INVESTIGATIONS 3
TRANSFORMING MATHEMATICS TEACHING & LEARNING

Math Content by Strand

Counting and The Number System

Counting

Kindergarten

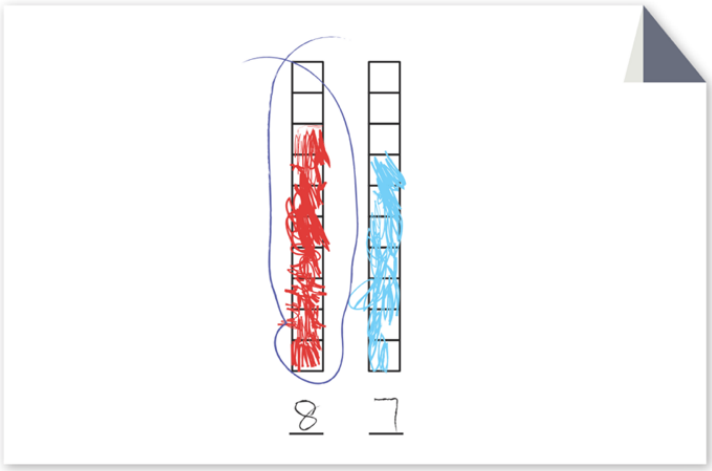
Counting is a central focus, as it is the basis for understanding the number system and for almost all the number work in the primary grades. Students hear and use the counting sequence (the number names, in order) in a variety of contexts. They have many opportunities to connect the number names (e.g., "five") with the written numbers (e.g., 5) and with the quantities they represent. They have repeated experiences counting sets of objects (e.g., "How many bears are there?"), making sets of a given size (e.g., "Can you show me 8 bears?"), and using spoken and written numbers to label those sets. Ten Frames offer students a tool for organizing sets of up to 10 objects and provide them with a structure for thinking about a 10 as two groups of 5 and, later, as a group of 10 ones—an idea critical in their work with teen numbers.

[This student used pictures, numbers, and words to show that he counted 13 nuts. He drew a circle for each nut and wrote both the number and the unit he counted—"13 nts." When he recounted his circles to check, he realized he had one too many, so he scribbled out one circle.]

As students count sets of objects and make sets, they begin to see the importance of counting each object once and only once and of having a system for keeping track of what has been counted and what remains to be counted. Students encounter increasing amounts over the course of the year, beginning with sets of up to 10, then 15, and ending the year fluently counting to—and counting out sets of—20. Because counting is a part of activities such as taking attendance, collecting data via Today's Question, and solving

problems about the class, students also have repeated experiences with an amount that may be greater than 20 (e.g., the number of students in their class). Similarly, they may encounter numbers in this range as they count to determine the lengths of objects and paths.

As students develop accurate counting strategies, they also build an understanding of how numbers in the counting sequence are related: Each number is one more (or one less) than the number before (or after) it. Students develop an understanding of the concepts of greater than, fewer than, and equal to; and they develop language for describing quantitative comparisons (e.g., bigger, more, smaller, fewer, less, same, larger, greater, equal) as they count and compare quantities.



[Students grab and then count the number of cubes in each of two handfuls. They represent the handfuls, use numbers to record the totals, and circle the amount with more.]

In addition to counting quantities, students practice writing the numbers to 20 and reciting the rote counting sequence to 100. They count by 1s from 1—and from numbers other than 1—using the number line to keep track of the numbers they are saying. They also practice the counting-by-10s sequences to 100, tying it to the context of counting students' fingers.

The Teacher Note **Algebra Connections in This Unit** in Unit 2 examines how students grapple with the idea that the number of objects in a set is fixed no matter how it is arranged or counted as they think about the question, "Does order matter when counting?"

MAIN MATH IDEAS

- Counting and representing quantities
- Comparing and ordering quantities
- Collecting, representing, describing, and interpreting data

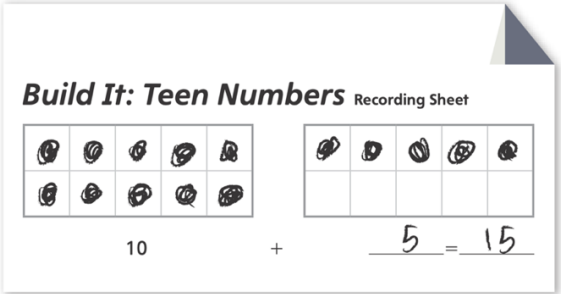
BENCHMARKS

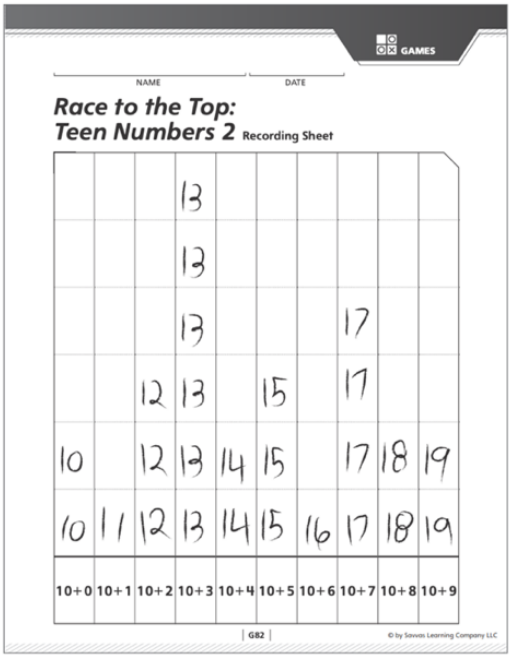
- Count and count out a set of up to 10 objects. (Unit 2)
- Compare two quantities up to 10 to determine which is greater. (Unit 2)
- Count and count out a set of up to 15 objects. (Unit 4)
- Count and count out a set of up to 20 objects. (Unit 6)
- Write the numbers to 10. (Unit 6) b Write the numbers to 20. (Unit 8)
- Rote count by 1s and 10s to 100; when counting by 1s, start from a number other than 1. (Unit 8)

The Number System

Kindergarten

Students' work composing and decomposing numbers in different ways culminates in a focus on ideas that lay the foundation for understanding place value, the base-10 nature of our number system. Students build on their familiarity with Ten Frames and use their structure to explore the two-addend combinations that make 10. This sense of 10 as an important number in our number system supports students in making sense of the teen numbers as ten 1s and some number of leftover 1s. Using equations to record such information (e.g., 15 = 10 + 5, 16 = 10 + 6) helps students notice important regularities in the structure of these numbers and the written numerals: The 1 in 15 refers to the group of ten 1s and the 5 refers to the number of leftover 1s.





MAIN MATH IDEA

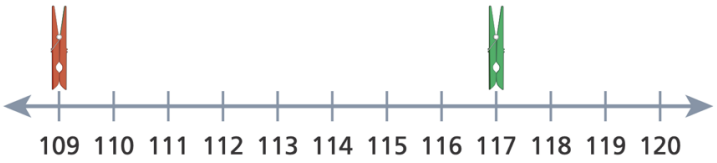
- Understanding place value

BENCHMARKS

- Figure out a missing addend when the sum is 10. (Unit 8)
- Represent the teen numbers as ten 1s and some number of 1s. (Unit 8)

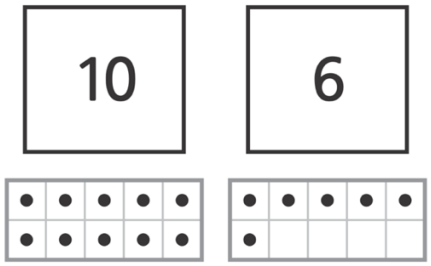
Grade 1

Students extend their understanding of counting and the number sequence, building a strong foundation for their work with place value and the operations of addition and subtraction. They practice the rote counting sequence to 120 and read and write numbers within this range. Students connect number names with written numbers and the quantities that they represent as they count—and count out sets of—objects. They develop reliable and efficient strategies for accurately counting, first by 1s, and eventually by groups of 5 and 10. Being able to count flexibly forward and back from any number, by 1s and eventually by 10s, supports students' work with addition and subtraction.



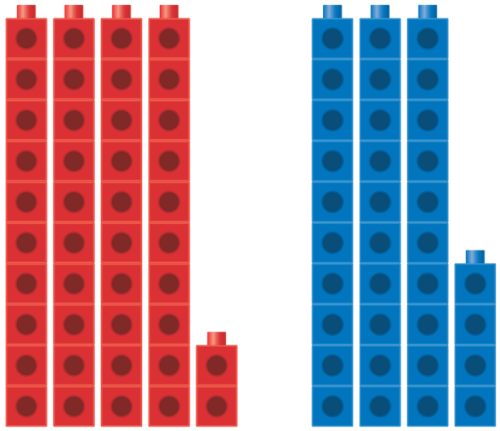
[In the *Start With/Get To* routine, students practice counting forward and back.]

Students transition from understanding 10 as 10 ones to understanding 10 ones as 1 ten—a foundational idea in the base-10 number system. Students extend this idea to the teen numbers, coming to see every teen number as one group of ten and some number of ones. They also examine the relationship between the written number and how it is represented with Ten Frame cards, a tool that represents and reinforces the structure of tens and ones, noticing how the written numeral also represents the tens and ones structure of our place-value system.



[10 + 6 = 16]

Students use this understanding as they think about the number of fingers in a group of students, cubes in so many towers of 10, and dots on so many Ten Frame cards, as well as how many there would be if one (or more) group(s) of 10 were added (or subtracted). This work helps students come to understand the way the counting-by-10 numbers are written. For example, they might reason, "If I build the number 40 with towers of 10, I need 4 towers. The 4 in the number 40 tells me that." With this knowledge of tens, students investigate the place value of 2-digit numbers. As they represent given numbers with cubes organized in towers of tens and ones, and as they determine the total when given groups of cubes, students come to understand that in a 2-digit number, the tens digit represents the number of tens and the ones digit represents the number of ones. They apply this knowledge as they revisit and use greater than and less than notation to compare 2-digit numbers, thinking about how the number of tens helps with the comparison.



[Some students use knowledge of the counting sequence to compare numbers. They might say, "42 is greater because it's higher on the number line. Others reason about tens and ones, thinking "42 has more groups of 10, so it's greater than 34."]

Composing and decomposing 2-digit numbers and representing 2-digit numbers as the sum of a multiple of ten and some number of ones (e.g., 22 = 20 + 2) lays the foundation for adding and subtracting larger numbers.

MAIN MATH IDEAS

- Understanding and extending the counting sequence
- Understanding place value

BENCHMARKS

- Understand ten ones as one ten, and the teen numbers as one ten and some number of ones. (Unit 3)
- Rote count, read, and write numbers to 120. (Unit 3)
- Understand that the multiples of 10 through 90 refer to 1-9 tens and 0 ones. (Unit 7)

- Use a numeral to represent a number of objects organized into tens
- and ones and, given a numeral, represent it with tens and ones. (Unit 7)
- Use standard notation (6, 7) to represent the comparison of two 2-digit
- numbers. (Unit 7)
- Add or subtract 10 to/from any 2-digit number. (Unit 7)

Grade 2

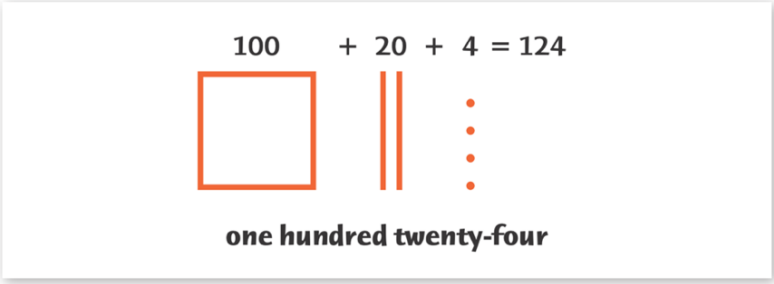
Students begin the year briefly reviewing the counting of large quantities and the number sequence. They learn to recognize and identify coins and their values and use equivalencies among them. Such models support students in thinking about and working with groups. As a review of place value, they investigate what happens when they count a set of objects by groups of 2, 5, and 10, focusing specifically on how the written numeral is another representation of the tens and ones structure of numbers in the base-10 number system.

Grouping by 10s

Total Number of Cubes	Number of Towers of 10	Number of Leftovers
13	1	3
17	1	7
23	2	3

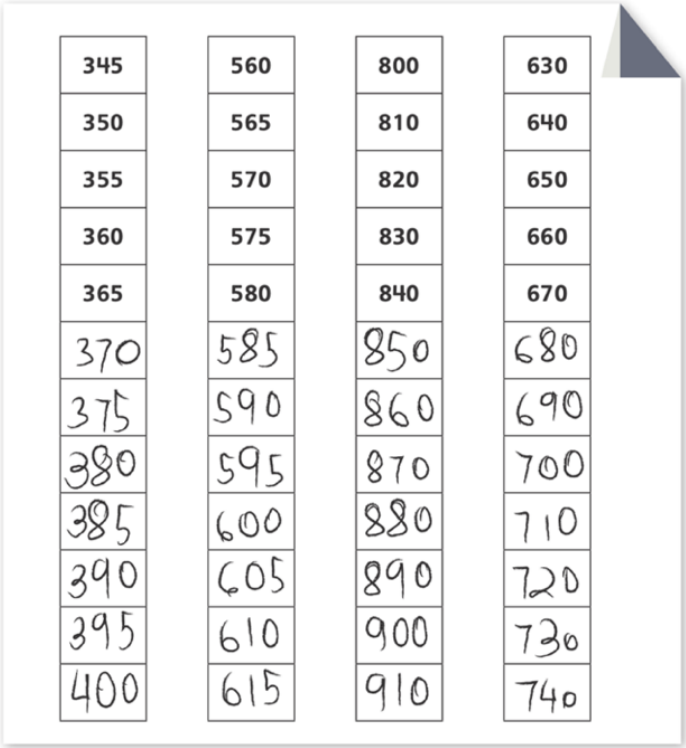
[Students count sets of cubes into groups of 10 and record the results.]

In addition to money and cubes in groups of 10, students work with a model for place value, Sticker Station, which sells stickers singly, in strips of 10, and in sheets of 100. This context—and the notation developed to represent it—is used to represent and visualize the composition of 2- and 3-digit numbers. Students come to see that 100 can be made from 100 ones, 10 tens, or 1 hundred and that numbers like 300 are made of 3 hundreds, or 100 + 100 + 100. This culminates in students being able to represent 3-digit numbers in sticker notation and in expanded form. They think about how the number of sheets, strips, and singles connect to the number of 100s, 10s, and 1s, and to the way a number is written in words, with numbers, and in expanded form.



[Students use expanded form, sticker notation, and words to represent the number 124.]

Students extend the rote sequence of numbers to 500, and then 1,000. They find and use patterns in the sequence of numbers in a 500 Book (made from five 100 charts) and a 1,000 Book (made from five 200 charts) as they read, write, and compare numbers, and as they practice counting within this range, by 1s, 5s, 10s, and 100s.



345	560	800	630
350	565	810	640
355	570	820	650
360	575	830	660
365	580	840	670
370	585	850	680
375	590	860	690
380	595	870	700
385	600	880	710
390	605	890	720
395	610	900	730
400	615	910	740

[Students skip count by 5s and 10s and record the sequences on Counting Strips.]

MAIN MATH IDEAS

- Understanding and extending the counting sequence
- Understanding place value

BENCHMARKS

- Recognize and identify coins and their values. **(Unit 1)**
- Understand that 100 can be seen as 1 hundred, as 10 tens, and as 100 ones. **(Unit 3)**
- Understand that multiples of 100 (e.g., 200, 300, 400, etc.) are made up of a number (2, 3, 4, etc.) of hundreds. **(Unit 3)**
- Understand that 3-digit numbers represent amounts of hundreds, tens, and ones. **(Unit 5)**
- Read, write, count, and compare numbers to 1,000. **(Unit 5)** b Count by 5s, 10s, and 100s within 1,000. **(Unit 5)**

Grade 3

Students focus on understanding and extending knowledge of place value and the number system and on adding and subtracting fluently within 1,000. Students use their deepening understanding of place value to build and refine strategies for fluently solving addition and subtraction problems with whole numbers. This includes decomposing numbers by place, rounding numbers to the nearest 10 or 100 to gauge the reasonableness of their solutions, and accurately adding and subtracting multiples of 10 and 100.

MAIN MATH IDEAS

- Using knowledge of place value to add and subtract

BENCHMARKS

- Use knowledge of place value to read, write, sequence, and round numbers up to 1,000. (Unit 3)

Grade 4

Students extend their knowledge of the base-10 number system up to 1,000,000, examining how the value of any digit is determined by its place and that any digit is ten times as great as the same digit one place to its right. Students read numbers, write them in expanded form, and round them. Special focus is given to the structure of 10,000 as 10 thousands, 100 hundreds, 1,000 tens, and 10,000 ones. Understanding these place-value ideas is fundamental to solving addition and subtraction problems fluently.

MAIN MATH IDEAS

- Extending knowledge of the number system to 1,000,000

BENCHMARKS

- Read, write, and compare numbers up to 1,000,000 and round them to
- any place. (Unit 5)