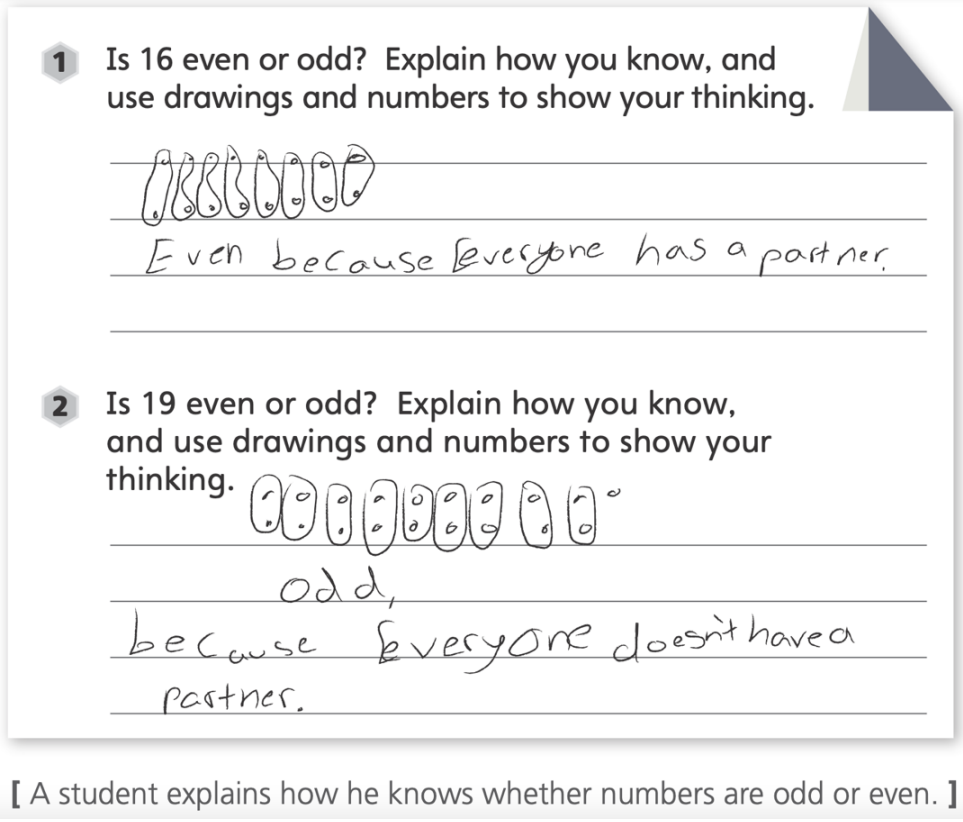
INVESTIGATIONS 3
TRANSFORMING MATHEMATICS TEACHING & LEARNING

Math Content by Strand

Multiplication and Division

Grade 2

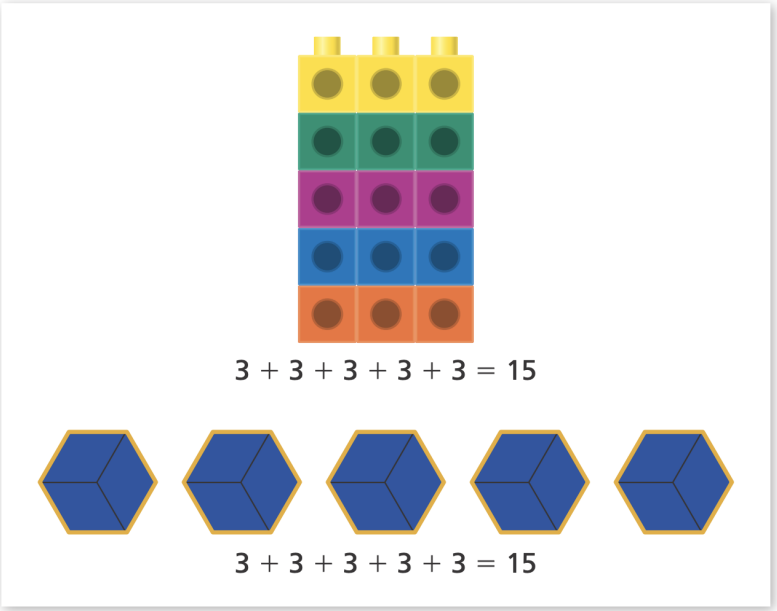
To lay the foundation for the study of multiplication, students build and model multiplicative situations that involve the accumulation of equal groups. They investigate odd and even numbers and multiplicative relationships in contexts where there are two equal groups (teams) and groups of 2 (partners). They use equations to represent even numbers as the sum of equal addends, and odd numbers as the sum of equal addends plus 1.



[A student explains how he knows whether numbers are odd or even.]

Work with multiplicative relationships also extends to adding multiple groups of 2 through 6 in two additional contexts. In the first context, students make cube buildings in the form of arrays, with an equal number of rooms per floor. They use equations to model their buildings, representing the total number of rooms as the sum of equal addends. In the second context, students investigate equal groups as they cover a certain number of a pattern block shape (e.g., hexagons) with another pattern block shape (e.g.,

rhombuses). Again, they use an equation to show the total number of blocks as the sum of equal addends. Students make observations about and compare the mathematical structure of these different-looking contexts and notice how different situations can have the same underlying mathematical relationship.



["Three rooms on each floor is the same as three rhombuses for one hexagon. There are 5 floors and 5 hexagons, so they both use the same equation."]

In all of these contexts, students explore multiplicative structures as they solve problems about adding equal groups, laying the foundation for later work with multiplication.

MAIN MATH IDEAS

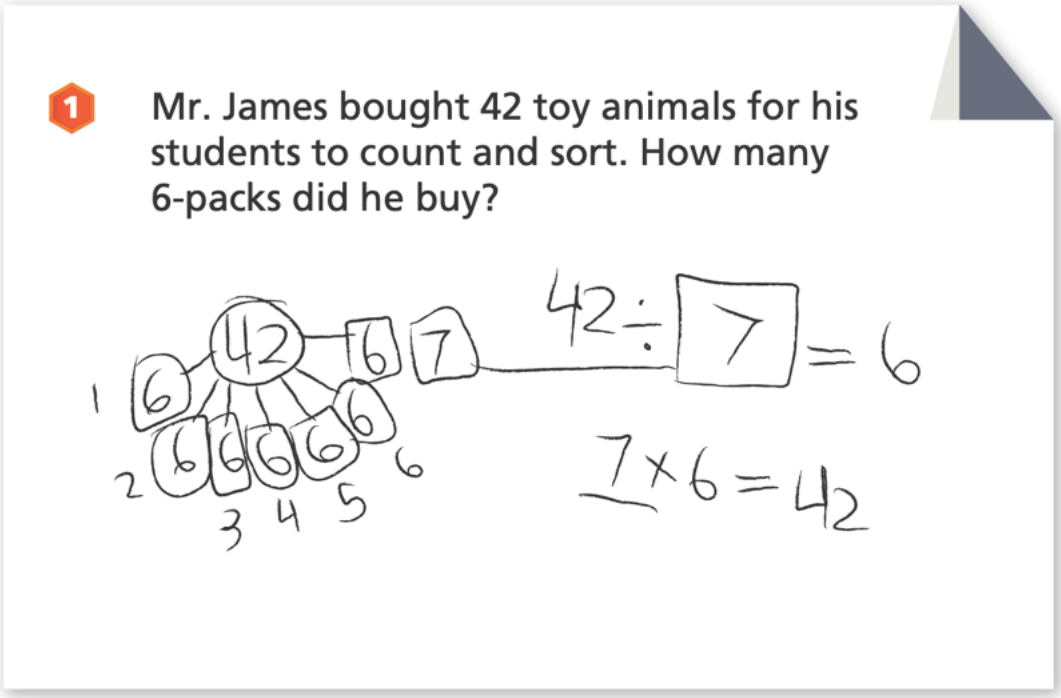
- Investigating odd and even numbers
- Visualizing equal groups in the structure of arrays
- Describing and representing equal groups as the foundation of multiplication

BENCHMARKS

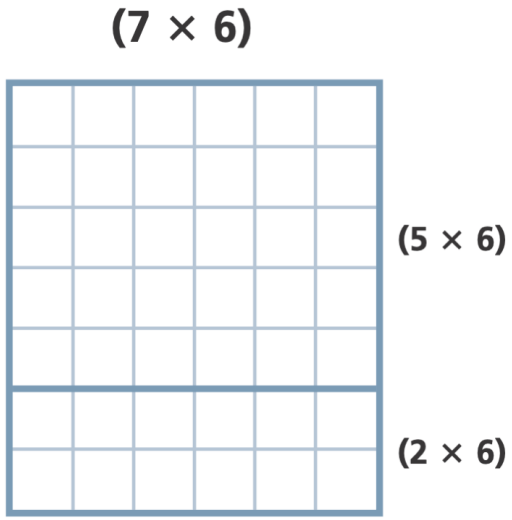
- Define even and odd numbers in terms of numbers that can/cannot be organized into groups of two or two equal groups. (Unit 7)
- Write an equation to express an even number as a sum of two equal addends. (Unit 7)
- Solve problems that involve equal groups. (Unit 7)
- Write an addition equation to express the total number of objects in a rectangular array. (Unit 7)

Grade 3

Students develop an understanding of the operations of multiplication and division, particularly focusing on multiplication and division as involving equal groups. Given problems in familiar contexts, students identify the components of these problems—the number of groups, the number in each group, and the number in all the groups. They identify where these components appear in multiplication and division equations. Division contexts involve two types of division situations: b Sharing (e.g., I have 18 balloons. I'm going to divide them evenly between my sister and me. How many balloons will each of us get?); b Grouping (e.g., I have 18 balloons. I'm going to tie them together in bunches of 2 to give to my friends. How many bunches can I make?). Creating a variety of physical models and other representations supports students' visualization of multiplicative relationships. Through working with such representations, students come to recognize the commutative and distributive properties and the relationship between multiplication and division.

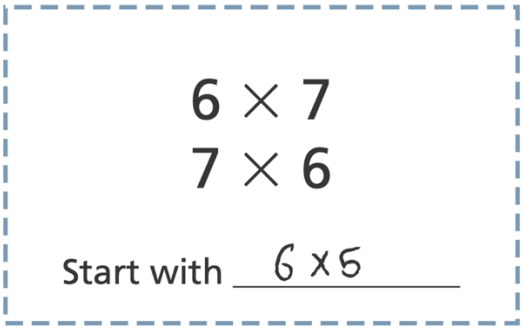


Key among the representations is the array—a rectangular arrangement of objects in equal rows and columns. Work with arrays also lays an important foundation for understanding why the dimensions of a rectangle can be multiplied to find the area of the rectangle.



$(5 \times 6) + (2 \times 6) = 7 \times 6$
$30 + 12 = 42$

Students develop fluency with multiplication facts to 10 × 10 and related division facts, using multiplication facts they already know to help them learn facts they are still working on. Visual images, including arrays and story contexts, help them see the relationships between known facts and unknown facts.



Multiplication with multiples of 10 is the focus of story problems set in the context of a toy factory that sells packages of toys in quantities of 1 through 9 and in multiples of 10. Students use representations to identify, express, and explain how an expression with single-digit factors, such as 3 × 4, is related to a problem involving a multiple of 10, such as 3 × 40. This concept, along with students developing fluency in multiplying in chunks (the distributive property), is the basis for solving multiplication and division problems within 100.

Students apply their growing understanding of multiplication and division to solve multi-step problems that involve more than one operation.

The **Algebra Connections in This Unit** Teacher Notes in Units 1, 5, and 8 show how the commutative, distributive, and associative properties of multiplication and the inverse relationship between multiplication and division are implicit in students' solution strategies. They also highlight how students identify and explain arithmetic patterns.

MAIN MATH IDEAS

- Understanding the meaning of multiplication
- Understanding and working with an array/area model of multiplication
- Learning the multiplication facts
- Developing strategies for division based on understanding the inverse
- relationship between multiplication and division
- Understanding the meaning and structure of multiplication and division and the relationship between them
- Solving multiplication and division problems, including multi-step problems and problems with multiple solutions
- Making sense of multiplying multiples of 10 by one-digit numbers
- Solving multiplication problems with 2-digit numbers
- Solving division problems b Learning division facts
- Identifying arithmetic patterns and solving multi-step problems

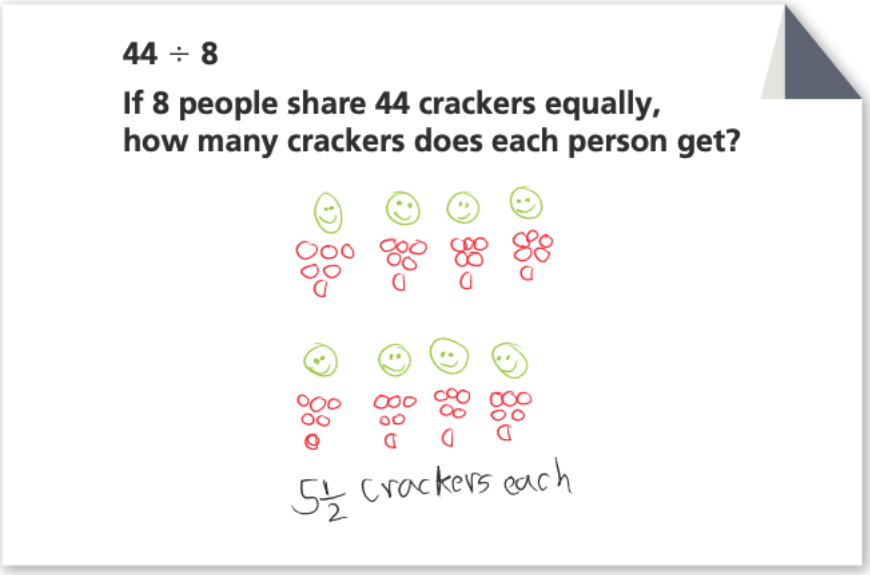
BENCHMARKS

- Demonstrate an understanding of multiplication and division as involving equal groups. (Unit 1)
- Solve multiplication and related division problems by using skip counting or known multiplication facts. (Unit 1)
- Interpret and use multiplication and division notation. (Unit 1)
- Demonstrate fluency with multiplication facts × 1, × 2, × 5, and × 10. (Unit 1)
- Represent and explain the relationship between multiplication and division. (Unit 5)
- Solve multiplication and division word problems and write equations to represent the problems. (Unit 5)
- Solve division problems (2-digit number divided by single-digit number). (Unit 5)
- Demonstrate fluency with multiplication facts to 10 × 10. (Unit 5)
- Multiply a single-digit number by a multiple of 10, up to 90. (Unit 5)
- Solve multi-step problems involving multiplication and addition. (Unit 5)
- Solve multiplication and division problems within 100. (Unit 8)
- Demonstrate fluency with the division facts. (Unit 8)
- Solve multi-step problems involving more than one operation. (Unit 8)
- Solve multiplication and division problems involving masses and volumes. (Unit 8)

Grade 4

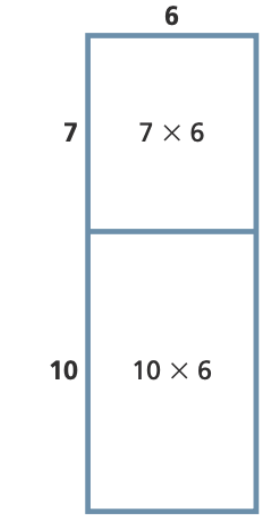
A major focus of the multiplication and division work in Grade 4 is developing strategies for operating with larger numbers: multiplying two 2-digit numbers and multiplying up to a 4-digit number by a 1-digit number; and dividing with up to a 4-digit dividend. This work builds on different aspects of understanding multiplication and division encountered in Grade 3, such as thinking in terms of equal groups, using arrays and area models to represent multiplication, multiplying by multiples of 10, and breaking multiplication or division problems apart to solve them.

Students continue to build on their understanding of multiplication and division as they solve story problems about equal groups, and they are introduced to, and solve, multiplicative comparison problems. For example, in Unit 1 students are asked to find the length of a python that is 4 times as long as a rattlesnake and to show their solutions using representations and equations. Students also solve division problems where they must make sense of what to do with a remainder in the context of the problem.



Students find factors and multiples of numbers and identify prime and composite numbers. As students become fluent and flexible in understanding the relationships between numbers and their factors, they can apply this knowledge to solving multiplication and division problems.

As students develop and refine strategies for multiplying numbers with two or more digits, being able to visualize how multiplication works is critical in applying the distributive property to solve problems. Students use story contexts, pictures, and unmarked arrays to help them keep track of the parts of the problem as they break the numbers apart or change the numbers.



$17 \times 6 = (10 \times 6) + (7 \times 6)$

Students' strategies for solving multiplication problems include breaking the numbers apart, changing one factor and adjusting, and creating an equivalent problem.

Breaking the numbers apart by addition

$48 \times 42 =$

Solution 1

$40 \times 40 = 1{,}600$
$40 \times 2 = 80$
$8 \times 40 = 320$
$8 \times 2 = 16$
$2{,}016$

Solution 2

$48 \times 40 = 1{,}920$
$48 \times 2 = 96$
$2{,}016$

Changing one factor and adjusting

$48 \times 42 =$

$50 \times 42 = 2{,}100$
$2 \times 42 = 84$
$2{,}100 - 84 = 2{,}016$

Creating an equivalent problem

$$48 \times 42 = 96 \times 21$$

$$96 \times 20 = 1{,}920$$
$$96 \times 1 = 96$$
$$1{,}920 + 96 = 2{,}016$$

Students solve division problems with up to 4-digit dividends, including problems with quotients that have a remainder. To solve division problems, students use one of two basic strategies. They either use multiplication to build up groups of the divisor to the dividend, or they use division to break the dividend into parts.

Using groups of the divisor

$$460 \div 8 =$$

$$\begin{aligned} 50 \times 8 &= 400 \\ 460 - 400 &= 60 \\ 7 \times 8 &= 56 \\ 60 - 56 &= 4 \\ 50 + 7 &= 57 \end{aligned}$$

57R4

Breaking the dividend into parts

$$460 \div 8 =$$

$$400 \div 8 = 50$$
$$60 \div 8 \text{ is } 7\text{R}4$$
$$460 \div 8 \text{ is } 57\text{R}4$$

Whichever computation strategy students apply for any operation, using multiples of 10, 100, and 1,000 is key to efficiency.

The **Algebra Connections in This Unit** Teacher Notes in Units 1, 3, and 7 show how the distributive property of multiplication and the inverse relationship between multiplication and division are implicit in students' work as they solve problems. These pages also highlight two generalizations about multiplication that students work on in Grade 4: a factor of a number is also a factor of that number's multiples; and if one factor in a multiplication expression is halved and another factor is doubled, the product is unchanged.

MAIN MATH IDEAS

- Visualizing multiplication
- Reasoning about numbers and their multiples and factors
- Solving multiplicative comparison problems
- Solving multiplication problems with 2-digit numbers
- Understanding and using the relationship between multiplication and division to solve division problems
- Understanding the meaning and structure of multiplication and division
- Solving measurement problems
- Solving multiplication problems
- Solving division problems

BENCHMARKS

- Use multiplication to solve multiplicative comparison problems. (Unit 1)
- Determine whether numbers up to 100 are prime or composite. (Unit 1)
- Find factors of numbers up to 100 and recognize multiples of 1-digit numbers. (Unit 1)
- Multiply 2-digit numbers by 1-digit and small 2-digit numbers (e.g., 12, 15, 20), using strategies that involve breaking the numbers apart. (Unit 3)
- Solve division problems (2-digit and small 3-digit numbers divided by 1-digit numbers), including some that result in a remainder. (Unit 3)
- Multiply a number by a multiple of 10. (Unit 3)
- Multiply two 2-digit numbers and up to a 4-digit number by a 1-digit
- number. (Unit 7)
- Solve division problems with up to 4-digit dividends and 1-digit divisors. (Unit 7)
- Solve measurement and conversion problems. (Unit 7)

Grade 5

Students consolidate their computational strategies for multiplication, building upon what they learned in Grades 3 and 4. Representations and story contexts help students to connect these strategies to the meaning of multiplication. By the end of Grade 5, all students should be able to carry out strategies based on the distributive property of multiplication over addition: specifically, those strategies that involve breaking one or both factors apart, multiplying each part of one factor by each part of the other factor, and then combining the partial products. They also practice notating their solutions clearly. Students learn the steps of the U.S. standard algorithm for multiplication, discuss the meaning of its notation, and practice using it.

Some students may also use other strategies, such as changing one factor and adjusting, or creating an equivalent problem by multiplying one factor by some number and dividing the other factor by the same number. As they solve computation problems with larger numbers, students are expected to use mental arithmetic, estimation (to determine if a result is reasonable), and a sound understanding of the operation of multiplication.

Examples of multiplication strategies

Breaking numbers apart by addition

Solution 1

148	
× 42	
4,000	40 × 100
1,600	40 × 40
320	40 × 8
200	2 × 100
80	2 × 40
16	2 × 8
6,216	

Solution 2

1 3
1
148
× 42
296
5,920
6,216

Solution 3

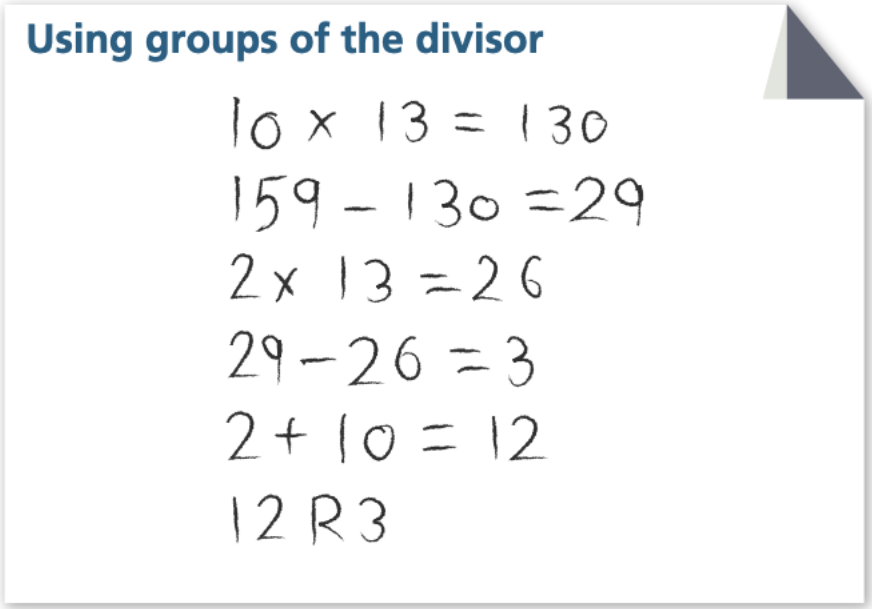
148 × 42 =
100 × 42 = 4,200
48 × 40 = 1,920
48 × 2 = 96
4,200 + 1,920 + 96 = 6,216

Changing one factor and adjusting

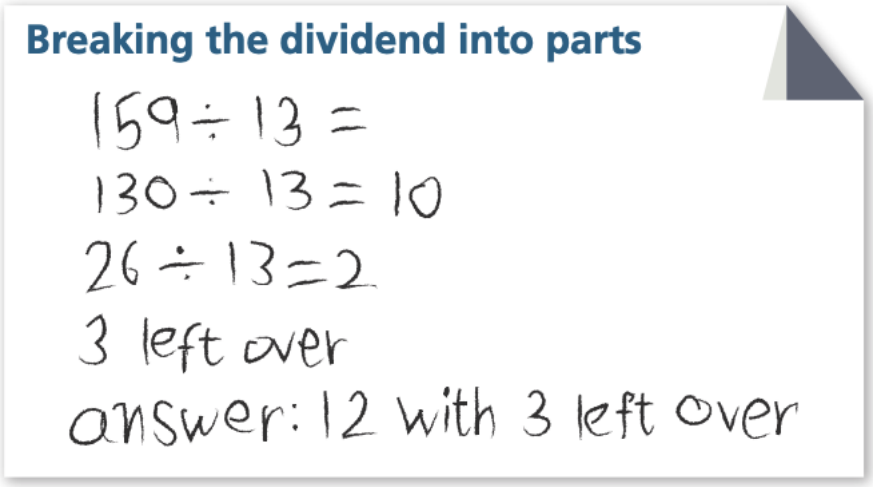
$148 \times 42 =$
$150 \times 42 = 6{,}300$ because
$6{,}300 = (100 \times 42) + (\frac{1}{2}$ of $100 \times 42)$
$2 \times 42 = 84$
$6{,}300 - 84 = 6{,}216$

Examples of division strategies for 159 ÷ 13

Using groups of the divisor



Students continue to develop ways to solve division problems fluently, focusing on the relationship between multiplication and division. They usually solve division problems either by building up groups of the divisor, or by breaking the dividend into convenient parts. As students refine their computation strategies for division, they find ways to use what they already know and understand well (familiar factor pairs, multiples of 10, relationships between numbers, etc.) to break apart the harder problems into easier problems. They also work on notating their solutions clearly and concisely.



Students are introduced to the order of operations and practice writing and interpreting expressions with grouping symbols (parentheses and brackets).

The **Algebra Connections in This Unit** Teacher Notes in Units 1 and 4 show how the distributive property and the relationship between multiplication and division are implicit in students' solution strategies. These pages also highlight what happens to the product when a factor is doubled, and how students use this idea when solving division problems.

MAIN MATH IDEAS

- Solving multiplication problems with 2-digit numbers
- Understanding and using the relationship between multiplication and
- division to solve division problems
- Writing and interpreting numerical expressions
- Solving multiplication problems fluently
- Solving division problems efficiently

BENCHMARKS

- Solve 2-digit by 2-digit multiplication problems efficiently. (Unit 1)
- Solve division problems with 1-digit and 2-digit divisors. (Unit 1)
- Use order of operations to solve computation problems. (Unit 1)
- Fluently solve multidigit multiplication problems using a variety of strategies including the U.S. standard algorithm. (Unit 4)
- Solve division problems with up to 4-digit dividends and 2-digit divisors efficiently. (Unit 4)