INVESTIGATIONS 3
TRANSFORMING MATHEMATICS TEACHING & LEARNING

Math Content by Strand

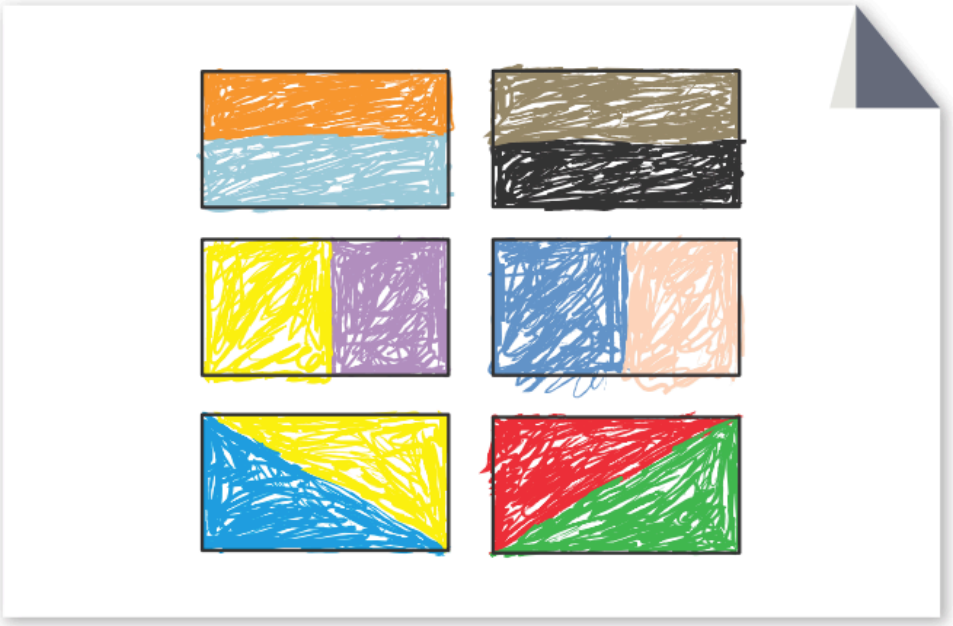
RATIONAL NUMBERS

Grade 1

As students learn to measure, they encounter lengths that are not a whole number of units long and times that are on the half hour. The fraction work connects to these experiences.

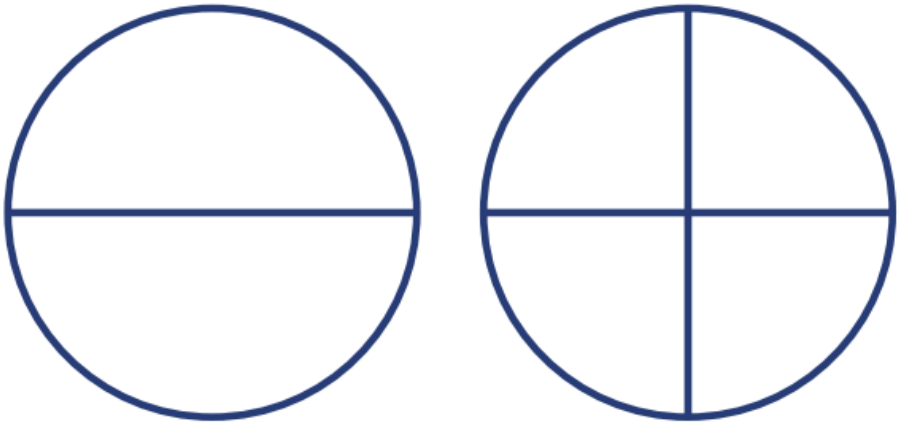
Using the context of designing "Fraction Rugs," students partition circles, rectangles, and squares into halves and fourths. The focus is on understanding that halves (or fourths) involve two (or four) equal parts. As they describe circular and rectangular rugs, they learn and use words like half, one half, halves, and half of, and the parallel language for fourths (e.g., one fourth, one quarter).

In their work with fractions, students partition familiar 2-D shapes into fractional parts, and in doing so, they decompose 2-D shapes into two or more smaller shapes. They attend to defining attributes of shapes as they notice the shapes that result when familiar 2-D shapes are divided into fractional pieces: half- and quarter-circles, triangles, squares, and rectangles.



[Students' half-and-half rugs]

Students come to see that the more parts a shape is partitioned into, the smaller the parts. In other words, one fourth of a shape is smaller than one half of the same shape, because fourths cut a shape into more, smaller pieces. Using the context of favorite kinds of pizza, they explain which piece they would rather have, one half or one fourth, and why.



[Would you rather have one half or one fourth of a pizza?]

MAIN MATH IDEA

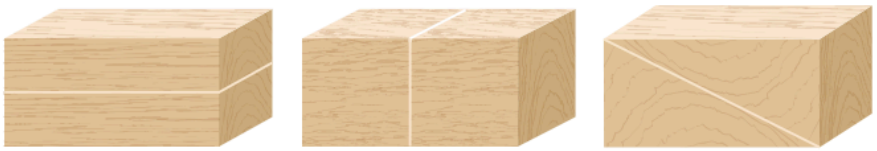
- Understanding halves and fourths

BENCHMARK

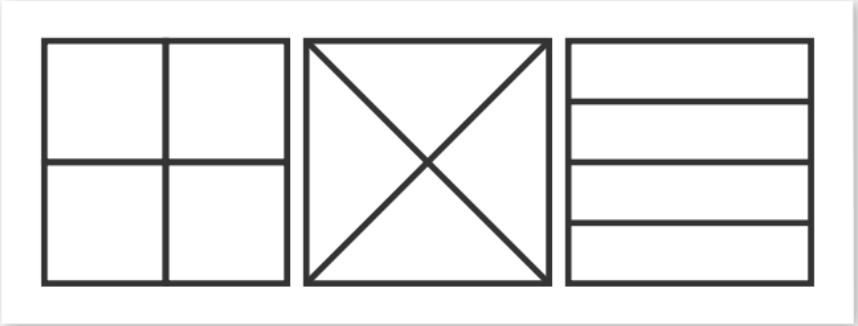
- Understand that halves or fourths (quarters) apply to wholes divided into two (four) equal parts; partition circles and rectangles into two and four equal parts. (Unit 4)

Grade 2

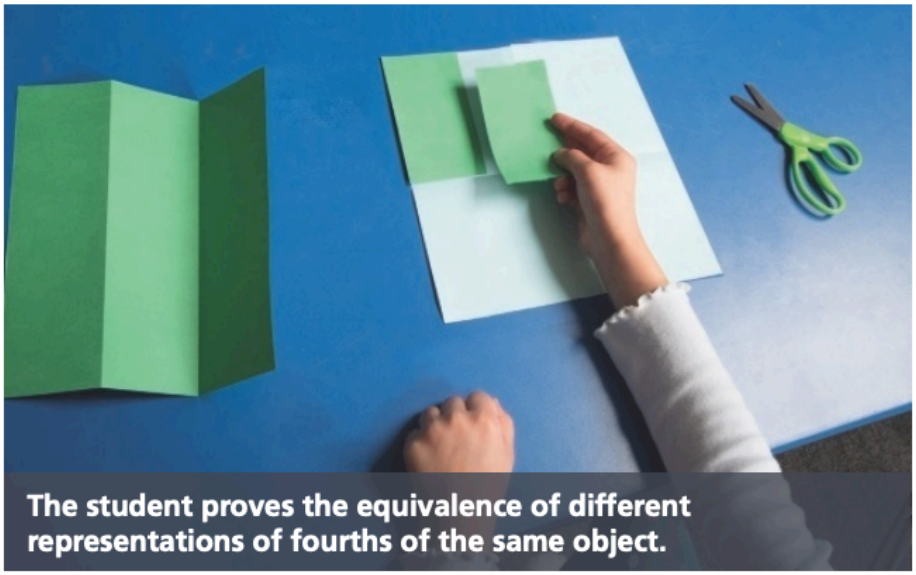
Work with fractions is directly related to geometry. As students fold paper squares, partition geometric shapes into equal parts on geoboards and dot paper, or compose and decompose GeoBlocks, they investigate halves, thirds, and fourths of squares, rectangles, circles, and prisms. They consider whether or not equal parts of the same whole (i.e., parts of equal area in 2-D shapes or equal volume in 3-D shapes) must look the same. They find ways to show that different-looking halves of a rectangular prism or fourths of a paper square are equal.



[Students investigate halves of a rectangular prism.]

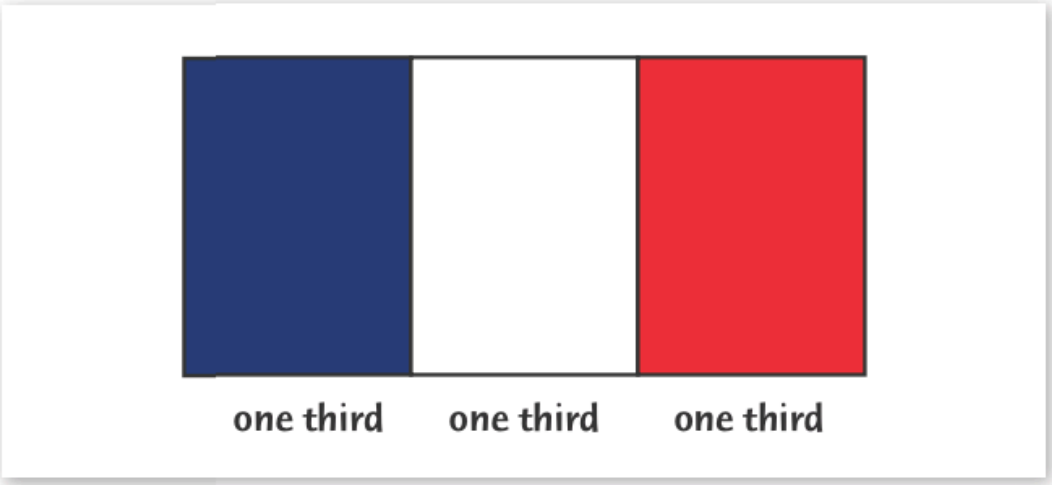


[Students investigate different-looking fourths.]



The student proves the equivalence of different representations of fourths of the same object.

Students use fractional language to identify regions of geometric shapes that are partitioned into equal parts. In addition to the terms one half, one third, and one fourth, they also hear and use language such as half of, halves, one third of, thirds, fourths, and quarters to describe these regions, as well as three thirds and four fourths to describe one whole. Like fraction words, fraction symbols and notation are used in many everyday situations and are therefore familiar to, but not necessarily understood by, many students. They see but are not expected to use such notation, though many choose to do so.



[Students color, discuss, and identify the three equal regions of a fraction flag.]

MAIN MATH IDEA

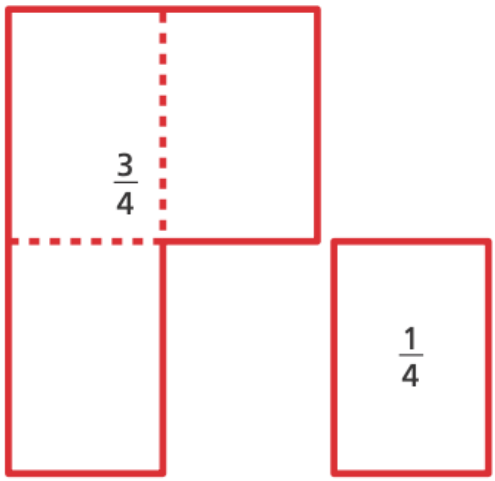
- Understanding equal parts of a whole

BENCHMARKS

- Recognize that [halves, thirds, fourths] of the same whole can look different. (Unit 2)
- Partition 2-D shapes into halves, thirds, and fourths and name the regions. (Unit 2)

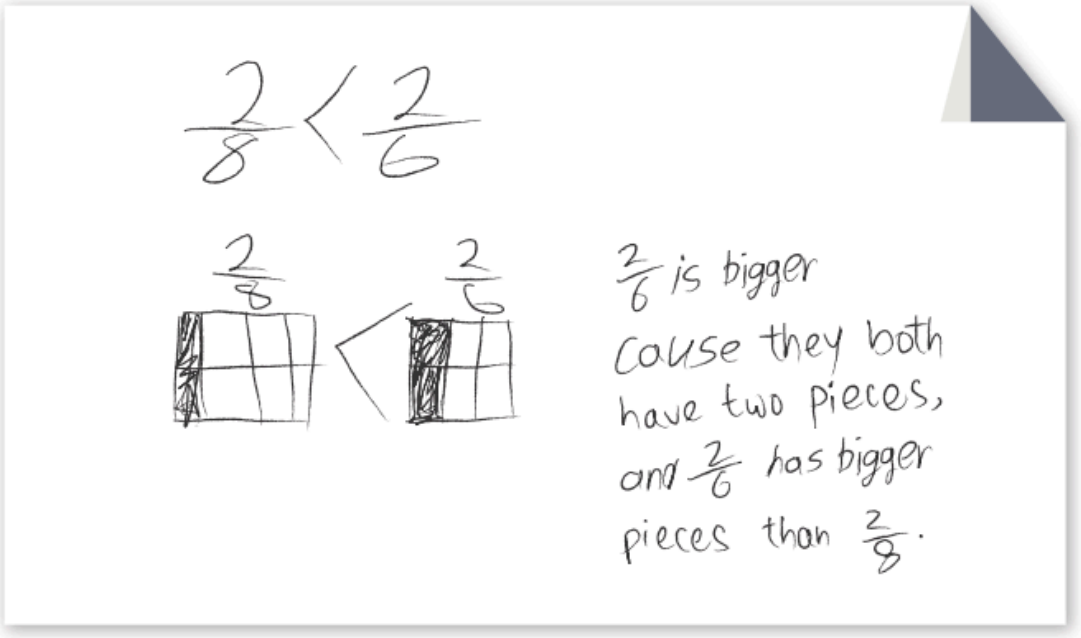
Grade 3

Students extend their understanding of the number system to include fractions. Students learn that when a whole is divided into equal parts, one of those parts is a unit fraction, and they come to recognize multiple equal parts as multiples of a unit fraction. For example, $\frac{1}{4}$ is a unit fraction—it is one part of a whole divided into fourths—and $\frac{3}{4}$ is three $\frac{1}{4}$s. The denominator in a fraction represents the number of equal parts a whole is divided into, and the numerator is the number of parts being used or identified.



Students work with fractions using an area model, representing halves, thirds, fourths, sixths, and eighths with a variety of contexts and representations, such as "brownies" (rectangles of a fixed size), fraction sets (made from sheets of $8\frac{1}{2}$ in. × 11in. paper), and pattern blocks. They also represent fractions on number lines, identifying how fractions relate to whole numbers and other fractions, thereby supporting their understanding of fractions as numbers. A story about ants walking city blocks provides a context that helps students think in terms of iterating unit fractions along a number line, naming fractional points between the whole numbers ($\frac{3}{8}$, $\frac{7}{4}$), and also naming whole numbers as fractions ($1 = \frac{2}{2} = \frac{3}{3}$, $2 = \frac{4}{2} = \frac{6}{3}$).

Students compare fractions and reason about their sizes, using both area models and number lines to explain how they know which is greater or if the fractions are equivalent. After comparing a number of fractions, students think through and explain how to determine which fraction is greater when two fractions have the same numerator or denominator. As students represent and compare fractions, they identify a number of common fraction equivalents.



MAIN MATH IDEAS

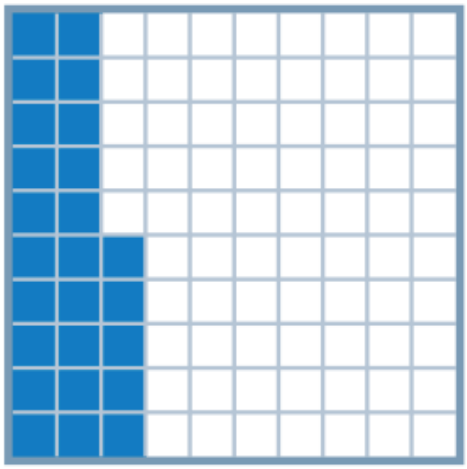
- Understanding the meaning of fractions as equal parts of a whole
- Understanding the meaning of fractions as numbers
- Comparing fractions and reasoning about fraction equivalencies with representations
- Modeling with fraction notation

BENCHMARKS

- Partition a quantity into equal parts, and name those parts as fractions. (Unit 6)
- Represent fractions as numbers on a number line. (Unit 6)
- Compare fractions with the same numerator or same denominator by reasoning about their size. (Unit 6)
- Identify equivalent fractions. (Unit 6)

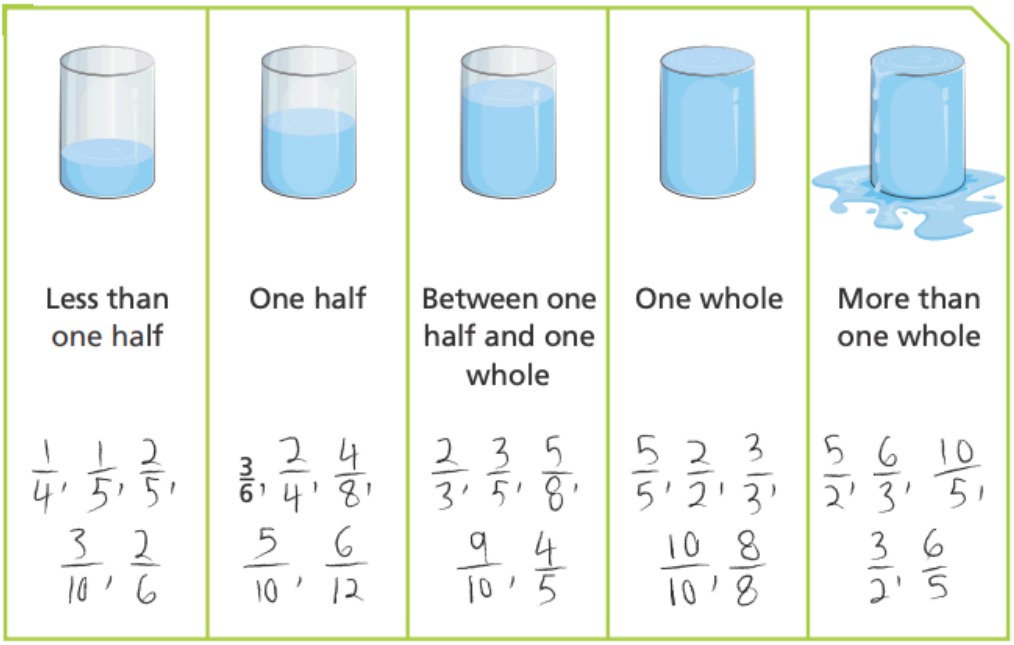
Grade 4

Students build their understanding of the meaning of rational numbers as they work with halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths. They are introduced to decimals and learn that tenths and hundredths can be written with both fraction and decimal notation. Using two different models—an area model (rectangles) and a linear model (number lines)—students visualize rational numbers and see how the numbers are related to each other. They see that in a given context the same fraction can represent different quantities ($\frac{1}{2}$ of 24 square units is 12 square units; $\frac{1}{2}$ of 60 square units is 30 square units), but that as a number, a fraction always has the same relationship to 1 and to other numbers. Students represent tenths and hundredths on rectangles divided into hundredths, name them as fractions and decimals, and recognize that quantities expressed as tenths can also be expressed as hundredths: 0.1 (or $\frac{1}{10}$) is equal to 0.10 (or $\frac{10}{100}$).



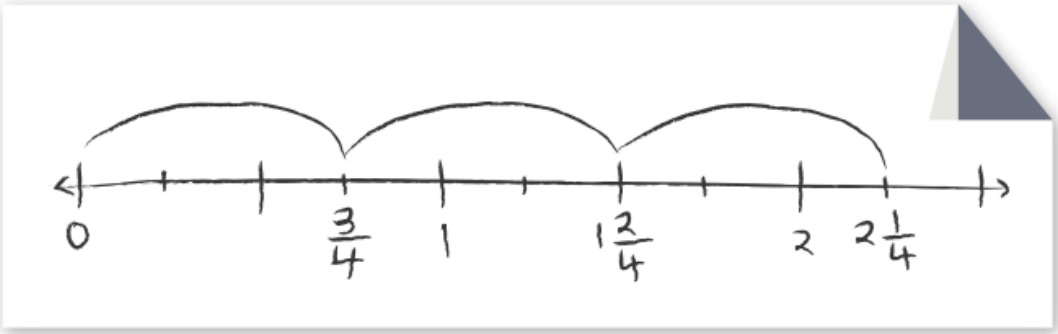
0.25 $\frac{25}{100}$

Students compare and order fractions using representations, their knowledge of fraction equivalents, and relationships to benchmarks such as $\frac{1}{2}$ and 1. Students compare decimals by representing them on rectangles divided into tenths and hundredths, and by placing them on a number line.



As students represent, compare, and order numbers, they find equivalent fractions or decimals, explain how they know the numbers are equivalent, and discuss how to generate equivalent fractions.

Students begin computing with rational numbers. They add and subtract fractions and mixed numbers (mostly with like denominators), add tenths and hundredths, and multiply whole numbers by fractions. In order to extend what they know about operations with whole numbers and to make sense of operations with rational numbers, students use contexts and representations such as rectangles (an area model) and number lines (a linear model), and they discuss what they understand about these operations and numbers.



$$3 \times \frac{3}{4} = 2\frac{1}{4}$$

MAIN MATH IDEAS

- Understanding the meaning of fractions and decimals
- Comparing the values of fractions and decimals
- Computing with rational numbers

BENCHMARKS

- Identify equivalent fractions and explain why they are equivalent. (Unit 6)
- Compare fractions with like and unlike denominators. (Unit 6)
- Add and subtract fractions and mixed numbers with like denominators. (Unit 6)
- Multiply a fraction by a whole number. (Unit 6)
- Read, write, and compare decimals in tenths and hundredths. (Unit 6)
- Add tenths and hundredths. (Unit 6)
- Represent data on a line plot including fourths and eighths. (Unit 6)

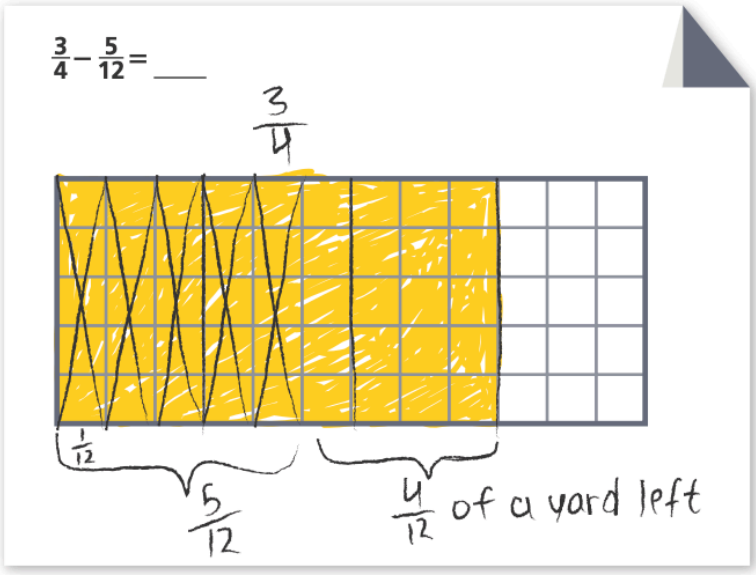
Grade 5

In Grade 5, there are three units in the Rational Numbers strand that focus on further developing and expanding students' understanding of the meanings of fractions and decimals, on the relationship between them, and on doing computation with all four operations. Students use various contexts and representations to compare fractions or decimals and to find equivalents. As students solve problems with fractions and decimals, they work on making sense of the magnitude of the numbers and of the operations of addition, subtraction, multiplication, and division. Their work with representations develops their number sense and supports their work with computation. This includes considering the reasonableness of their answers.

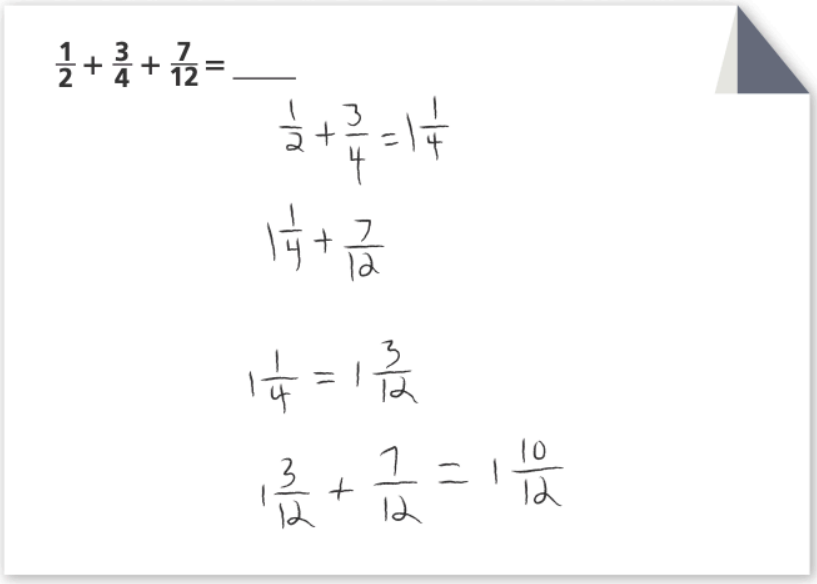
Fractions: Addition and Subtraction

Students compare fractions, identify equivalent fractions, and add and subtract fractions and mixed numbers. They compare fractions using fraction equivalents and the relationship of fractions to landmarks, such as $\frac{1}{2}$, 1, and 2, to decide which of two fractions is greater. Understanding equivalent fractions and how to find them is the basis for the approaches students develop for adding and subtracting fractions with unlike denominators.

Students use representations such as rectangles, rotations on a clock, and a number line, to visualize and reason about fraction equivalents and relationships. They use these same representations as the basis of students' initial work on adding and subtracting fractions.



Based on what they understand about adding and subtracting whole numbers, as well as what they see in their representations, students develop procedures for adding and subtracting fractions and mixed numbers without the use of representations. Their procedures involve finding equivalent fractions with common denominators and adding or subtracting the numerators.



MAIN MATH IDEAS

- Finding equivalents and comparing fractions
- Adding and subtracting fractions

BENCHMARKS

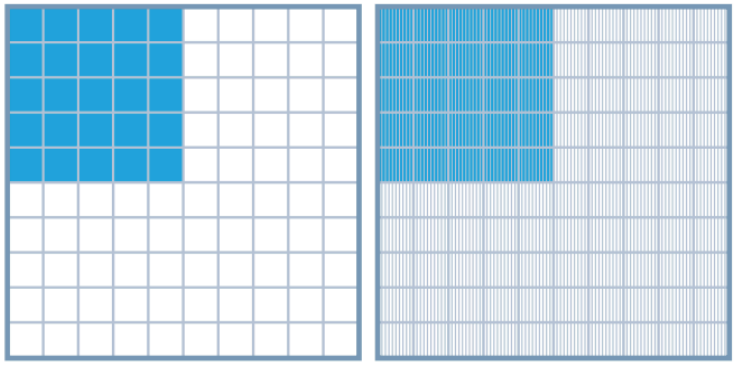
- Add fractions with unlike denominators. (Unit 3)
- Subtract fractions with unlike denominators. (Unit 3)
- Represent data including fractions on a line plot and solve addition and subtraction problems about the data. (Unit 3)

Decimals: Addition and Subtraction

Students represent, write, read, compare, and add and subtract decimals to thousandths.

Students represent tenths, hundredths, and thousandths on hundredths or thousandths grids (grids divided into 100 squares or 1,000 rectangles) and on number lines. These two forms of representation, and connecting them to fraction equivalents, allow students to visualize the relationships among decimals and to see how these numbers extend the structure of our base-10 number system to include places with values less than 1. Students also use hundredths grids to help them compare decimals and to see equivalencies among tenths, hundredths, and thousandths. Doing this work with representations is an important foundation as students begin to add and subtract decimals.

As students represent decimals on the grids, they also write them with numerals. Students learn how to interpret the zeroes in decimals and learn that the way decimal numbers are said and written is related to the meaning of the numbers.

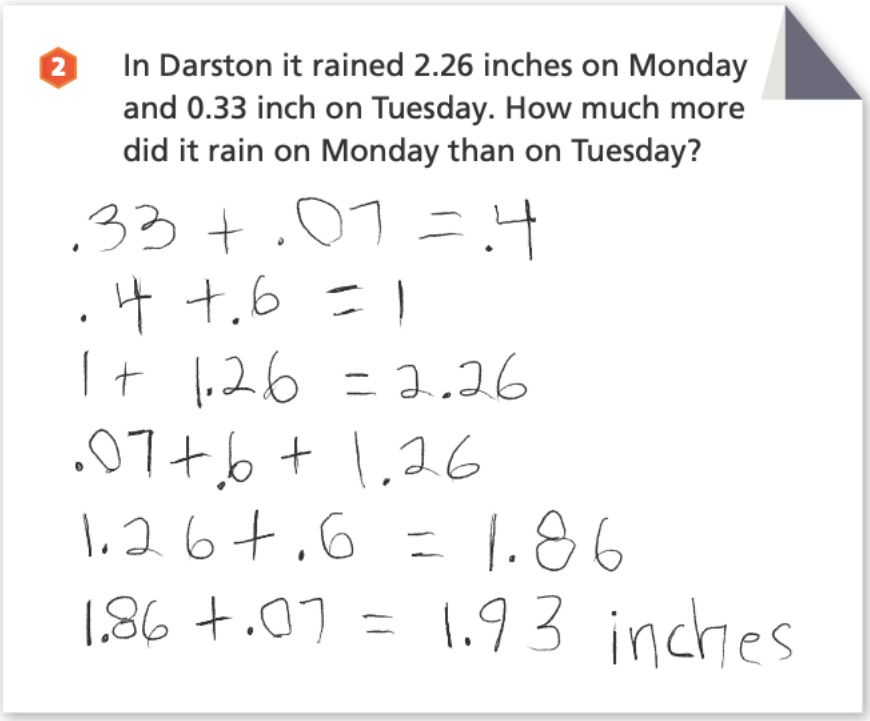


Fractions: $\frac{25}{100}$ or $\frac{1}{4}$
Decimal: 0.25

Fractions: $\frac{250}{1000}$ or $\frac{1}{4}$
Decimals: 0.250 or 0.25

To add and subtract decimals, students apply their understanding of the operations of addition and subtraction with whole numbers, their understanding of place value, and their understanding of decimals. Students begin by representing the numbers on the hundredths grids in order to visualize the operation with decimals and to help them carefully identify the values of the digits in each number.

As students internalize the images, they move to adding and subtracting decimals without using representations, applying strategies similar to those they use with whole numbers, such as keeping one number whole and adding or subtracting the other number in parts.



2 In Darston it rained 2.26 inches on Monday and 0.33 inch on Tuesday. How much more did it rain on Monday than on Tuesday?

.33 + .07 = .4
.4 + .6 = 1
1 + 1.26 = 2.26
.07 + .6 + 1.26
1.26 + .6 = 1.86
1.86 + .07 = 1.93 inches

MAIN MATH IDEAS

- Understanding the meaning of decimals
- Comparing decimals
- Adding and subtracting decimals

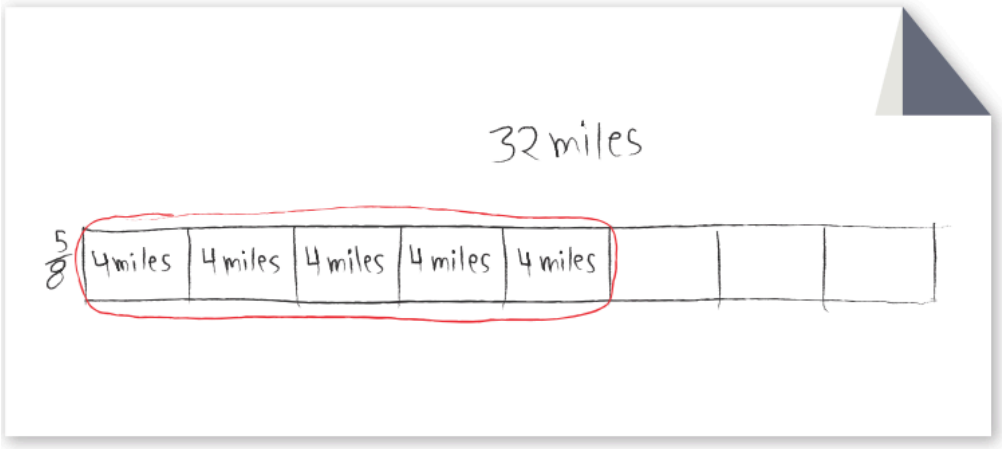
BENCHMARKS

- Write, compare, and round decimals to thousandths. (Unit 6)
- Add and subtract decimals. (Unit 6)

Fractions: Multiplication and Division

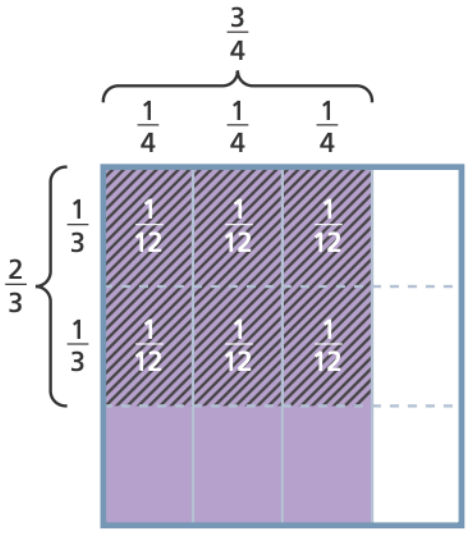
Students build on their understanding of the operations of multiplication and division and their understanding of fractions to multiply and divide with fractions. The emphasis is on using contexts and representations to solve multiplication and division problems that involve fractions and mixed numbers, and on helping students expand their understanding of multiplication and division to include fractions.

On Tuesday, Margaret biked $\frac{5}{8}$ of a 32-mile bike path. How many miles did she bike?



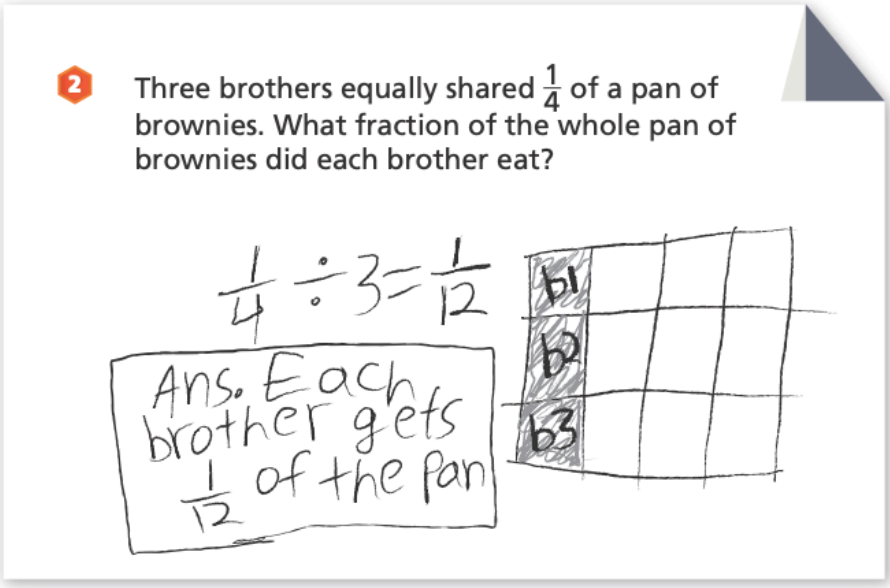
$$\frac{5}{8} \times 32 = 20$$

Students solve problems that involve multiplying a whole number by a fraction or a mixed number, as well as problems that involve multiplying two fractions. They use representations (especially fraction bars and arrays) and equations to relate problems with fractions or mixed numbers to whole-number problems situated in the same context. Study of the relationship between the size of the factors and the size of the product—when a problem includes fractions less than 1, fractions greater than 1, or mixed numbers—helps students evaluate the reasonableness of their answers.



$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

Division problems are limited to dividing a whole number by a unit fraction and dividing a unit fraction by a whole number. To solve problems that involve dividing a whole number by a unit fraction, students draw on their understanding of division as how many of a number fit into another number. Solving problems that involve dividing a unit fraction by a whole number fits more readily into students' ideas of breaking the fraction into equal parts. Students represent these problems with equations and solve them by drawing representations.



Students are also introduced to the idea that fraction notation ($\frac{6}{5}$) can also be interpreted as division notation (6 ÷ 5).

MAIN MATH IDEAS

- Multiplying and dividing fractions, mixed numbers, and whole numbers
- Interpreting fractions as division
- Converting measurements

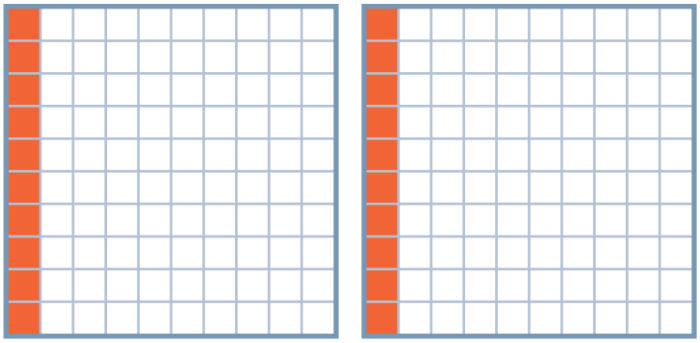
BENCHMARKS

- Multiply fractions, mixed numbers, and whole numbers. (Unit 7)
- Compare the size of the factors and the size of the product and explain their relationship. (Unit 7)
- Divide a unit fraction by a whole number and a whole number by a unit fraction. (Unit 7)
- Solve division problems with two whole numbers resulting in a fraction or a mixed number. (Unit 7)
- Solve measurement conversion problems including multistep word problems. (Unit 7

Decimals: Multiplication and Division

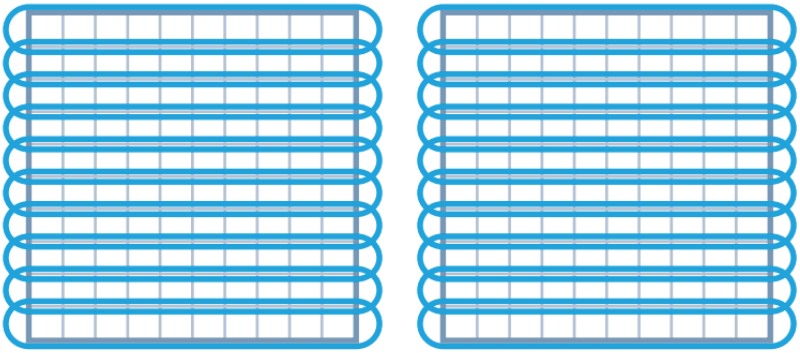
Students extend their understandings of place value and whole-number multiplication and division to solve multiplication and division problems with decimals. Similar to their work with fractions, as students multiply and divide with decimals, they realize that multiplication does not always make things "bigger," and division does not always make things "smaller."

Students use hundredths grids and number lines to make sense of what is happening when they multiply or divide with numbers less than 1. Similar to whole number multiplication, when students represent multiplying 2 × 0.1 or 2 × 0.01, they make 2 copies of one-tenth or of one-hundredth.



2 × 0.1 = 0.2

When dividing 2 , 0.1 or 2 , 0.01, students find how many one-tenths or one-hundredths there are in 2.



2 ÷ 0.1 = 20
20 × 0.1 = 2

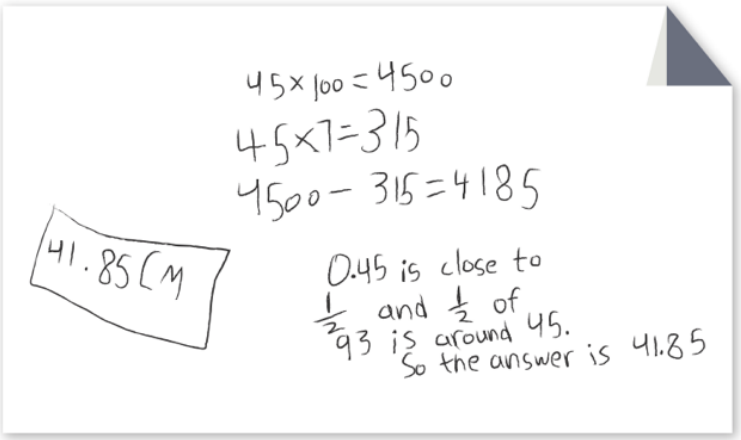
Students solve a series of problems where they multiply and divide whole numbers by powers of 10 (0.01, 0.1, 1, 10, 100) to examine and discuss the patterns they see in the location of the decimal point, how the magnitude of the answer changes, and how the place value of digits in the problems changes.

As students work with these representations and patterns, they more easily recognize place-value relationships—that a digit in one place represents 10 times as much as it represents in the place to its right, and $\frac{1}{10}$ of what it represents in the place to its left.

Students solve multiplication and division problems involving decimals mostly by operating on the numbers as if they were whole numbers, and then reasoning about the magnitude of the answer to determine the correct product or quotient. By using this method to multiply and divide with decimals, students apply their understanding of place value, the magnitude of numbers, and the operations of multiplication and division to solve problems.

See the **Measurement and Data** section for more information on how students use knowledge of multiplication and division of fractions and decimals to solve measurement conversion problems.

The Flickerbill makes jumps of 0.45 centimeter.
How far does the Flickerbill go if it makes 93 jumps?



MAIN MATH IDEAS

- Multiplying with decimals
- Dividing with decimals
- Converting measurements

BENCHMARKS

- Recognize and use place-value relationships to explain patterns when multiplying or dividing by powers of 10 including placement of the decimal point. (Unit 7)
- Multiply and divide decimals to hundredths. (Unit 7)
- Solve measurement conversion problems including multi-step word problems. (Unit 7)