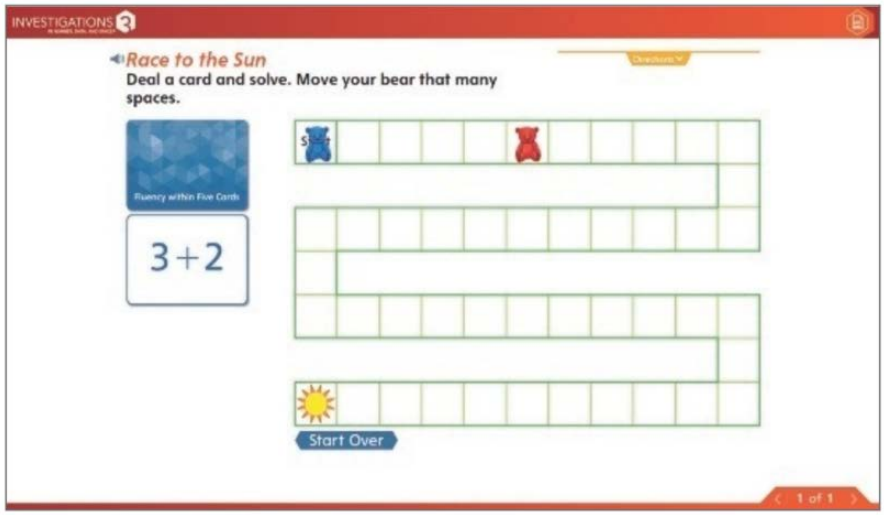
[Students record their arrangements of 6 tiles and indicate how they know there are 6 tiles in all.]

Students also encounter story problems about adding to, taking from, putting together, and taking apart. They retell the stories, act them out, model them with objects, solve them in the whole group and on their own, and eventually show their work on paper. They use mathematical tools and representations to model and solve problems and to clarify and communicate their thinking. They are just beginning to learn how to represent their mathematical work on paper and are encouraged to do so in ways that make sense to them. Many use a combination of pictures, words, numbers, and notation, building on the ways they have observed their teacher representing their strategies for the class throughout the year.



[This student draws a group of 2, a group of 4, and determines that she has 6 in all.]

Having had many opportunities to determine the expression that matches a given situation, students tell stories to match a given addition or subtraction expression. They play games where such expressions determine the number of spaces to move or counters to take. Such repeated practice leads to being able to fluently add and subtract within 5.



[Students turn over a card, determine the answer to the expression (e.g., 3 + 2), and move their bear that many spaces.]

In the units that focus on addition and subtraction (Units 4, 6, and 8), the **Algebra Connections in This Unit** Teacher Notes show how students apply the commutative property of addition, encounter the relationship between addition and subtraction, and generalize about the operations of addition and subtraction.

MAIN MATH IDEA

- Understanding, representing, and solving addition and subtraction problems

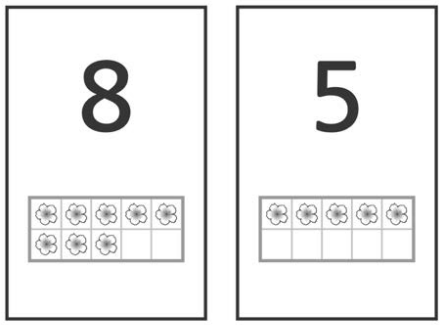
BENCHMARKS

- Figure out what is one more or one less than a number. (Unit 4)
- Represent and solve addition story problems within 10. (Unit 6)
- Decompose a number into two addends in more than one way. (Unit 6)
- Represent and solve subtraction story problems within 10, with result
- unknown. (Unit 8)
- Add and subtract fluently within 5. (Unit 8)

Grade 1

Students make an important connection between counting and quantity and the operations of addition and subtraction. They come to recognize that counting on (or back) 1 or 2 (or any amount) from a given number is the same as adding (or subtracting) 1 or 2 (or any amount) to (or from) that given number. Such discussions begin students' work with addition and subtraction within 20. Activities, Classroom Routines, and games that ask students to combine two or more quantities, to remove one quantity from another, and to compose/decompose numbers in different ways provide repeated practice with these ideas throughout the year and support the development of fluency within 10. Discussions focus on strategies for efficiently adding and subtracting numbers, with a particular focus on using what students know about the combinations of 10 and the relationship between addition and subtraction.*

Students come to understand that 6 = 4 + 2, 2 + 2 + 2 = 6, and 2 + 4 = 3 + 3 are all valid equations because the equal sign signifies equivalence—whatever is on one side of the equation has to balance or have the same value as whatever is on the other side. They have many opportunities to see, discuss, and use a variety of equation formats as they solve and discuss story problems and record their work on games (e.g., Dot Addition) and in activities (e.g., Today's Number). Students are also explicitly asked to consider given equations, to decide whether they are true or false, and to explain why they think so. This understanding can help students think about and make sense of helpful strategies, such as making a 10, to solve an addition problem (e.g., 8 + 5 = 10 + ___).



["Move two from the 5, and put them with the 8 to make 10. 10 plus 3 is 13."]

* When solving subtraction problems, some students count or add up to find the difference. They are turning a subtraction problem into an unknown addend problem. While not a benchmark in a single place, teachers are encouraged to look for and track evidence of such strategies over the course of the year.

Understanding and solving a variety of story problem types, with unknowns in all positions, is a major focus. Throughout, the emphasis is on making sense of the problem and what it is asking. Students listen to and retell the story and consider whether the resulting amount will be more or less than the initial amount. They solve the problem,

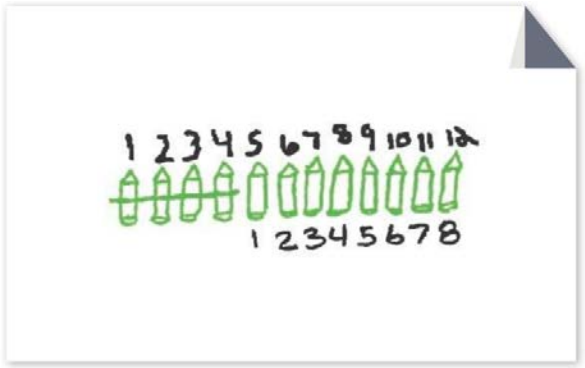
using available tools as needed (e.g., cubes, Ten Frames, and number lines), and show their work so that someone else could tell how they solved it. Most use a combination of numbers, symbols, pictures, and words. Finally, they share and compare solution strategies. Notation is introduced and modeled, with a focus on understanding how the quantities and actions described in a story problem can be represented with an equation. Note that some of this work is in units focused on Measurement or Data, which contain contexts that especially lend themselves to adding and subtracting. Students also solve related problems that encourage them to notice the relationship between problems (often involving a property of addition or the relationship between addition and subtraction) and to use one problem to solve the other. *

Problem Type	Description and Location
Add To	In these problems, two amounts are joined. The unknown can be the result (U1), the amount added (U5), or the starting amount (U6).
Take From	In these problems, one amount is removed from another. The unknown can be the amount remaining (U1), the amount removed (U5), or the initial amount (U6).
Put Together/ Take Apart	These problems involve two groups, but no action (e.g., joining or removing). The unknown can be the total (U1), the number in one group (U5), or the number in both groups (U3).
Comparison	These problems involve comparing two amounts. The unknown can be the difference (U4) or either of the amounts (bigger or smaller) (U6, U7).

[For more information, see **Teacher Note 8:** Types of Story Problems, Unit 1.]

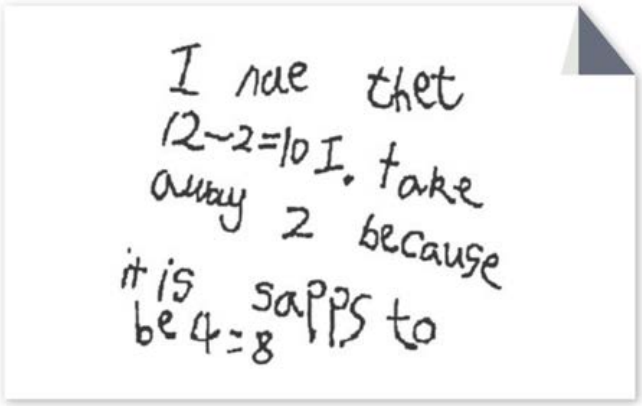
* When solving problems about combining, separating, and/or comparing, some students use strategies that involve the properties of operations. While not a benchmark in a single place, teachers are encouraged to look for and track evidence of such strategies over the course of the year.

Depending on the problem type and the time of year, students' strategies typically fall into three categories: counting all of the quantities in the problem by 1s; counting on (or back); or using something they know about a particular fact, or about the operation, to solve the problem.



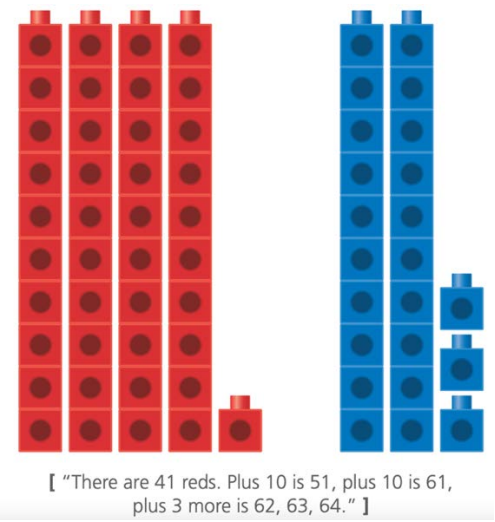
[To solve 12 – 4, this student showed all 12 of the pencils, crossed out 4, and then counted the ones that were left.]

[This student *counted back* 4 from 12.]

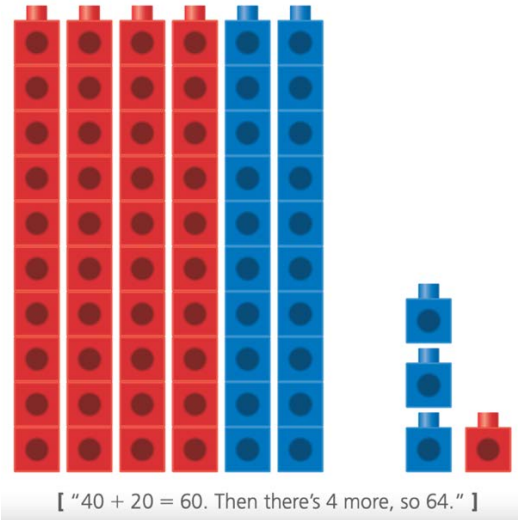


[This student used something he knew to subtract 4 from 12. First he subtracted 2, to get to 10, and then he subtracted 2 more.]

Students apply their understanding of place value and the operations as they add and subtract larger numbers. They use a place-value model (i.e., cubes in towers of 10) to add 2-digit numbers and to subtract multiples of 10 from multiples of 10. Students approach this work in a variety of ways, almost all of which rely on place value. To solve 41 + 23, some students add one number on in parts (e.g., 41 + 10 = 51, 51 + 10 = 61, and 61 + 3 = 64), while others add by place (e.g., 40 + 20 = 60, 1 + 3 = 4, and 60 + 4 = 64). They encounter situations where there are more than 9 ones and consider what happens to the answer in those situations. The steps of students' strategies are modeled with equations, and discussions focus on relating the numbers and symbols to the model and actions and thinking about what part of the number changes and why.



["There are 41 reds. Plus 10 is 51, plus 10 is 61, plus 3 more is 62, 63, 64."]



["40 + 20 = 60. Then there's 4 more, so 64."]

In units that focus on addition and subtraction, the **Algebra Connections in This Unit** Teacher Notes illustrate the kinds of generalizations students make about addition and subtraction, show how students explicitly encounter the commutative and associative properties of addition, show how students' strategies depend on properties of the operations, and demonstrate how they come to understand the relationship between addition and subtraction.

MAIN MATH IDEAS

- Understanding, representing, and solving problems involving addition and subtraction
- Understanding equivalence
- Using knowledge of place value to add and subtract

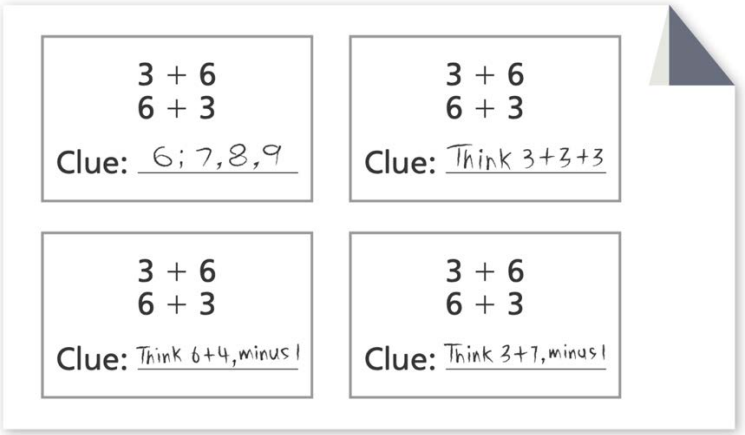
BENCHMARKS

- Understand that you can count on/back to add/subtract 1 or 2. (Unit 1)
- Fluency with the +1, +2, -1, -2 facts. (Unit 1)
- Determine which of two pairs of numbers to 10 is greater. (Unit 1)
- Solve a take from story problem with result unknown. (Unit 1)
- Solve an add to story problem with result unknown. (Unit 1)
- Solve a put together story problem with total unknown. (Unit 1)
- Understand that you can count on/back to add/subtract. (Unit 3)
- Find at least 5 solutions to a put together/take apart problem with both addends unknown. (Unit 3)
- Solve story problems with 3 addends. (Unit 3)
- Represent numbers with equivalent expressions. (Unit 3)
- Solve comparison story problems with the difference unknown (how many more and how many fewer). (Unit 4)
- Fluency with addition and subtraction within 10. (Unit 5)

- Solve a put together/take apart problem with one addend unknown. (Unit 5)
- Understand the meaning of the equal sign. (Unit 5)
- Determine the unknown in an addition or subtraction equation relating 3 numbers (e.g., 5 + ___ = 8). (Unit 5)
- Solve add to and take from problems with unknown change. (Unit 5)
- Solve comparison story problems with a bigger or smaller unknown. (Unit 6)
- Subtract multiples of 10 from multiples of 10 using concrete models that represent tens and ones. (Unit 7)
- Add within 100 using concrete models that represent tens and ones. (Unit 7)

Grade 2

Students develop fluency with addition facts to 10 + 10 and the associated subtraction facts over the course of the year. Games, activities, and a Classroom Routine provide repeated practice, as do Fact Cards, which students get in related sets and sort into envelopes of facts they know and facts they're still working on. Throughout, this work focuses on using known facts as "clues" to learn other facts (e.g., 6 + 8 = 6 + 6 + 2 or 14 - 6 = 14 - 4 - 2; or "I know 8 + 3 = 11, so 11 - 3 = 8."). The development of fluency is based on what students know and understand, rather than on memorization.



[Four students' clues for remembering 3 + 6 and 6 + 3.]

As in Grade 1, students make sense of and solve a variety of story problem types involving addition and subtraction, using a variety of contexts (objects, stickers, money, measurements), with unknowns in all positions.

Problem Type	Description and Location
Add To	In these problems, two amounts are joined. The unknown can be the result (U1), the amount added (U3), or the starting amount (U3).
Take From	In these problems, one amount is removed from another. The unknown can be the amount remaining (U1), the amount removed (U3), or the initial amount (U3).
Put Together/ Take Apart	These problems involve two groups, but no action (e.g., joining or removing). The unknown can be the total (U1), the number in one group (U3), or the number in both groups (U3).
Comparison	These problems involve comparing two amounts. The unknown can be the difference (U1), or either of the amounts (bigger or smaller) (U5, U8).

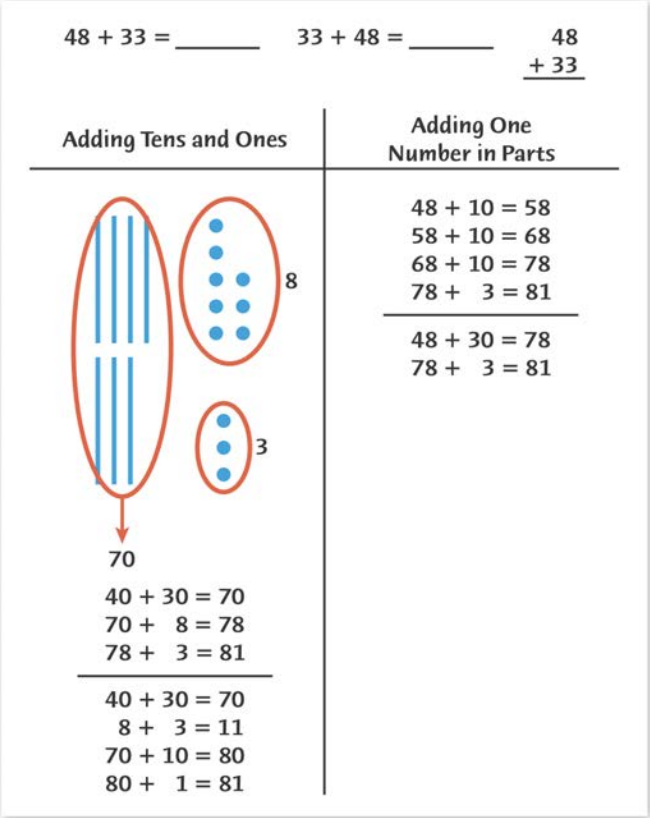
[For more information, see Teacher Note 10: Types of Story Problems, Unit 1.]

Students use mathematical tools and representations (e.g., number lines,* sticker notation) to model and solve problems and to clarify and communicate their thinking.** They are encouraged to show their mathematics work on paper in ways that make sense to them; many use a combination of pictures, words, numbers, and equations. The focus is on developing ever more efficient strategies for solving problems and for recording their work.

* When students solve problems about combining, comparing, or removing, some use the number line to represent the problem, solve the problem, and/or show their work. While not a benchmark in a specific unit, teachers are encouraged to look for and track evidence of students' understanding of the number line throughout the year.

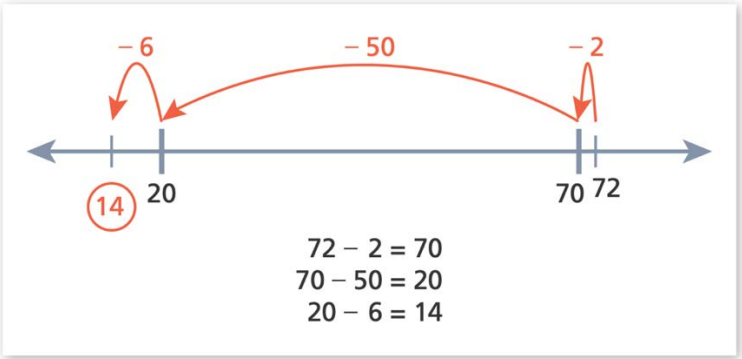
** Throughout *Investigations*, students are asked to show their work. As they describe their strategies, they are often asked to consider and explain why their strategies work. While not a benchmark in a specific unit, there are many identified opportunities to look for and track evidence of students' abilities throughout the year.

While students see and discuss a range of methods, there is a focus on two strategies for addition: adding tens and ones and adding one number in parts.

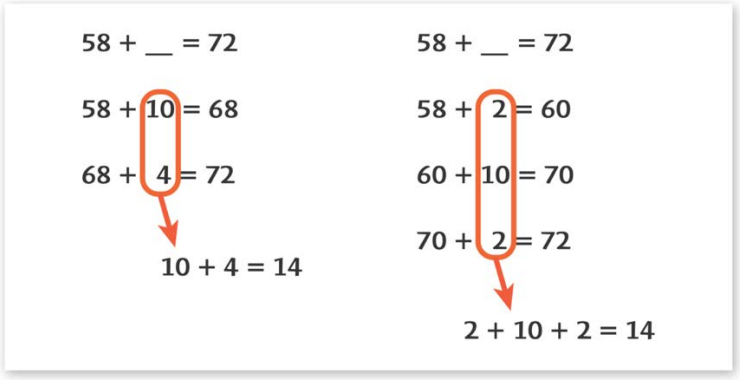


[The teacher records students' strategies for solving a story problem about 48 + 33.]

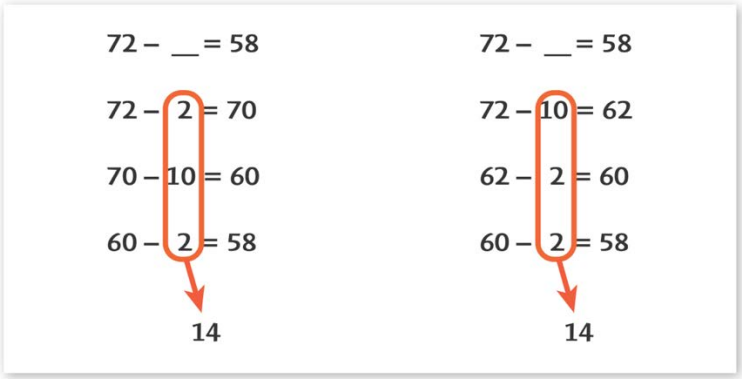
Similarly, students focus on two main strategies for subtraction: subtracting in parts and adding up/subtracting back.



[A student solves 72–58 by subtracting the 58 in parts.]

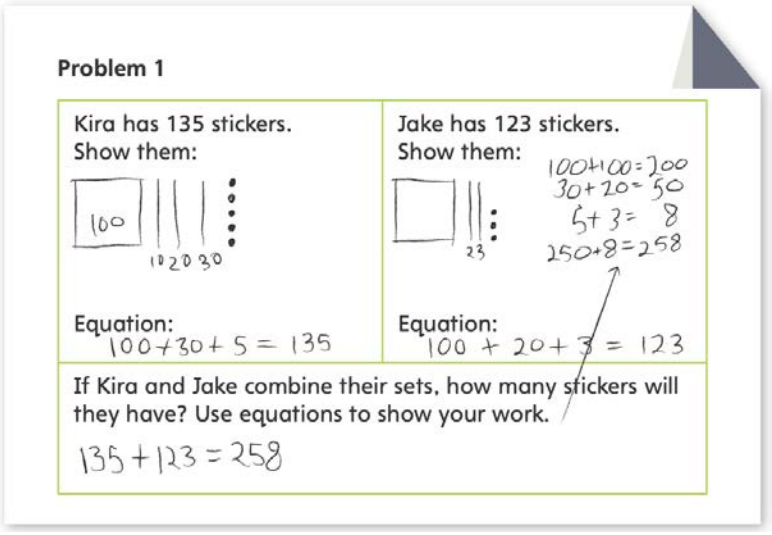


[These students add up to find the difference between 58 and 72.]



[These students subtract back to find the difference between 72 and 58.]

All these strategies depend on an understanding of place value. Students end the year with at least one strategy for accurately and efficiently adding and subtracting within 100. Students apply their understanding of place value and the operations, as well as their strategies for adding and subtracting 2-digit numbers, as they work with larger numbers. They use place-value notation to model and solve problems about adding and subtracting 3-digit numbers. They encounter addition problems where there are more than 9 ones (and/or 9 tens), and subtraction problems where a hundred (and/or a ten) needs to be broken apart; and they reflect on how the digits in the numbers change in those situations. Class discussions focus on using equations to model the steps of students' strategies and on relating the numbers and symbols to the place-value representation (e.g., sticker notation or towers of 10 cubes) and to components of the problem.



Problem 1

Kira has 135 stickers.
Show them:

100 | 10 20 30 ⁞

Equation:
100 + 30 + 5 = 135

Jake has 123 stickers.
Show them:

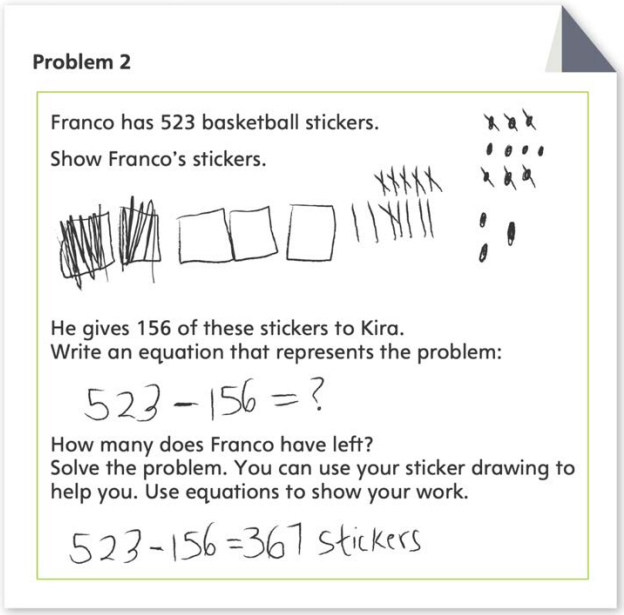
23

100+100=200
30+20=50
5+3=8
250+8=258

Equation:
100 + 20 + 3 = 123

If Kira and Jake combine their sets, how many stickers will they have? Use equations to show your work.

135 + 123 = 258



Problem 2

Franco has 523 basketball stickers.

Show Franco's stickers.

He gives 156 of these stickers to Kira.
Write an equation that represents the problem:

523 − 156 = ?

How many does Franco have left?
Solve the problem. You can use your sticker drawing to help you. Use equations to show your work.

523 - 156 = 367 stickers

In the curriculum units that focus on addition and subtraction, the **Algebra Connections in This Unit** Teacher Notes illustrate the kinds of generalizations students use as they add and subtract, show how students are applying the commutative and associative properties of addition, and highlight how students study the relationship between addition and subtraction.

MAIN MATH IDEAS

- Fluency within 20
- Understanding, representing, and solving problems involving addition and subtraction
- Using knowledge of place value to add and subtract

BENCHMARKS

- Use known combinations to add several numbers in any order. (Unit 1)
- Solve a comparison story problem with the difference unknown. (Unit 1)
- Solve put together/take apart story problems with the total unknown and add to and take from story problems with the result unknown. (Unit 1)
- Solve a put together/take apart story problem with both addends unknown and find all the possible combinations. (Unit 3)
- Solve a put together/take apart story problem with one addend unknown. (Unit 3)
- Solve two-step story problems about money. (Unit 3)
- Solve story problems with an unknown change. (Unit 3)
- Solve story problems with an unknown start. (Unit 3)
- Solve a 2-step story problem that involves finding the difference between a 2-digit number and 100. (Unit 5)
- Add/subtract 10 or 100 to/from numbers within 1,000. (Unit 5)
- Add fluently within 100. (Unit 5)
- Solve comparison story problems with a bigger unknown. (Unit 5)
- Solve a comparison story problem with a smaller unknown. (Unit 8)
- Fluently subtract two 2-digit numbers. (Unit 8)
- Fluently add and subtract within 20. (Unit 8)
- Represent and solve addition and subtraction problems with 3-digit numbers. (Unit 8)

Grade 3

Students deepen their understanding of the meaning of addition and subtraction and the relationship between the operations through visualizing and solving story problems. Among the types of problems students encounter are adding to, taking from, putting together/taking apart, and comparison problems. Contexts include liquid volume, mass, money, and distance. Problems may involve more than two addends or situations that require more than one step.

To solve addition and subtraction problems, students must understand the meaning of the operations and have a good mental model of what is happening in the problem. The ability to visualize what is happening in these different types of story problems is an important foundation for students in understanding the operations and for making good decisions about choosing strategies for computation.

As students solve a variety of addition and subtraction problems, they discuss, refine, and compare strategies. Two addition strategies receive emphasis: 1) breaking apart both numbers by place and adding like places, and 2) breaking apart one number and adding it in parts. Students also examine the strategy of changing the numbers (either changing

both numbers and adjusting or creating an equivalent problem) to make them easier to add.

Breaking the numbers apart

Adding by place

$349 + 175 =$

Solution 1	Solution 2
$300 + 100 = 400$	$300 + 100 = 400$
$40 + 70 = 110$	$(30 + 70 = 100)$
$9 + 5 = 14$	$10 + 100 = 110$
$400 + 110 + 14 = 524$	$9 + 5 = 14$
	$400 + 110 + 14 = 524$

Adding one number in parts

$349 + 175 =$

Solution 1	Solution 2
$349 + 100 = 449$	$349 + 100 = 449$
$449 + 70 = 519$	$449 + 50 = 499$
$519 + 5 = 524$	$499 + 25 = 524$

Changing the numbers

Changing the numbers and adjusting

$349 + 175 =$

Solution 1	Solution 2
$350 + 175 = 525$	$349 + 200 = 549$
$525 - 1 = 524$	$549 - 25 = 524$

Creating an equivalent problem

$349 + 175 =$

Solution 1	Solution 2
$324 + 200 = 524$	$400 + 124 = 524$

Subtraction strategies that are emphasized are subtracting a number in parts, adding up, and subtracting back.

Subtracting in parts

$251 - 187 =$

Solution 1

$251 - 187 =$
$251 - 100 = 151$
$151 - 80 = 71$
$71 - 7 = 64$

Solution 2

$$\begin{array}{r} 251 \\ -\ 100 \\ \hline 151 \\ -\ 50 \\ \hline 101 \\ -\ 30 \\ \hline 71 \\ -\ 7 \\ \hline 64 \end{array}$$

Adding up or subtracting back

Adding up

$251 - 187 =$

Solution 1

$187 + \underline{13} = 200$
$200 + \underline{51} = 251$
$13 + 51 = 64$

Solution 2

$187 + \underline{20} = 207$
$207 + \underline{40} = 247$
$247 + \underline{4} = 251$
$20 + 40 + 4 = 64$

Subtracting Back

$251 - 187 =$

Solution 1

$251 - \underline{51} = 200$
$200 - \underline{13} = 187$
$51 + 13 = 64$

Solution 2

$$\begin{array}{r} 251 \\ -\ 51 \\ \hline 200 \\ -\ 10 \\ \hline 190 \\ -\ 3 \\ \hline 187 \end{array}$$

$51 + 10 + 3 = 64$

The **Algebra Connections in This Unit** Teacher Notes in Units 3 and 7 show how the associative and commutative properties of addition and the relationship between addition and subtraction are implicit in students' work.

MAIN MATH IDEAS

- Using knowledge of place value to add and subtract
- Adding and subtracting fluently
- Understanding different types of addition and subtraction problems

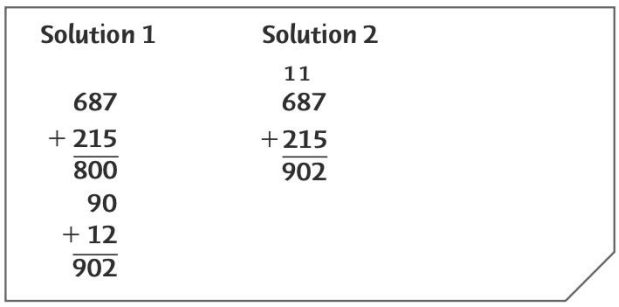
- Solving problems involving measurement of liquid volume and mass
- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers

BENCHMARKS

- Use knowledge of place value to read, write, sequence, and round
- numbers up to 1,000. (Unit 3)
- Solve addition problems with 3-digit numbers (up to 400) by using strategies that involve breaking each number apart by place, or by adding on one number in parts. (Unit 3)
- Solve subtraction problems with 2- and 3-digit numbers (up to 300) by
- using strategies that involve either subtracting one number in parts, adding up, or subtracting back. (Unit 3)
- Solve addition and subtraction problems involving masses or volumes. (Unit 7)
- Solve 3-digit addition problems using at least one strategy fluently. (Unit7)
- Solve 3-digit subtraction problems fluently. (Unit 7)

Grade 4

Students practice a variety of addition and subtraction strategies, most of which should be familiar from Grade 3. Students discuss, refine, and compare strategies for addition and subtraction, including the U.S. standard algorithms. Students learn the steps of the U.S. standard algorithms for addition and subtraction, discuss the meaning of the notation, and practice the algorithms. By explaining, refining, and comparing the addition and subtraction strategies that they are using, students are adding to the repertoire of strategies they can use for flexible and fluent computation.



Solution 1

$$\begin{array}{r} 687 \\ +\,215 \\ \hline 800 \\ 90 \\ +\,12 \\ \hline 902 \end{array}$$

Solution 2

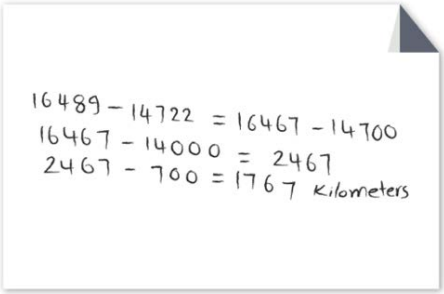
$$\begin{array}{r} {\scriptstyle 11} \\ 687 \\ +\,215 \\ \hline 902 \end{array}$$

Students work with place value of large numbers, and their study of addition and subtraction strategies are combined as they add and subtract numbers in the thousands and ten thousands. Students are encouraged to look carefully at the numbers in the problem, and then choose from a variety of strategies they can use confidently, including the U.S. standard algorithms, to solve problems such as the following:

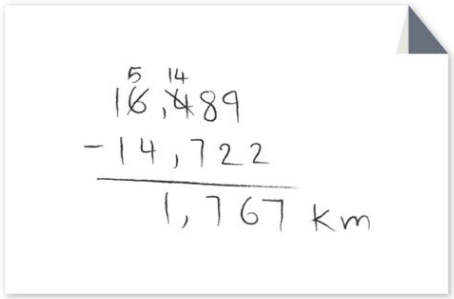
Ursula traveled from San Francisco, California, to Cape Town, South Africa. It is 16,489 kilometers from San Francisco to Cape Town. She then traveled from Cape Town to Tokyo, Japan. It is 14,722 kilometers from Cape Town to Tokyo.

How much farther is it from San Francisco to Cape Town than from Cape Town to Tokyo? Show how you solved the problem.

Solution 1



Solution 2



The Algebra Connections in This Unit Teacher Note in Unit 5 shows how students are applying the inverse relationship between addition and subtraction as they solve problems. It also highlights the algebraic idea that given an addition expression, if one addend increases by some amount and the other addend decreases by the same amount, the sum is unchanged—a concept students may use to create equivalent addition problems that are easier to solve (e.g., 597 + 375 = 600 + 372).

MAIN MATH IDEAS

- Extending knowledge of the number system to 1,000,000
- Adding and subtracting fluently
- Describing, analyzing, and comparing strategies for adding and subtracting whole numbers

BENCHMARKS

- Read, write, and compare numbers up to 1,000,000 and round them to any place. (Unit 5)
- Fluently solve multidigit addition and subtraction problems using a variety of strategies, including the U.S. standard algorithms. (Unit 5)
- Use addition and subtraction to solve word problems involving measurement. (Unit 5)